Classical Waves – TRIUMF Saturday Morning Lecture – October 25th, 2008

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An Introduction … Who am I?

- I am taking a Ph.D. in nuclear astrophysics at UBC and TRIUMF, my fourth UBC degree in 30 years.
- I am a former Naval Officer on the inactive service list: My first job in the mid-1980's - learning how to search for and sink Soviet nuclear powered, nuclear armed submarines.
- I have twenty years experience in the high technology sector and have worked on 300 projects worth around $ 250 million
- I am an experienced and qualified teacher and have taught high school physics and as an instructor at the Emily Carr School of Art and Design, and here at UBC as a Teaching Asst.

What we will look at ...

- Mathematics as the language of nature.
- Those who made waves through history
- Waves are everywhere in nature and the need to learn Calculus.

Important advice from my high school physics teacher

1) Mathematics is the language best able to describe nature.

2) Every physical object or process can be described and understood through measurement and numbers

3) If you study the measurements that describes an object or process, patterns emerge ... patterns are everywhere in nature.

4) Physics is the quest to understand these physical objects and processes through these patterns

How new is this advice?

- This advice sounds new ....
- But in fact it goes back a very long time .....
- How far back does this advice go?

Galileo Galilei (1564-1642)

- "… the universe stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written.

- It is written in the language of mathematics … without which it is humanly impossible to understand a single word of it; without this, one is wandering about in a dark labyrinth."

Galileo Galilei

Who inspired Galileo? ... the two Leonardos did

- There was Leonardo da Vinci (1452-1519) one hundred years before Galileo.
- Leonardo da Vinci was the pre-eminent Renaissance artist and scientist ... He observed nature closely and tried to understand its form and function...
- Leonardo da Vinci was the first "Applied Scientist."

From Leonardo’s Notebook 1508

- “knowledge is vain and full of errors, which are not born of esperienza, mother of all certitudes and which do not terminate in cognition (nota esperienza), that is whose origin or middle or end does not come through one of the five senses ...”

True Science ...

- “True science ... is indicated in the elementary mathematical sciences, that is number and measure, called arithmetic and geometry, which treat with absolute truth discontinuous and continuous quantity....”

The infinite works of nature …

- “The eye, which is called the window of the soul, is the principal means by which the central sense can most completely and abundantly appreciate the infinite works of nature.”
- But who was the other Leonardo that influenced Galileo Galilei?

Leonardo da Pisa (1170-1240)

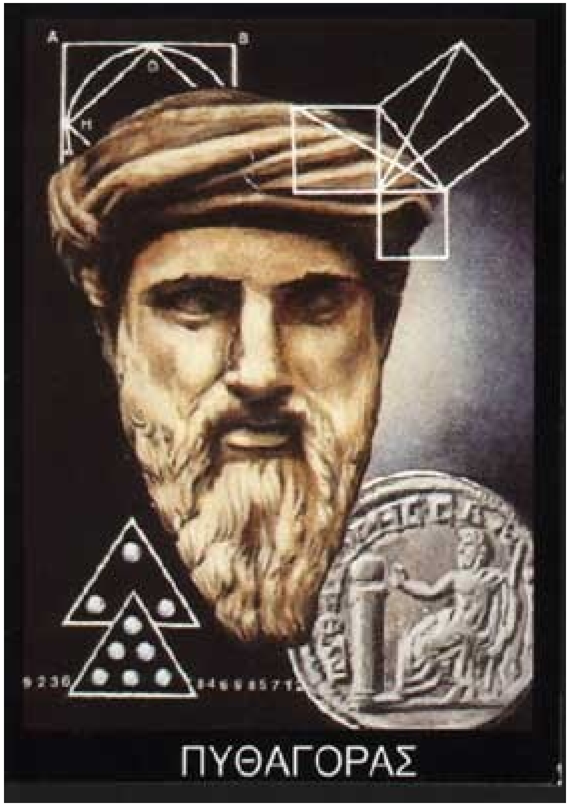
- He was Leonardo da Pisa also known as Fibonacci – the son of Bonacci.
- He initiated the tradition of the *Maestri d'Abaco*, which is an expert in arithmetic.
- Mathematics flourished in Pisa beginning in the 13th century. Fibonnacci's successors in Pisa included great mathematicians like Cardano and Galileo.

Leonardo of Pisa, who was called Fibonacci
Courtesy of Columbia University Libraries

- but who ultimately inspired Leonado da Pisa?

Pythagoras (582-500 BC)

- Pythagoras of Samos is best known for the geometric theorem bearing his name.
- Pythagoras believed that every physical object and process can be represented and understood through numbers.
- We can ultimately trace the beginnings of mathematics and physics to Pythagoras.



Pythagoras … Prime mover …

- "If one man more than another is to be credited with starting the mathematical and physical sciences on their course from antiquity to the present it is Pythagoras. And if 'western civilization' means the technology and commerce of recurrent industrial revolutions *resulting from* the application of experiment and mathematics to the physical world, Pythagoras was its prime mover."

The Magic of Numbers,

Eric Temple Bell, Historian of Mathematics

Pythagoras made waves …

- Pythagoras' notion that everything was numbers *made waves* amongst Greek philosophers … They didn't understand the idea and so they tried to suppress it.

- To explain his idea, Pythagoras demonstrated the relationship between musical harmonies and numbers. He studied waves!

Waves are everywhere!

- Mathematicians and physicists have been studying waves for 2,500 years!
- When you look closely at natural objects and physical processes you find waves are everywhere!
- Today ... we still study many types of waves: sound waves, seismic waves, light waves, waves in fluids, and waves in gravity, quantum and particle physics.
- Our advice to you: to be a good mathematician and physicist you have to understand waves.

Galileo made waves …

- Galileo's father Vincenzo, was a musician. They both played the lute.
- When he was growing up, Galileo wanted to be a great artist like da Vinci.
- Vincenzo introduced his son to the ancient theory of musical harmony and mathematics!
- They found the ancient theory of numerical harmony was in error!
- Galileo did his own experiments to support their theory and better understand waves!
- Galileo was also a superb instrument maker, be it a lute or a telescope! Like Galileo, you should not be afraid to build things and do experiments!

Galileo observed vibrations in nature…

- Galileo himself wrote later in life:
- “Thousands of times I have observed vibrations, especially in churches where lamps, suspended by long cords, had been inadvertently set into motion.”

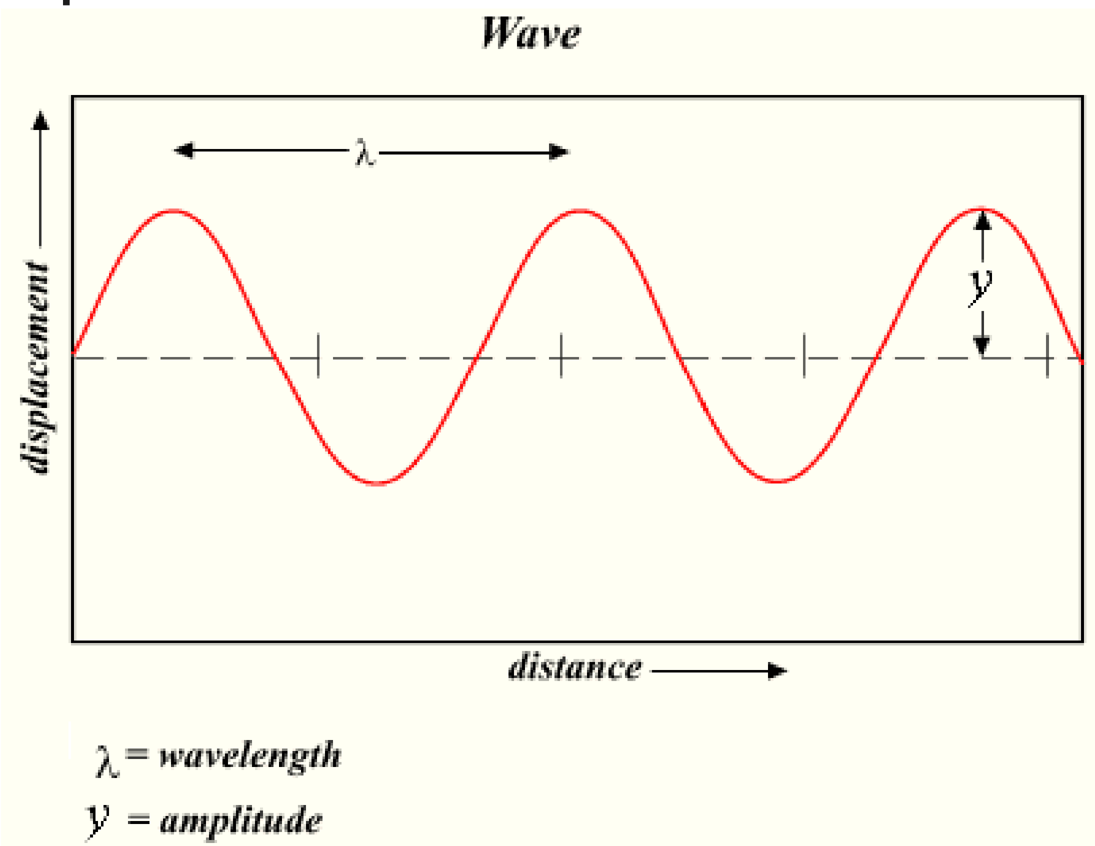
What is a wave?

- What do we learn about waves through observation?

- We know there are periodic waves ... And aperiodic waves.

- An aperiodic wave has a structure that changes constantly...

If a wave is periodic ...



- A – amplitude

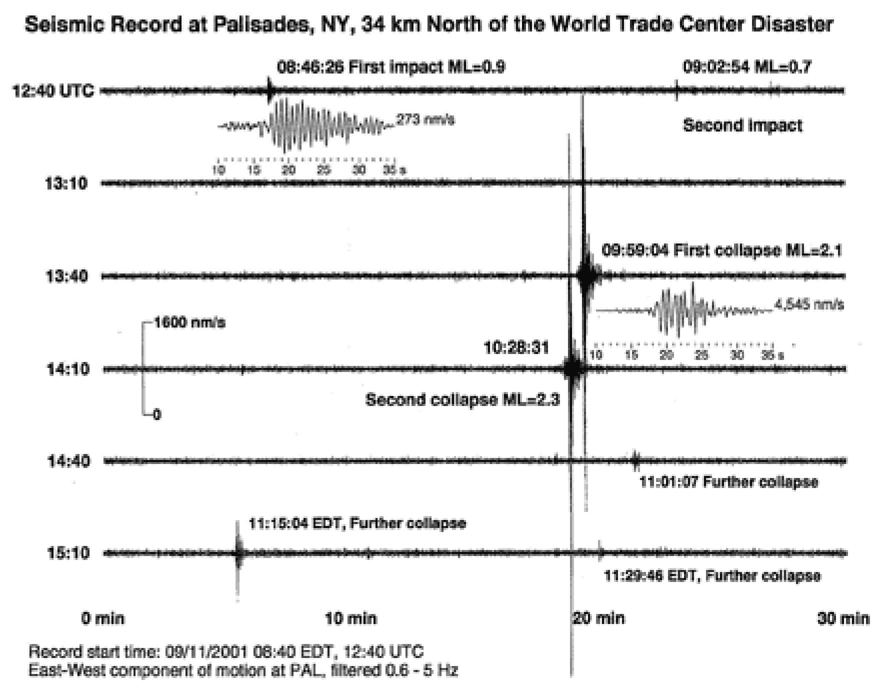
- λ- wavelength
- f - frequency
- v – speed

$\lambda f = v$

this graph shows
a Cosine function.

Most waves are aperiodic …

- Not all waves repeat with a constant wavelength or frequency.
- Aperiodic waves don't repeat in a regular way. An example is a seismic wave … here is a seismic record from Sept. 11th, 2001.



- Seismos means earthquake in Greek..

Stationary and Traveling Waves …

- Stationary waves, waves that do not travel from one point to another, tend to be periodic. Dr. Waltham will talk about standing waves in music.

- Waves that travel from one point to another are called traveling waves.

Think about traveling Waves !

- What two important measurable quantities do traveling waves carry from one point to another?
- What is the physical attribute we would most naturally want to measure for a traveling wave?

- {demo: traveling wave sent down a string with a small bell at the other end}

Waves carry momentum and energy and have a velocity

- The wave travels along the string carrying the energy I put in.
- The movement of the bell also tells me that the wave carries momentum.
- We would naturally want to measure the speed of a traveling wave.

Important characteristics of a string ...

- Let us just recognize that energy and momentum is carried by all waves ...
- Our measurement shows that the disturbance traveled at over a 100 km/hr down the string!
- Let us look at how the velocity of a traveling wave is related to the characteristics of a string.

Tension and linear density

- As we change the tension on the string the velocity obviously changes.
- Experiment shows that the velocity of a traveling wave on a string is related to
- The Tension of the string (T) and
- μ the linear density of the string.

The velocity expression ...

- The velocity of a traveling wave on a string is given by

$$v = \sqrt{[T / \mu]}$$

If we have time at the end I will derive this (or look in any good textbook).

The velocity term ...

- Since a traveling wave has a velocity, somewhere in the mathematical description of the wave will appear

$$(x - v\,t)$$

- why? .... because

$$dx = v\,dt$$

Waves Forms ...

- Of possible wave forms the cosine wave form

$$f(x,t) = A \cos[\,(2\pi/\lambda)(x - vt) + \delta]$$

- is the most familiar, where A is the amplitude,
- λ is the wavelength,
- v is the velocity and
- δ is a phase constant.

Euler Wave formulation

- Another less familiar way of describing a wave form is by Euler's wave formulation

$$\exp[i\,\Theta] = \cos\Theta + i\sin\Theta$$

- where i is the imaginary number $\sqrt{(-1)}$, and Θ is the angle, and where

$$\cos\Theta = \text{Real part of } \{\exp[i\,\Theta]\} = \mathrm{Re}\{\exp[i\,\Theta]\}$$

“Euler's Treat”

- This means

$$f(x,t) = A \cos[\,(2\pi/\lambda)(x - vt) + \delta]$$
$$= A^* \,\mathrm{Re}\{\, \exp(i\,(2\pi/\lambda)(x - vt) + \delta)\,\}$$

- See “Euler's Treat” by Andrew Jarvis, (April 2008 edition of the Canadian Undergraduate Physics Journal, www.cupj.ca)

the simplest example of a Classical Wave

- For the waves on a string, the Euler wave formulation is easiest to use.

- For the Incident wave we have

$$f(x,t)_{Inc} = A_{Inc} * Re\{exp(i\ (2\ \pi/\lambda_{Inc})(x - v_{Inc}\ t) + \delta_{Inc})\}$$

- For the reflected and transmitted waves you have similar wave functions, $f(x,t)_{Ref}$ and $f(x,t)_{Trans}$.

Where two strings meet ...

- Since the wave is continuous, at the junction point where two strings meet,
- ... the wave function is connected (continuous) and
- ... the slope is the same on both sides of the junction.

Related amplitudes and phases ...

- You can use this fact to solve for the way the incident, reflected and transmitted amplitudes and phases are related:

$$A_{Ref} \exp(i\,\delta_{Ref}) = [(v_{Trans} - v_{Inc}) / (v_{Trans} + v_{Inc})] * A_{Inc} \exp(i\,\delta_{Inc})$$

$$A_{Trans} \exp(i\,\delta_{Trans}) = [2\, v_{Trans} / (v_{Trans} + v_{Inc})] * A_{Inc} \exp(i\,\delta_{Inc})$$

Second string is Lighter …

- If the second string is lighter, all three waves Incident, reflected and transmitted have the same phase, and so

$$A_{Ref} = [(v_{Trans} - v_{Inc}) / (v_{Trans} + v_{Inc})] * A_{Inc}$$

$$A_{Trans} = [2 v_{Trans} / (v_{Trans} + v_{Inc})] * A_{Inc}$$

Second String is Heavier …

- If the second string is heavier, the reflected wave is out of phase. In this case the reflected wave is “inverted” compared to the incident wave.

$$A_{Ref} = -1 * [(v_{Trans} - v_{Inc}) / (v_{Trans} + v_{Inc})] * A_{Inc}$$

$$A_{Trans} = [2\, v_{Trans} / (v_{Trans} + v_{Inc})] * A_{Inc}$$

What if the string is tied down?

- This means that $v_{Trans} = 0$ so that

$$A_{Ref} = A_{Inc}$$

$$A_{Trans} = 0$$

- which means that all the incident wave is reflected back. This is what we observe!

To really understand waves ...

- The really understand waves you need to understand Calculus ...
- While most of you will take Calculus in High school ...
- Try to begin to learn Calculus as early as you can! (I did this in grade 9 ...)

The Wave Equation …

- There is a second order differential equation that describes all Classical Waves …

- Here’s a simple derivation using infinitesimals …

A simple derivation ...

- We know

$$dx = v\ dt$$

where dx is a small measure of distance and dt is a small measure of time, v is velocity.

For some arbitrary small change [d] to a function f

$$[d\ ]\ dx = v\ [d\ ]\ dt$$

Simple Operator Algebra

- In terms of the infinitesimals, this means

$$[d\ ]/\ dt = v\ [d\ ]\ /dx$$

This operator applied twice on a function f gives

$$d^2 f / d^2t = v^2 d^2 f / d^2x$$

which is the Classical Wave equation.

Solution to the wave equation …

- A function like Euler's equation

$$f(x,t) = A^*\{ \exp(i\,(2\,\pi/\lambda)\,(x - vt) + \delta) \}$$

is obviously a solution to the wave equation, as is the Cosine function.

Some parting thoughts ...

- As you go about your daily lives ...
- be like Pythagoras, Leonardo da Pisa, Leonardo da Vinci and Galileo.
- Take note of the waves around you. Do experiments!
- Try to better understand their form and function, and
- Try to learn the mathematics needed to more fully understand what it is you are observing ...

Thank you

- The website for the Canadian Undergraduate Physics Journal is www.cupj.ca. (I am the Editor in Chief of the CUPJ)

- You can also contact me at

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