Physics of and in Ion Traps

TRIUMF, Vancouver

June 2012

Proposed Topics:

- **Basics of Paul- and Penning-traps**
  (equ. of motion, trap geometries,
  influence of trap imperfections,)
- **Ion detection and cooling**
  (Buffer gas cooling, resistive cooling,
  Laser Doppler- and sideband-cooling,
  sympathetic cooling, ion crystallization)
- **Zeeman spectroscopy (g factor determinations)**
- **Hyperfine spectroscopy**
- **Atomic clocks**
- **Mass spectrometry in Paul- and Penning-traps**
- **Quantum computing with trapped ions**

Why particle trapping?

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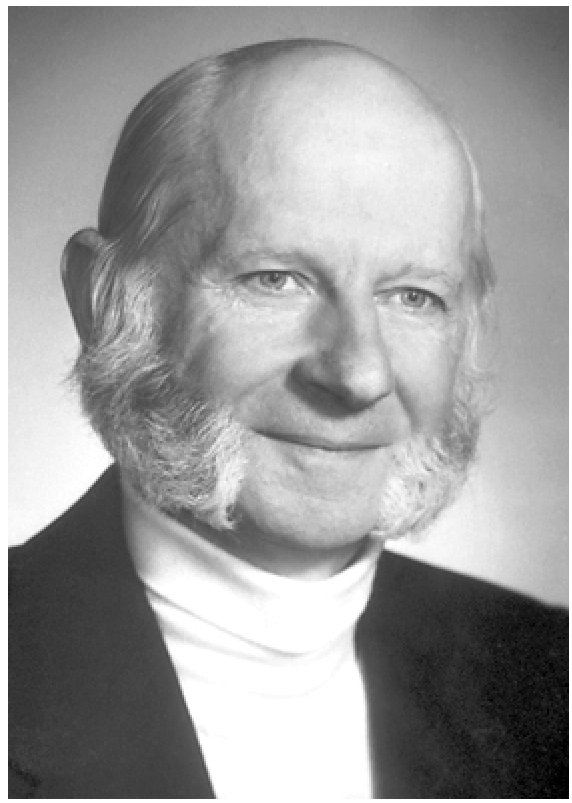
Approximation:

A single particle at very low momentum confined for long times by well known forces in a small volume in space would be a desirable object

Key words for trap spectroscopy

- Sensitivity
- Precision
- Control

Pioneers of ion trapping



Wolfgang Paul Hans Dehmelt

Nobelprize 1989

Basics of Ion Traps

Trapping of charged particles by electromagnetic fields

***Required*: 3-dimensional force towards center**

$$F = -\, e \text{ grad } U$$

***Convenience:* harmonic force** $F \propto x,y,z$

→ $U = ax^2 + by^2 + cz^2$

Laplace equ.: $\Delta(eU) = 0$

→ **a,b,c can not be all positive**

***Convenience:* rotational symmetry**

→ $$\boxed{U = (U_0/r_0^2)(x^2+y^2-2z^2)}$$

Quadrupole potential

Equipotentials: Hyperboloids of revolution

Problem:

No 3-dimensional potential minimum because of different sign of the coefficients in the quadrupole potential

Solutions:

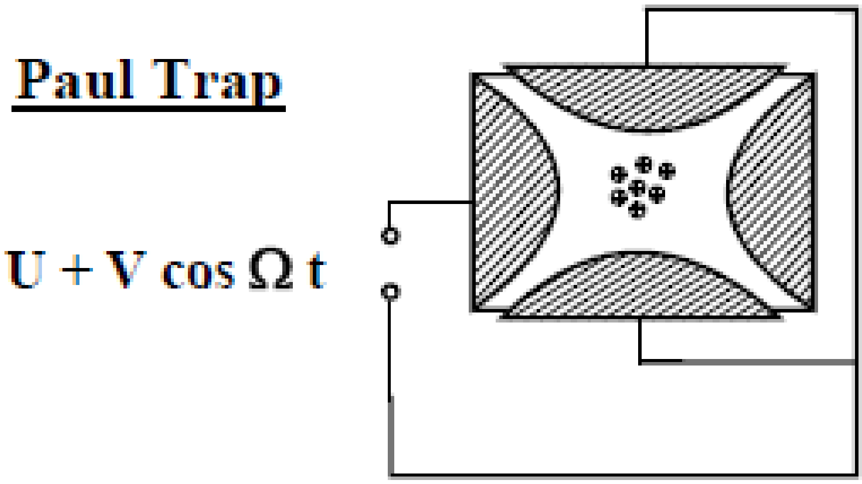
- Application of r.f. voltage: dynamical trapping

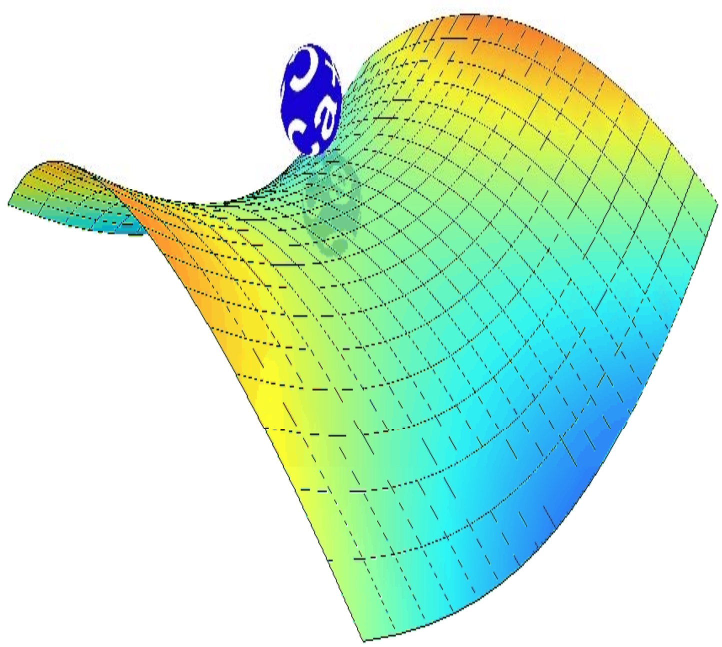
  Paul trap

- d.c. voltage + magnetic field in z-direction:

  Penning trap

The ideal Paul trap





Time-averaged potential minimum

Equation of motion for a single particle

Potential: $U=(U_0+V_0\cos\Omega t)(r^2-2z^2)/r_0^2$

Using:

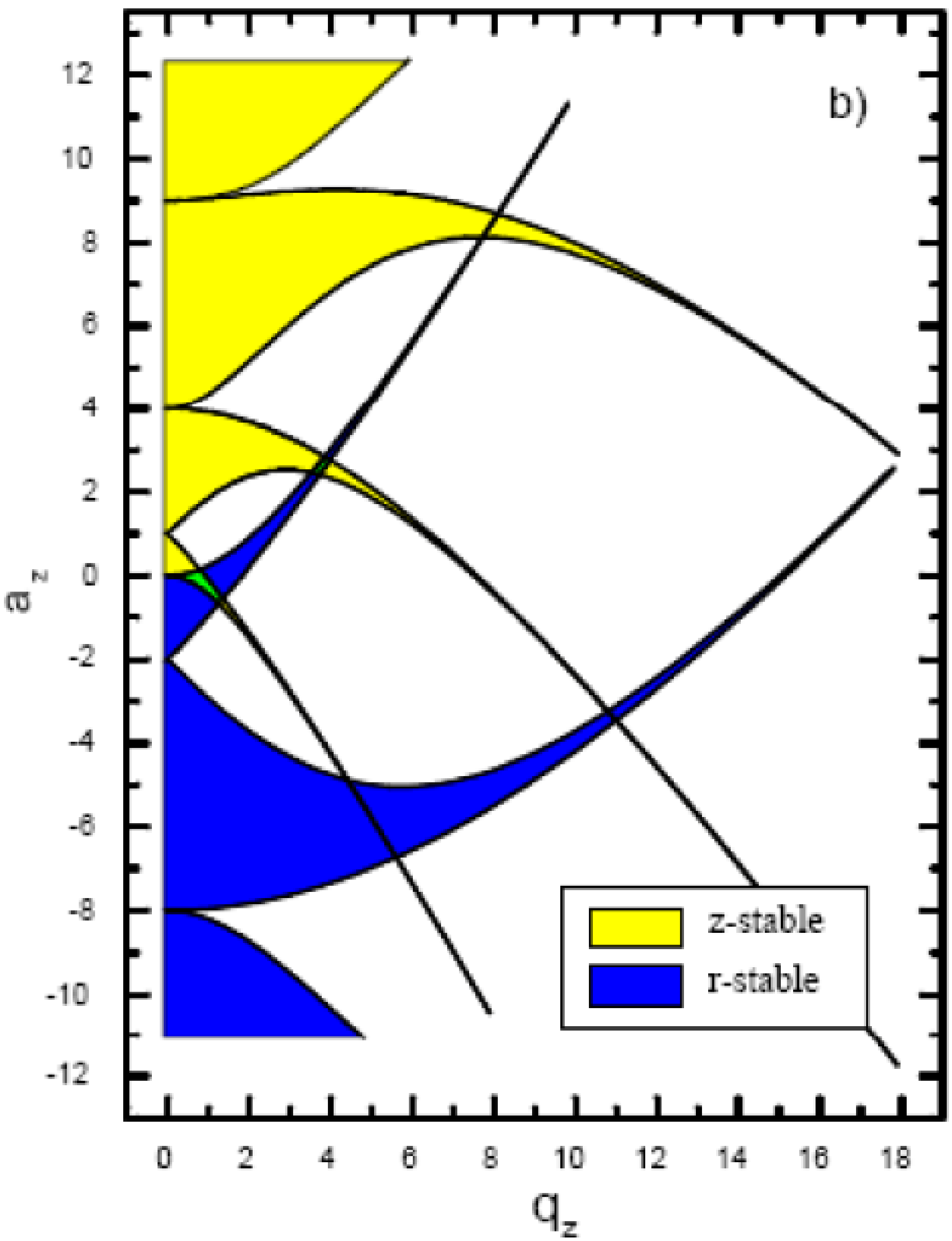
$$a_z = \frac{8eU_o}{mr_0^2\Omega^2} = -2a_r$$
$$q_z = \frac{4eV_0}{mr_0^2\Omega^2} = 2q_r$$
$$u = r, z$$
$$\tau = \Omega t/2$$

We obtain the normalized Mathieu differential equation

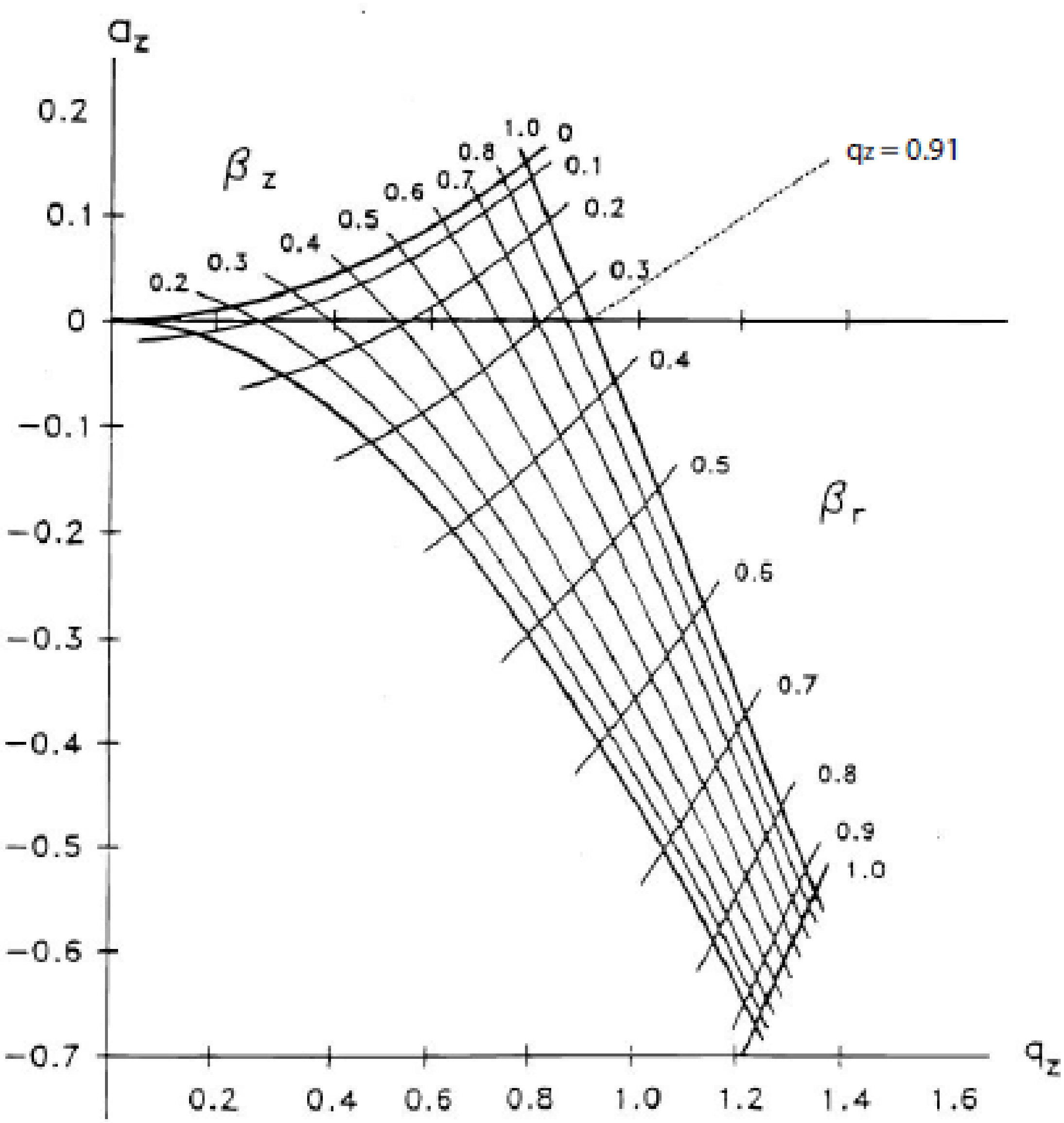
$$\frac{d^2u}{dt^2} + (a - 2q\cos 2\tau)u = 0$$

The solutions are well known and depend on the size of the parameters *a* and *q*:
When *u* remains finite in time:stable solutions
When *u* goes to infinity: unstable solutions

Stable solutions of the Mathieu equation



First stability area



For a=0: $$\left|\frac{Q}{M}\right| < \frac{0{,}908}{2} \cdot \frac{r_0^2 \Omega^2}{U_{ac}}$$

solution of the equation of motion:

$$u(t) = A\sum_{n=0}^{\infty} c_{2n} \cos(\beta + 2n)(\Omega t / 2)$$

$$\beta = \beta(a, q)$$

$$c_{2n} = f(a, q)$$

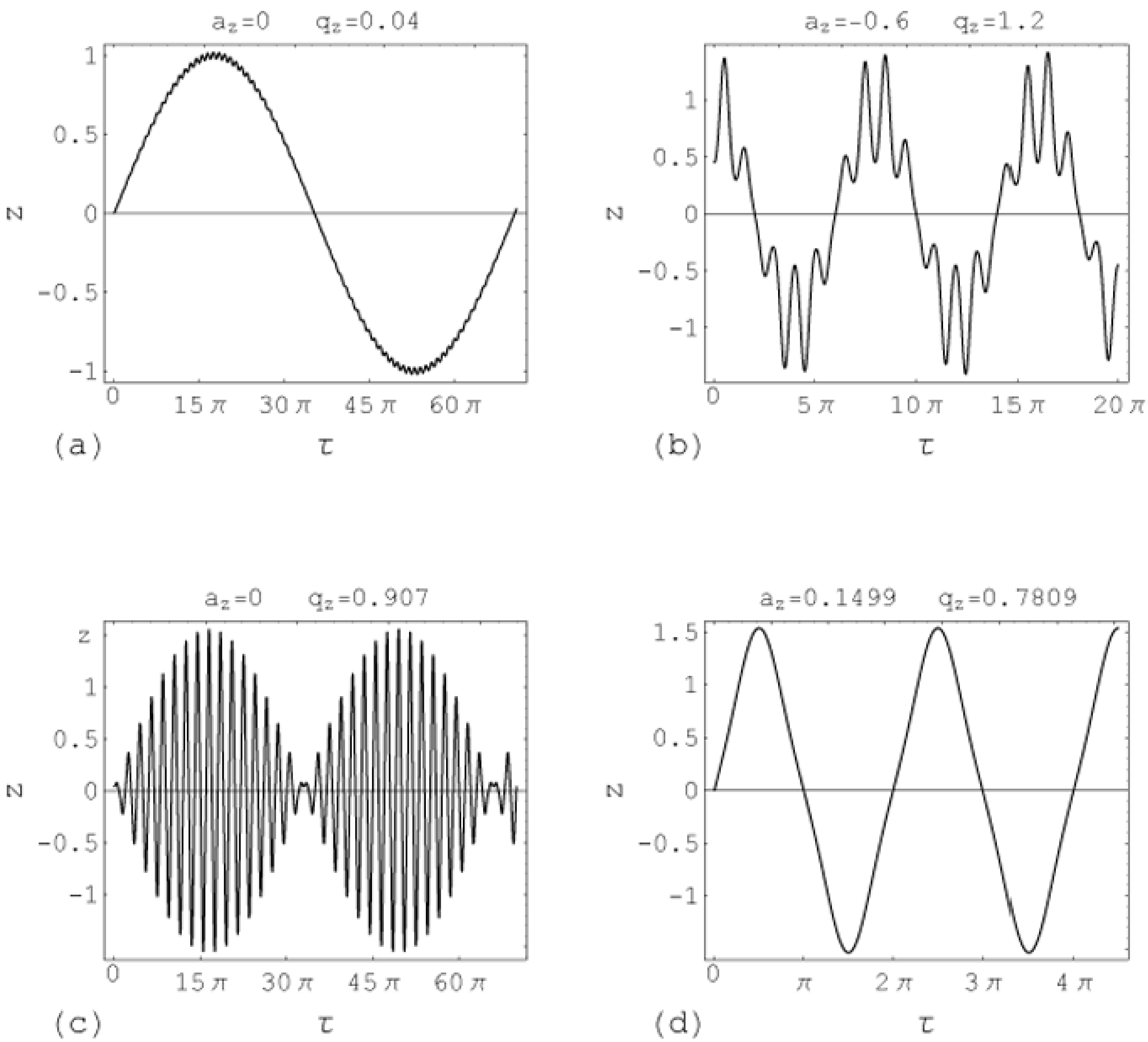
Approximate solution for a,q<<1:

$$u(t) = A[1 - (q/2)\cos \omega t]\cos \Omega t$$

$$\beta^2 = a + q^2/2$$

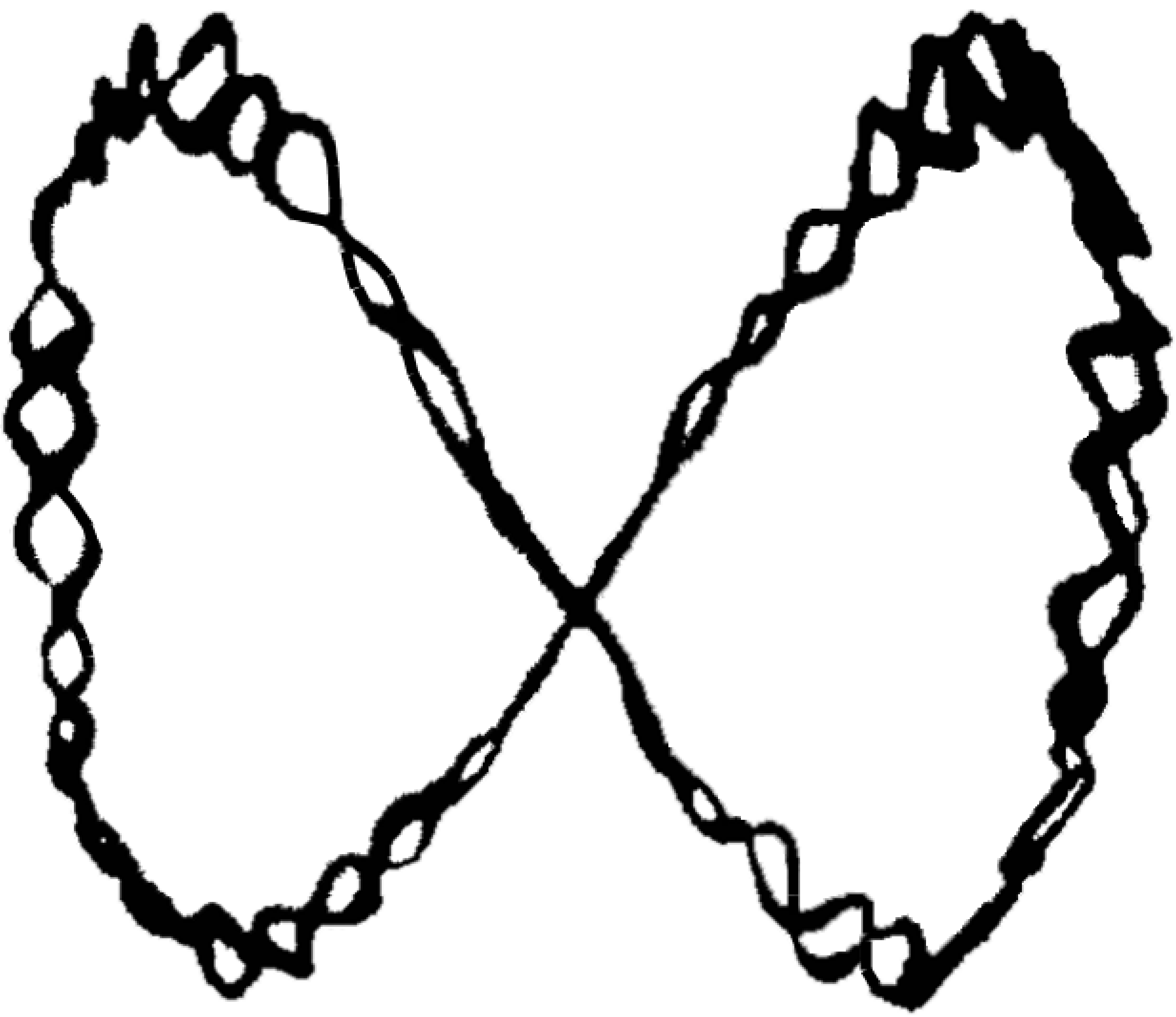
This is a harmonic oscillation at frequency Ω (micromotion) modulated by an oscillation at frequency ω(macromotion)

Ion trajectories at different operating conditions



Trajectory of a single microscopic particle

(Wuerker 1959)



Time averaged potential depth:

$$\overline{D_i} = \frac{m}{8}\Omega^2 r_0^2 \beta_i^2$$

<u>Numerical example:</u>

m=50

$\Omega/2\pi$=1 MHz

r_0=1 cm

ß=0.3

D = 25 eV

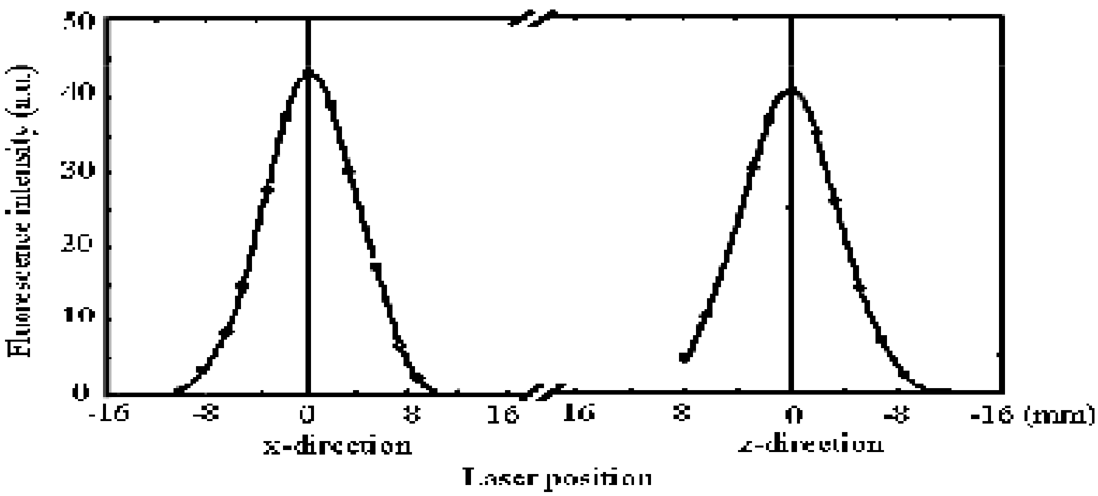
Maximum ion density, when space charge potential equals trapping potential depth

$n_{max} \approx 1o^6$ cm^{-3}

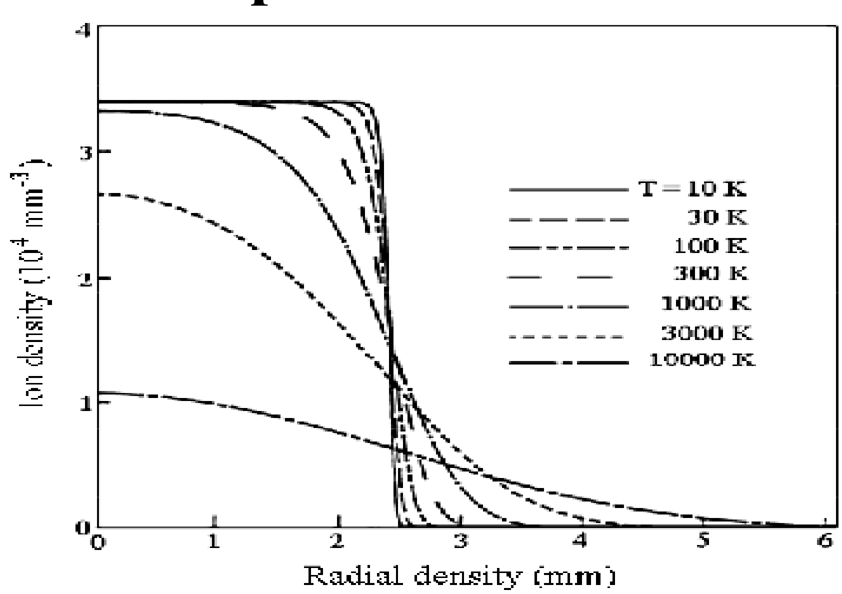
Mean kinetic ion energy (no cooling) ≈ 1/10 D

Density distribution of an ion cloud in a Paul trap

Experimentally measured distribution for uncooled ion cloud

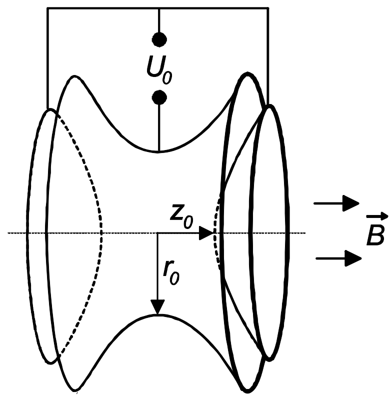


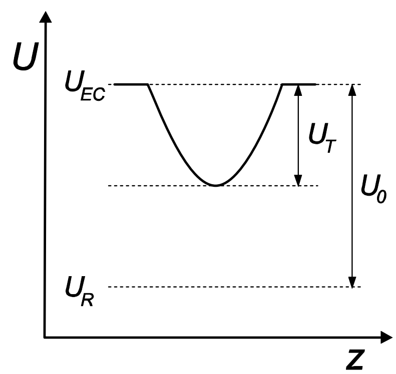
Calculated density distribution for different temperatures



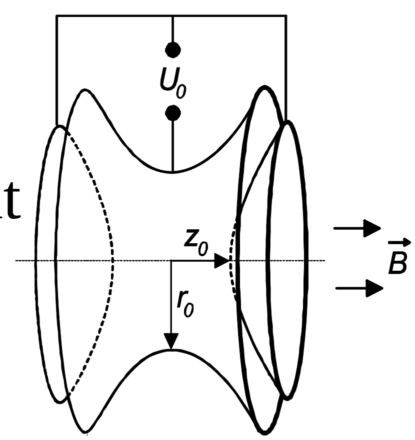
The ideal Penning trap

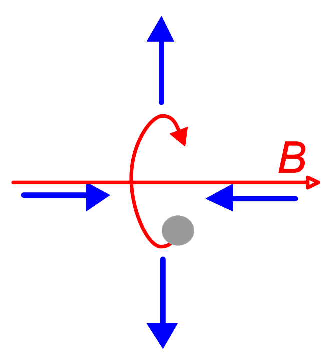
Axial harmonic potential





Radial confinement by magnetic field





Equations of motion

Electric quadrupole field and homogeneous magnetic field in axial direction

Force acting on charged particle in 3D:

$$\vec{F} = -e \nabla \Phi + e(\vec{v} \times \vec{B})$$

$$\Phi = \frac{U_0}{2d^2}(2z^2 - x^2 - y^2)$$

Equations of motion:

$$\frac{d^2x}{dt^2} - \omega_c \frac{dy}{dt} - \frac{1}{2}\omega_z^2 x = 0 \quad (1)$$

$$\frac{d^2y}{dt^2} + \omega_c \frac{dx}{dt} - \frac{1}{2}\omega_z^2 y = 0 \quad (2)$$

$$\frac{d^2z}{dt^2} + \omega_z^2 z = 0 \quad (3)$$

$$\omega_c = \frac{e}{m} B$$

Solutions:

3 harmonic oscillations

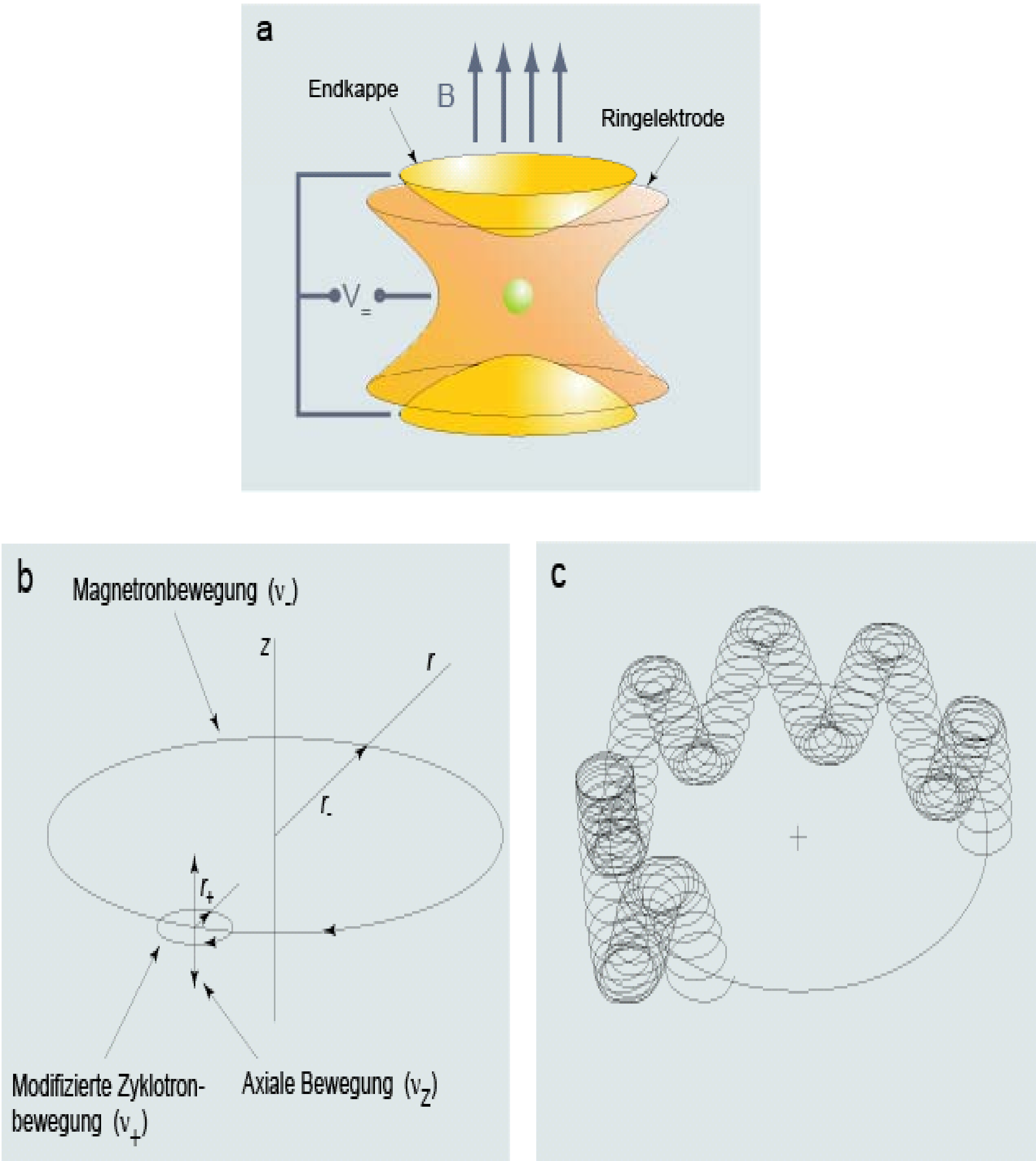
$$\omega_z = \left[\frac{4eU_0}{md^2}\right]^{1/2}$$ axial

$$\omega_+ = \frac{1}{2}(\omega_c + \omega_1)$$ perturbed cyclotron

$$\omega_- = \frac{1}{2}(\omega_c - \omega_1)$$ magnetron

$$\omega_1 = \sqrt{\omega_c^2 - 2\omega_z^2}$$

Ion Motion in a Penning Trap



important relations

$$\omega_c = \omega_+ + \omega_-$$

$$\omega_c^2 = \omega_+^2 + \omega_z^2 + \omega_-^2$$

L.S. Brown, G. Gabrielse, Rev. Mod. Phys. 58, 233 (1986).

Stability limit:

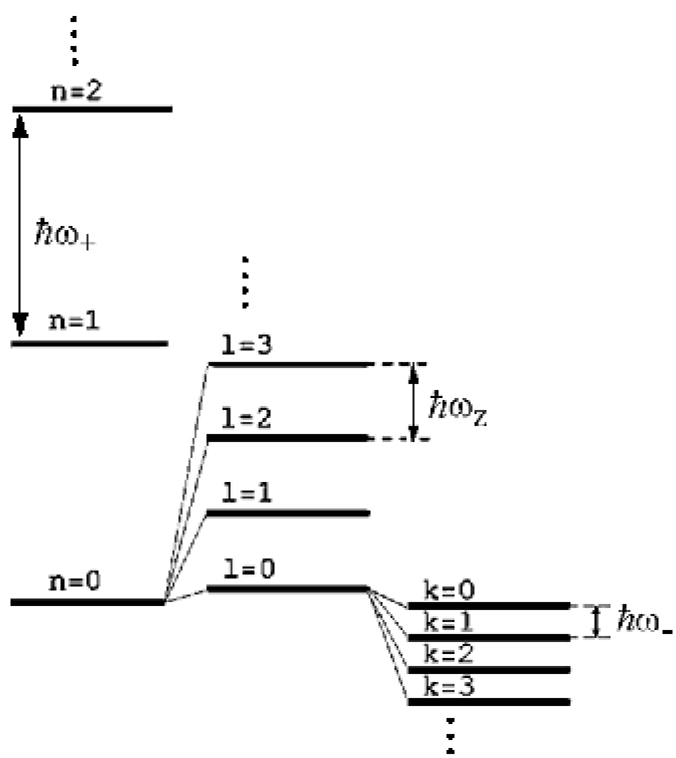
$$\omega_c{}^2 \geq 2\omega_z{}^2$$

$$\frac{e}{M} B^2 \geq \frac{8U}{d^2}$$

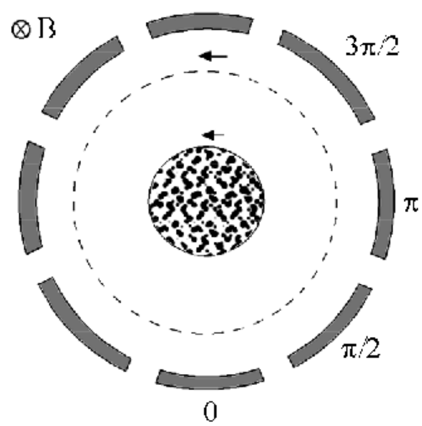
Quantum mechanical energy levels of a particle in the Penning trap:

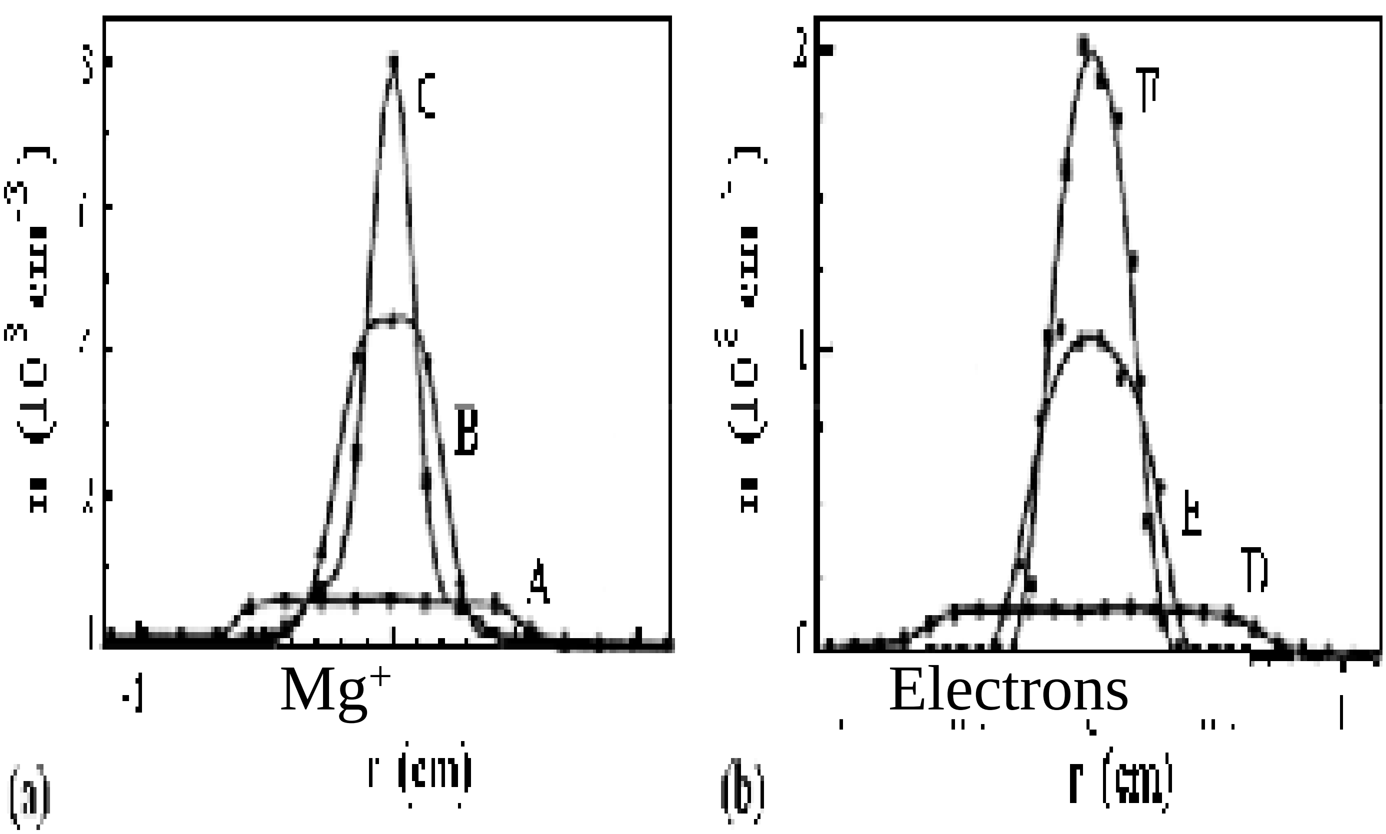
$$E = (n_+ + 1/2)\hbar\omega_+ - (n_- + 1/2)\hbar\omega_- + (n_z + 1/2)\hbar\omega_z$$

Negative sign for magnetronenergy indicates metastability of motion



Rotating wall compression of ion clouds





Comparison of Paul- and Penning traps

Paul traps	Penning traps
• Simple set up (no big magnet required) • Three dimensional potential well • Simultaneous trapping of both signs of charge • Limited range for different charge states • Rf heating • All frequencies subject to space charge shift	• No rf heating effects • High mass resolution • Better stability for higher charge states • cyclotron frequency insensitive to space charge • Expensive (big magnet required) • Trapping for only one sign of charge

References

W. Paul, Electromagnetic Traps for Charged Particles, Rev. Mod. Phys. 62, 531 (1990)

P.K. Ghosh, Ion Traps, Clarendon, Oxford (1995)

H. Dehmelt, in: Advances in: Atom. Molec. Phys. Vol 3 (1967)

F.G. Major, V. Gheorghe, G. Werth Charged Particle traps, Springer (2005)

G. Werth, F.G. Major, V. Gheorghe, Charged Particle traps II, Springer (2009)

G. Werth, Trapped Ions, Contemporary Physics 26, 241 (1985)

G. Savard and G. Werth, Precision Nuclear Measurements with Ion Traps, Ann. Rev. Nucl. Part. Science 50, 119 (2002)

Real Traps

Deviations from ideal harmonic potential caused by:

Truncation of trap electrode
Imperfect electrode shape
Misalignements
Space charge from simultaneously trapped ions

Dealing with imperfections:

Expansion of the trapping potential in spherical harmonics:

$$\Phi(r,\vartheta) = \Phi_0 \sum_{n=2}^{\infty} c_n \left(\frac{r}{d}\right)^n P_n(\cos\vartheta)$$

Coordinate dependence of some higher order contributions:

$$n = 2 : -r^2 + 2z^2$$

$$n = 3 : -3r^2 z + 2z^2 r$$

$$n = 4 : 3r^4 - 24r^2 z^2 + 8z^4$$

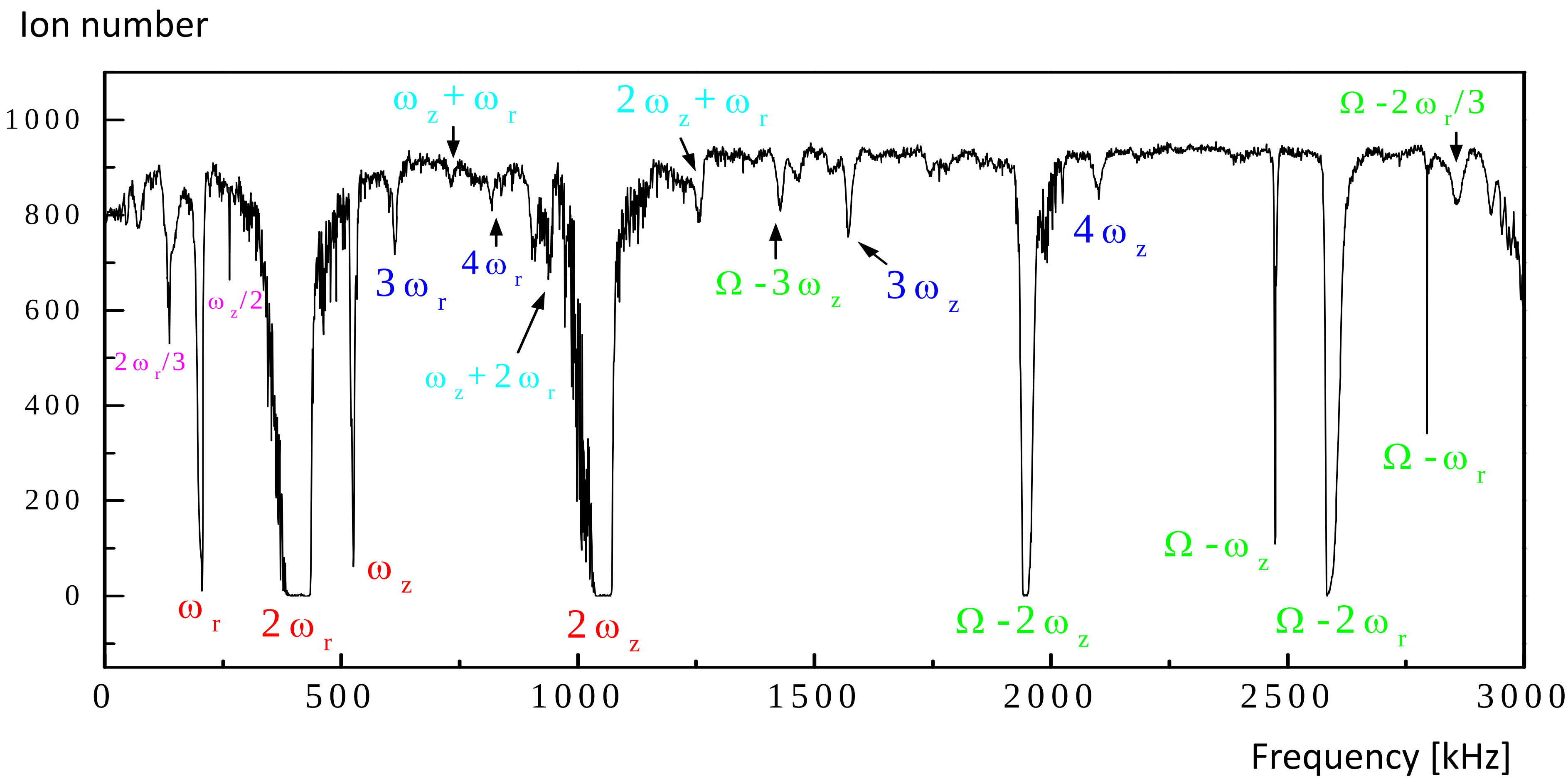
$$n = 6 : -5r^6 + 90r^4 z^2 - 120r^2 z^4 + 16z^6$$

n=2: quadrupole, n=3: hexapole, n=4. octupole, n=6: dodekapole

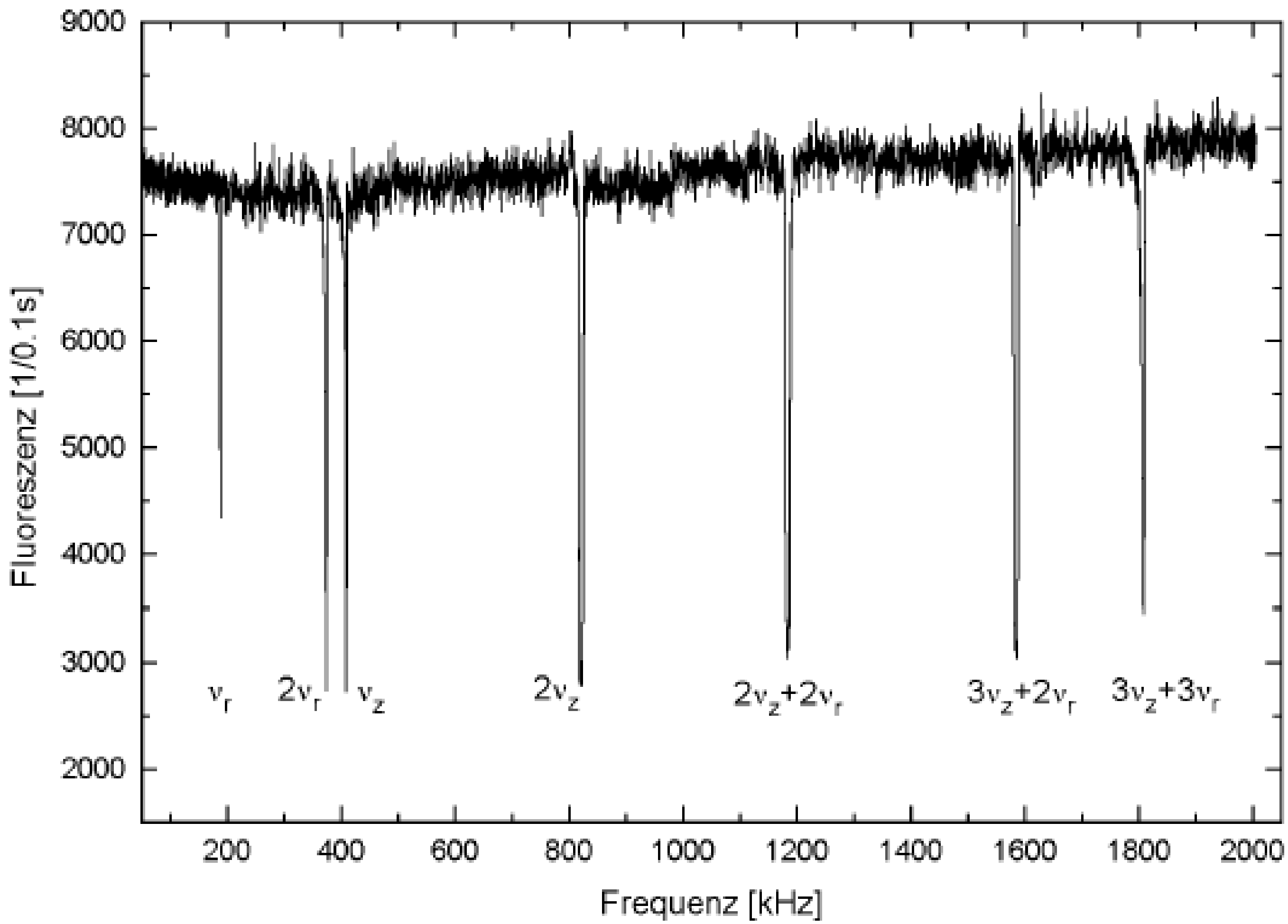
Effects of trap imperfections:

- Coupling of oscillation modes
- Shift of eigenfrequencies
- Asymmetry of resonances
- Collective and noncollective oscillations
- Instabilities of ion trajectories

Coupling of ion oscillation modes of a stored ion cloud in a Paul Trap

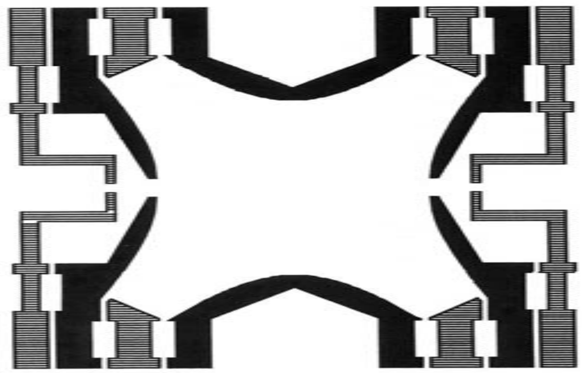


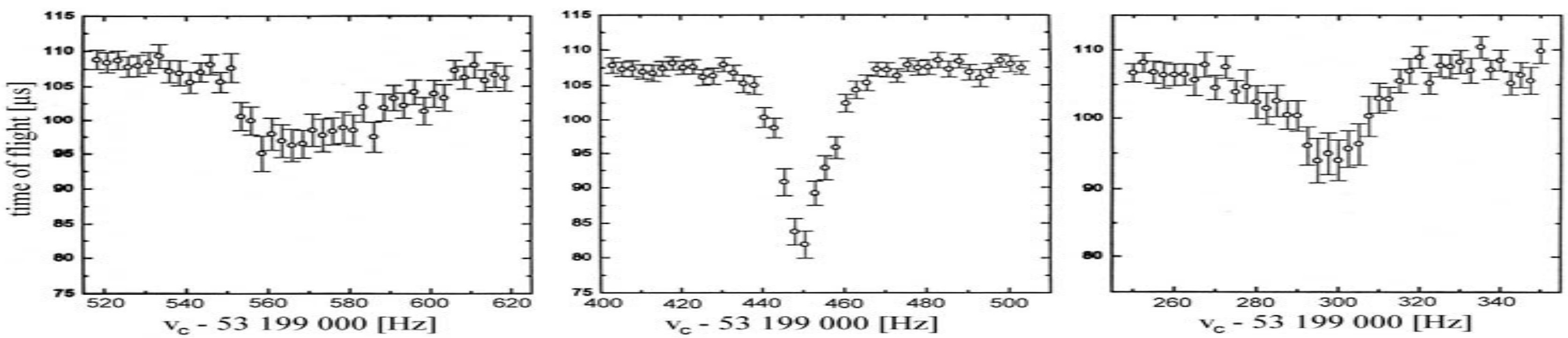
Motional spectrum of ions in a Paul trap measured by laser induced fluorescence



Minimizing trap imperfections

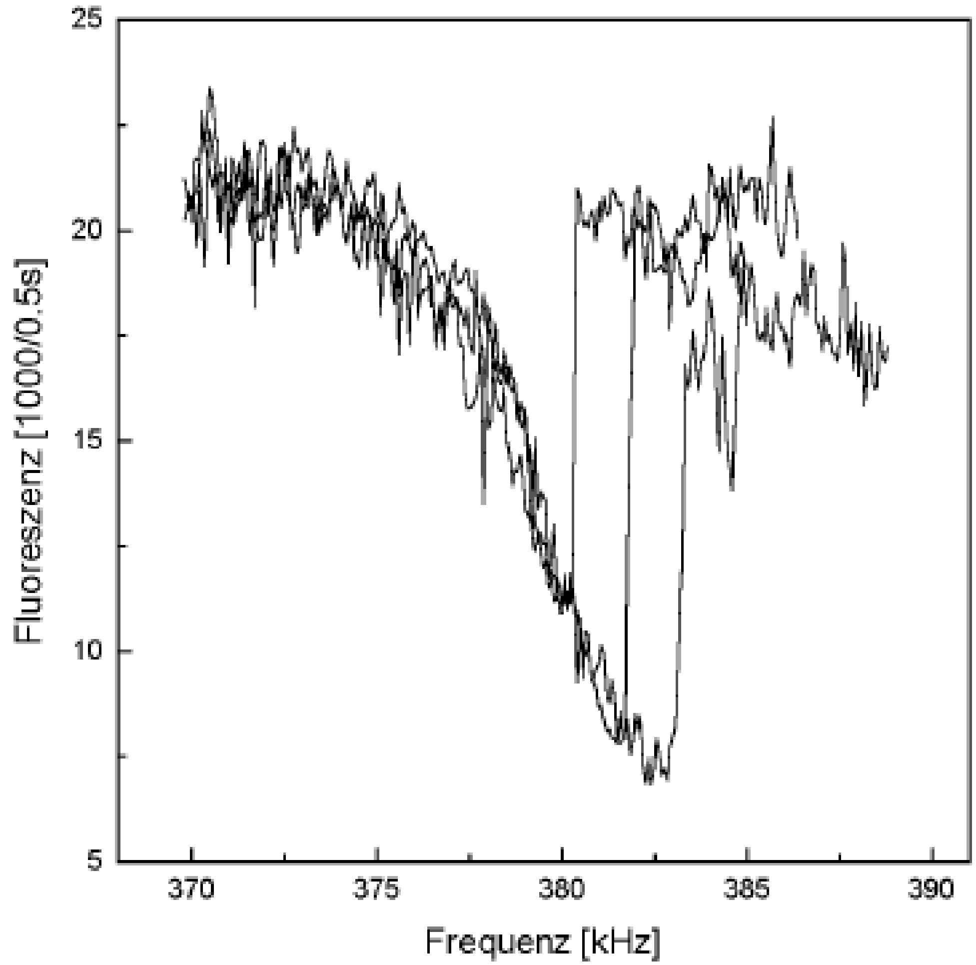
Additional electrodes between ring and endcap



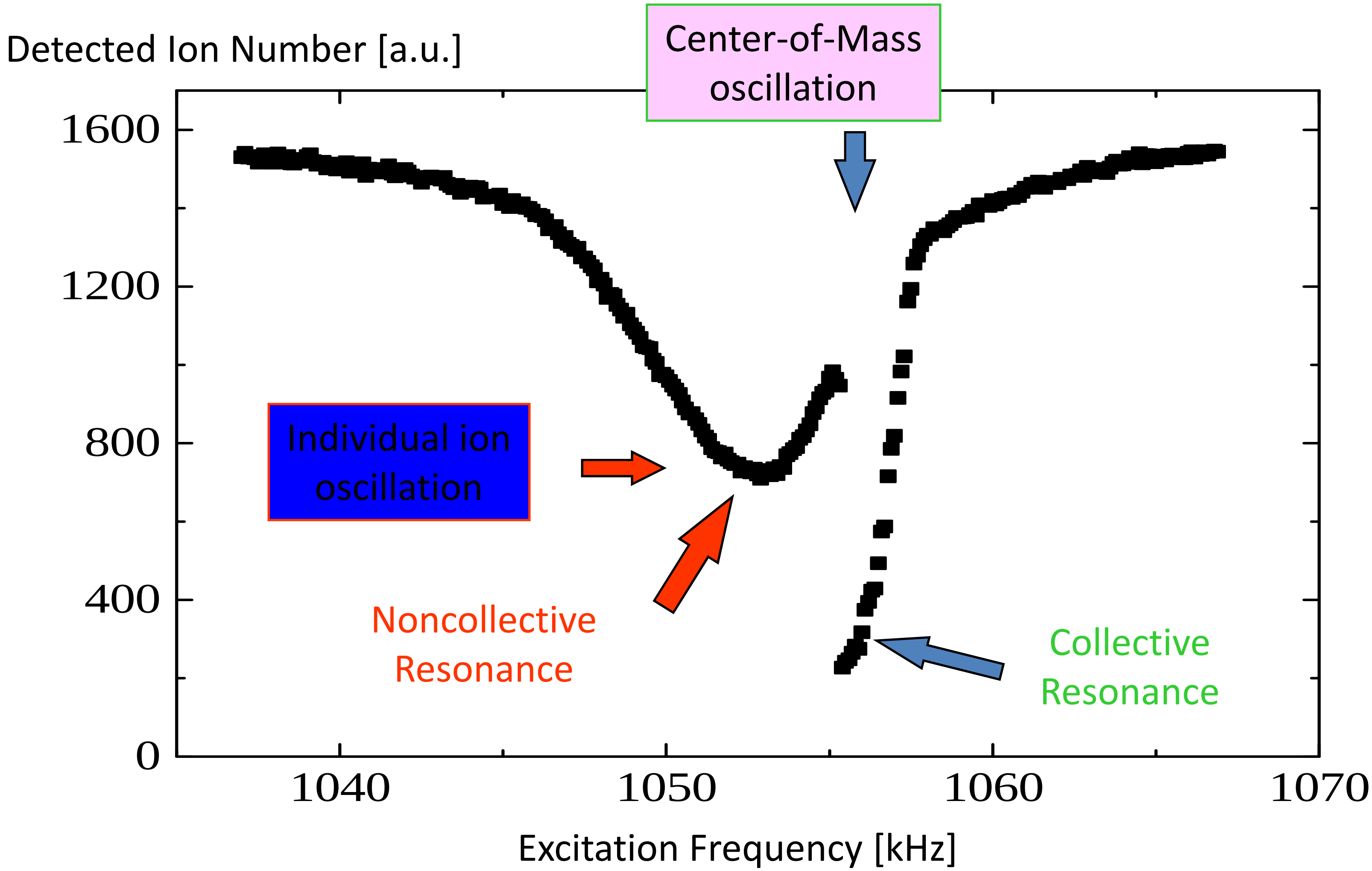


Cyclotron resonance at different values of the correction voltage

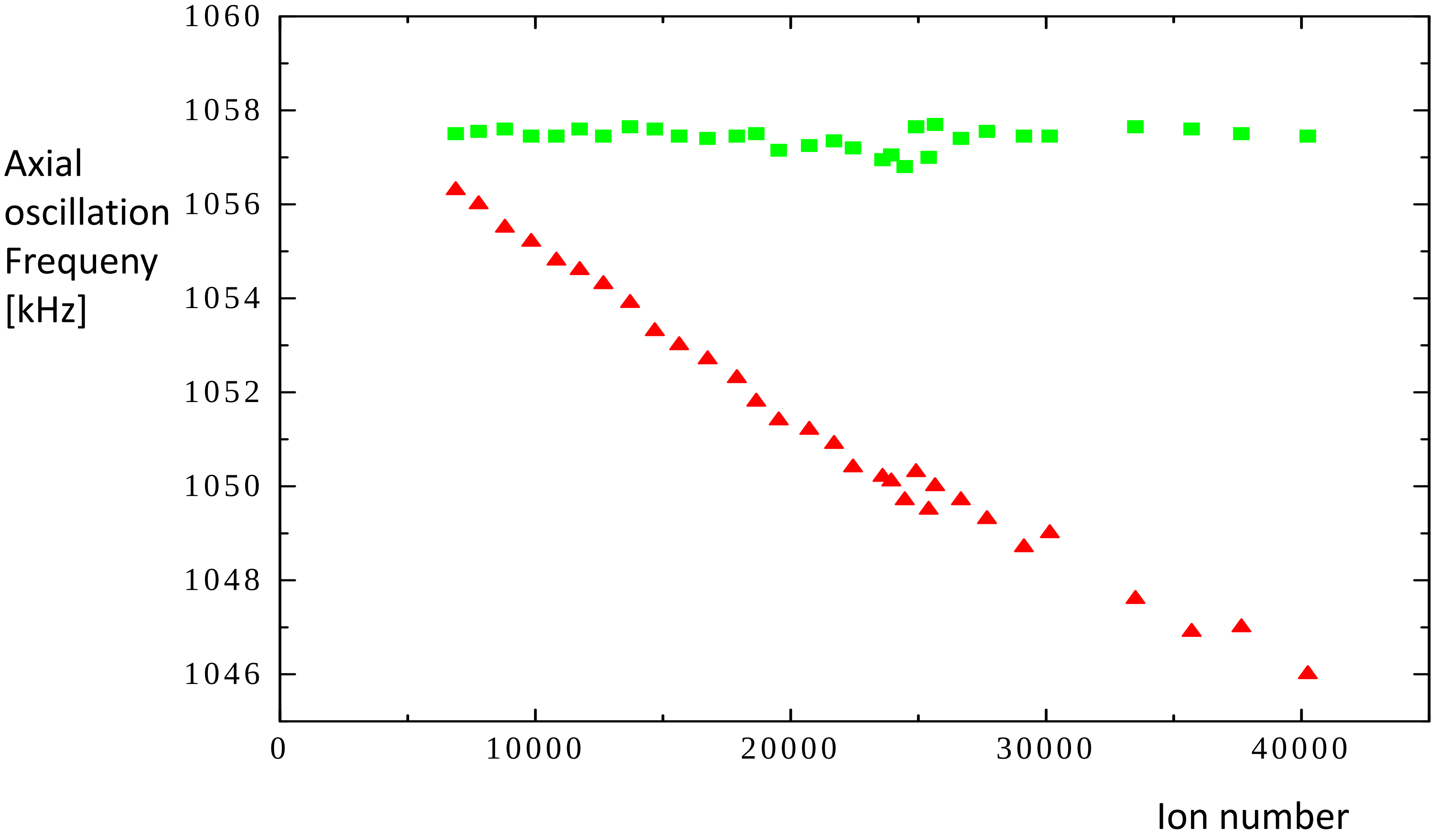
Asymmetry of axial resonance
(taken at different excitation amplitudes)



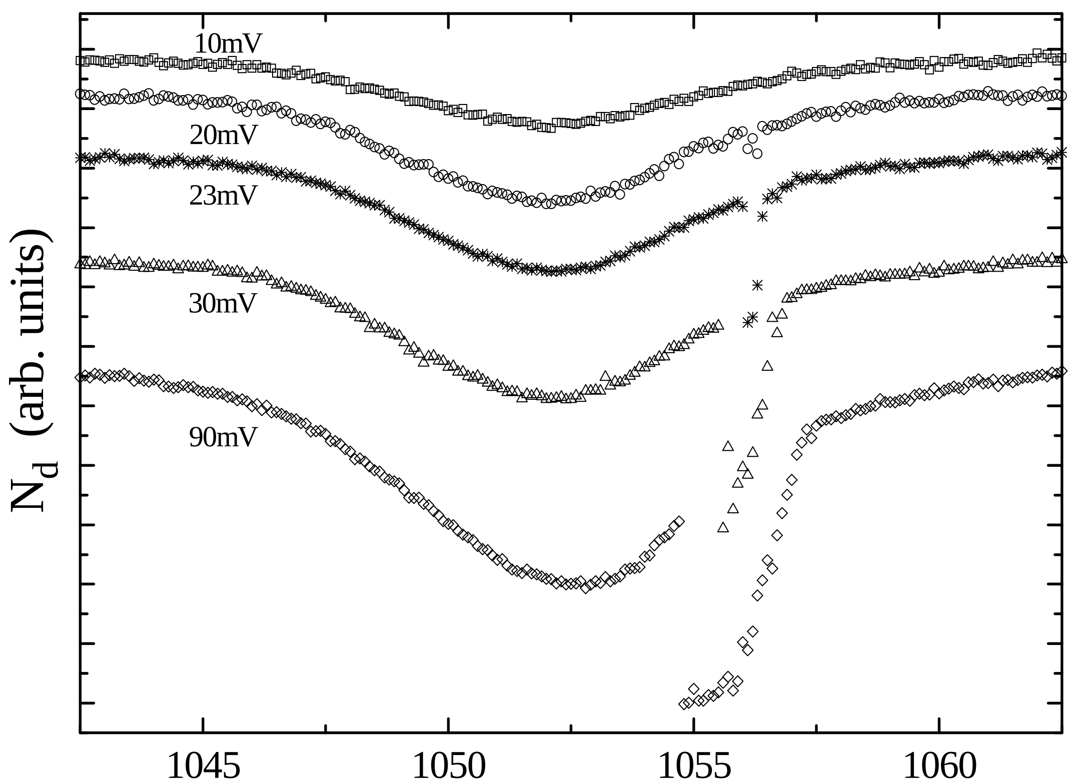
Individual and center-of-mass oscillation of axial motion



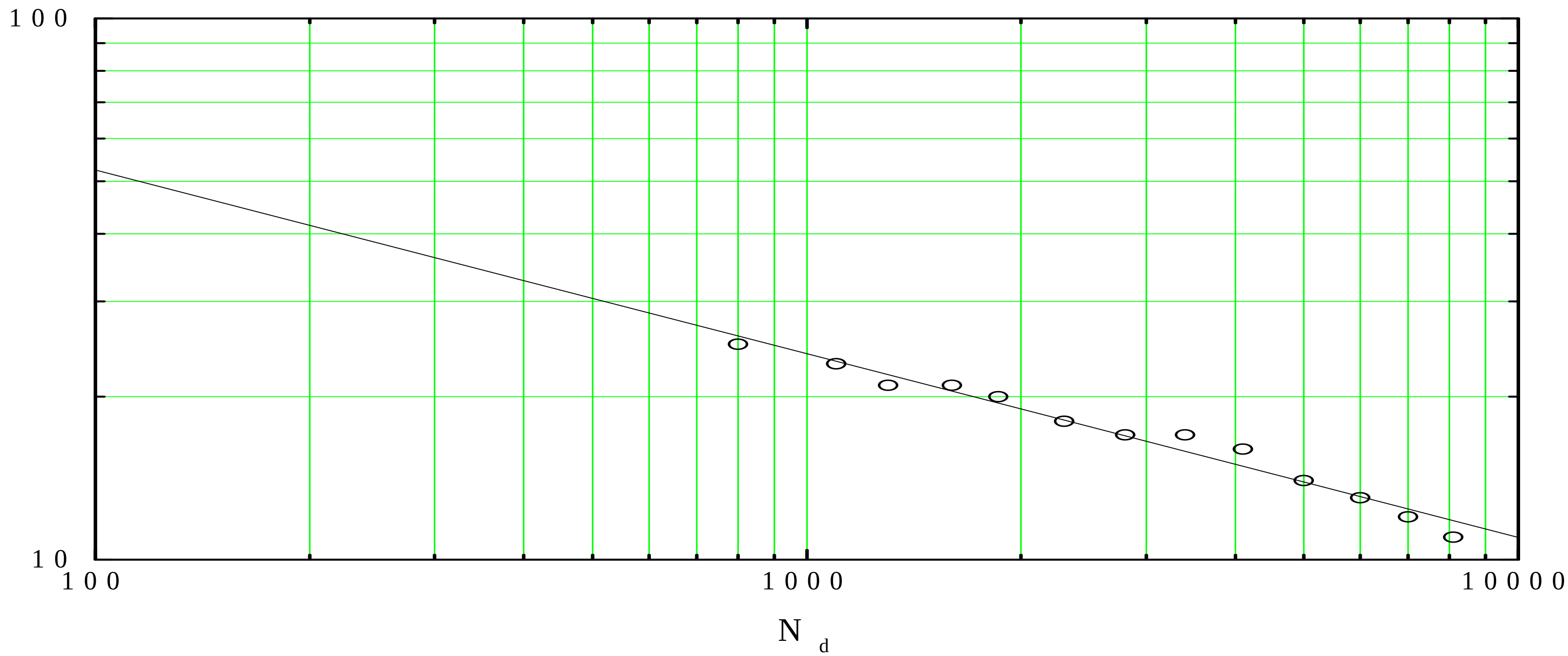
Space charge shift of axial resonance



Threshold behaviour of center-of mass resonance

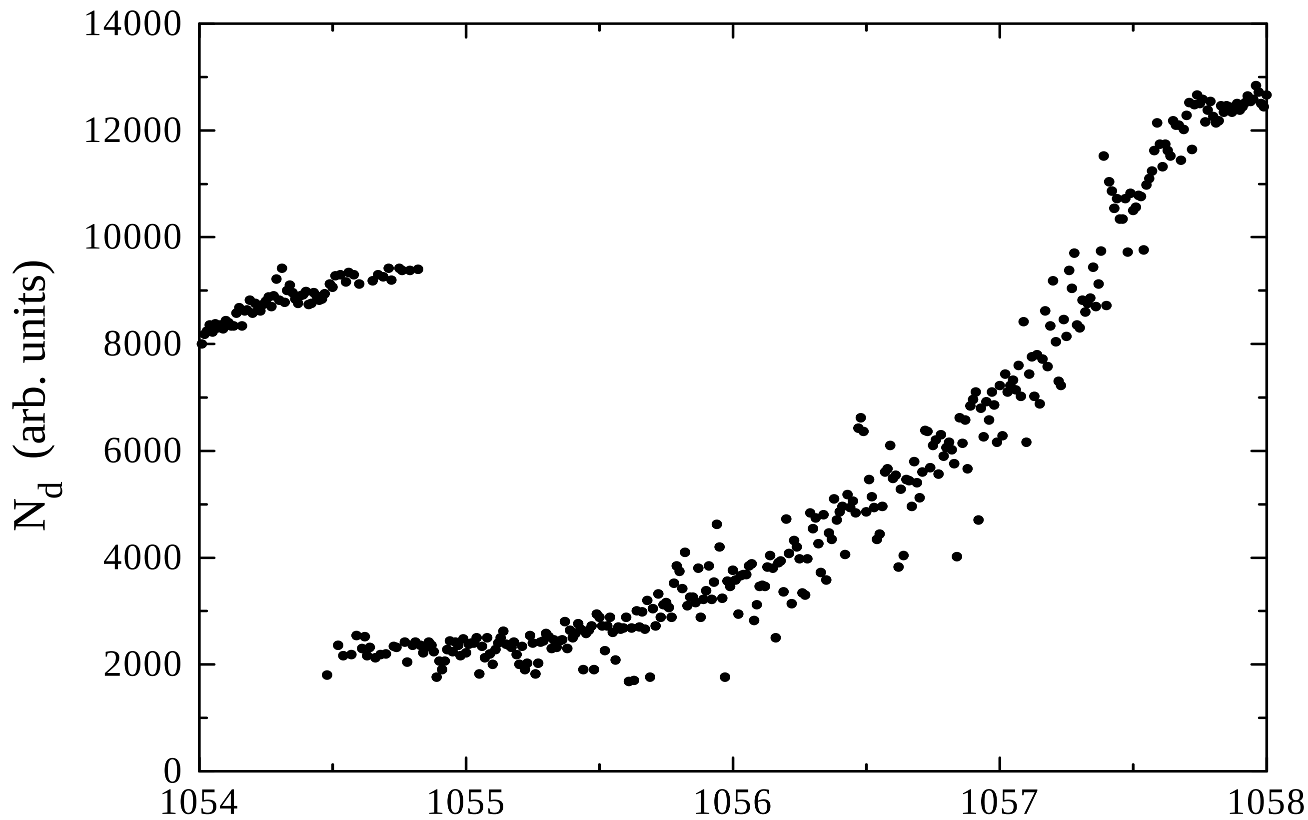


Dependence of thresghold amplitude on ion number



$V_{th} \sim N^{-2/3}$

Bistability in the excitation of motional resonances



Instabilities of the ion motion in a Paul trap occur when the ion oscillation freqencies ω_r, ω_z are linear dependend on the traps driving frequency Ω

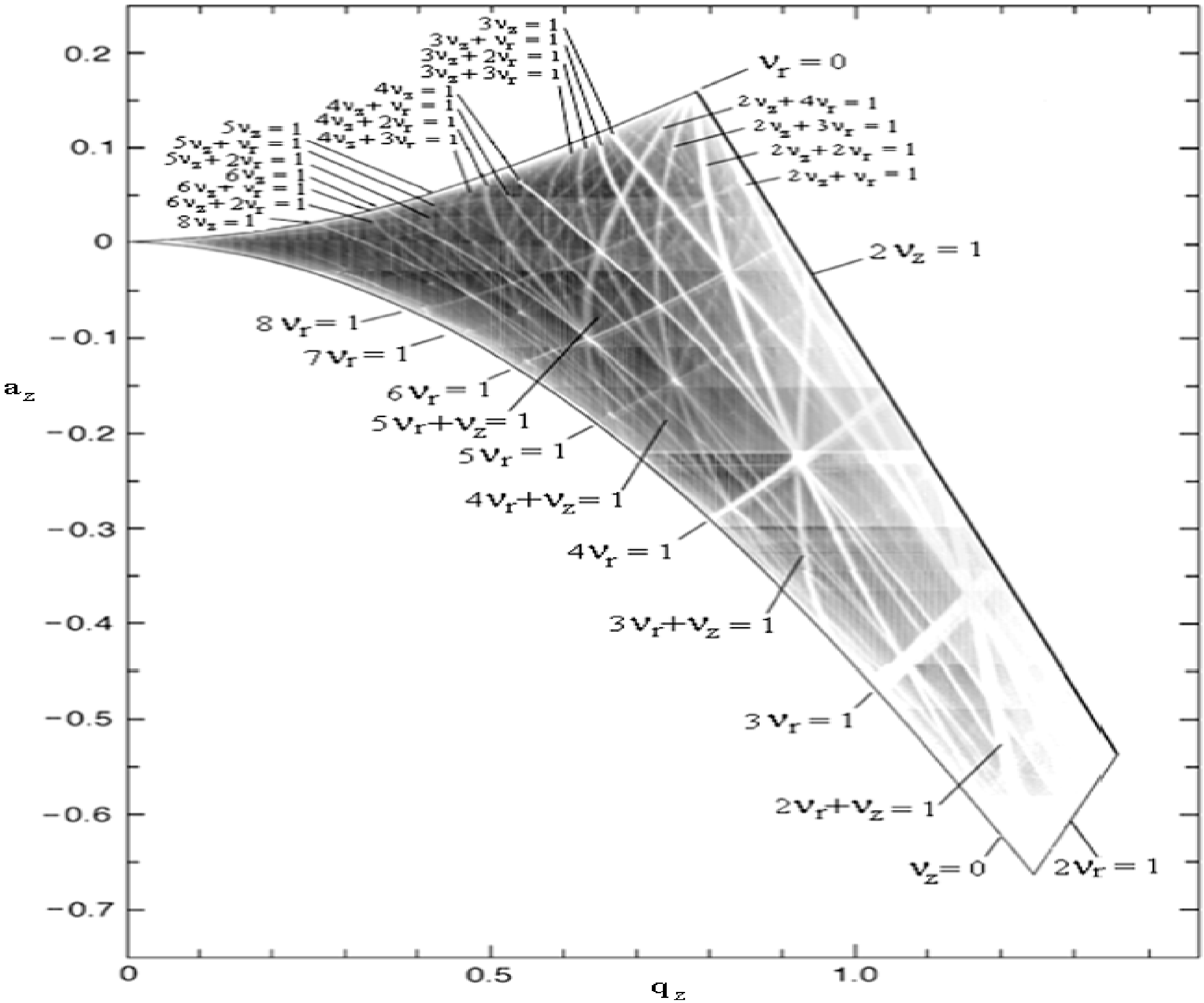
$$\boxed{n_r\omega_r + n_z\omega_z = k\,\Omega}$$

n_r, n_z, k integer

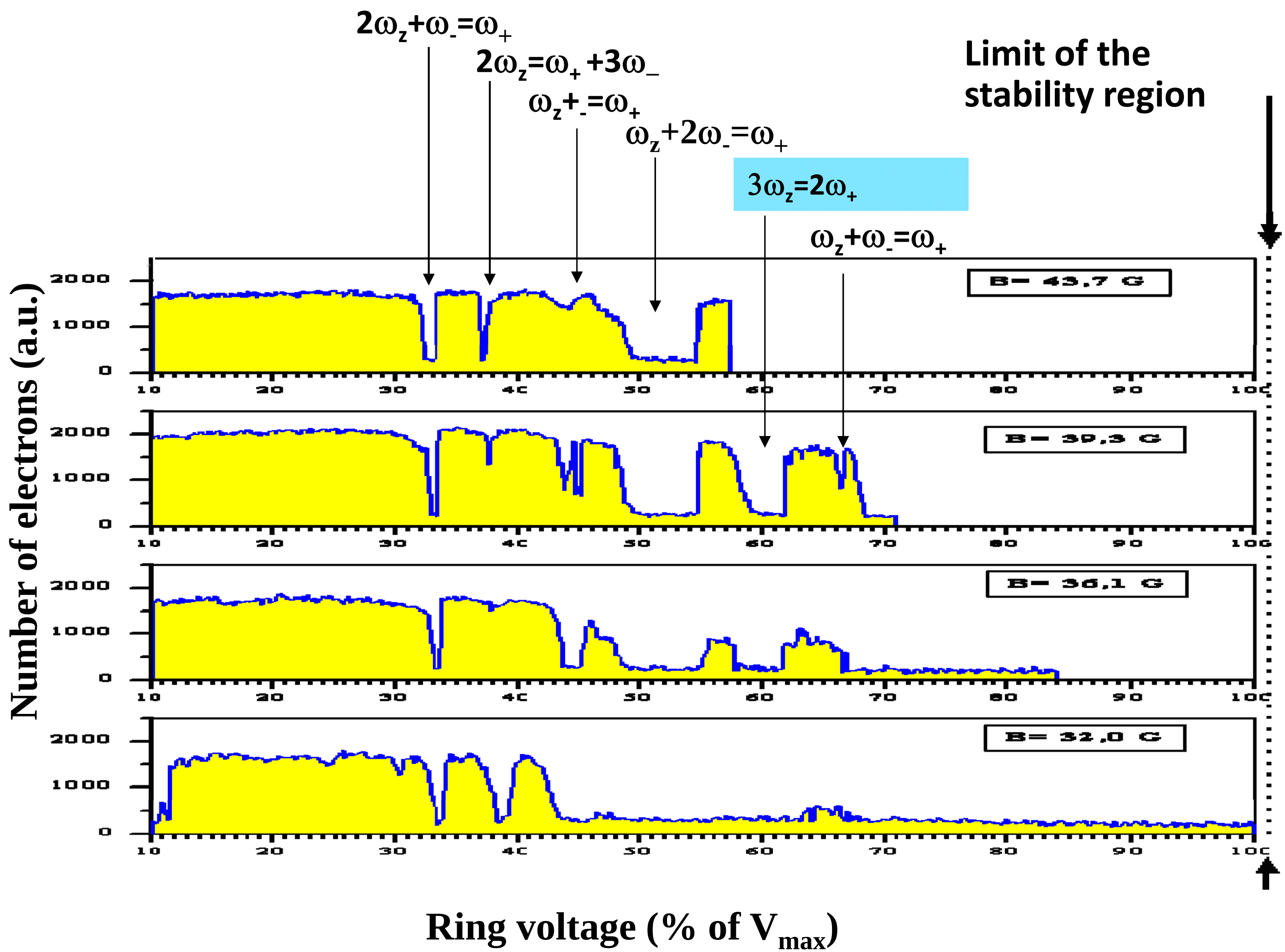
$n_r + n_z = N$

2N = order of perturbation

Instabilities of ion motion in a Paul trap



Observed Instabilities on electrons in a Penning trap



Instabilities of electrons stored in a Penning trap for different storage times