TRIUMF

Canada's national laboratory for particle and nuclear physics
Laboratoire national canadien pour la recherche en physique nucléaire
et en physique des particules

Giant and pigmy dipole resonances in medium-mass nuclei from chiral nucleon-nucleon interactions

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Collaborators:

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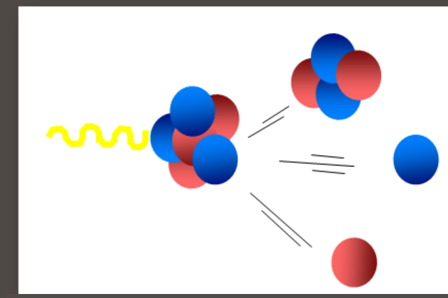
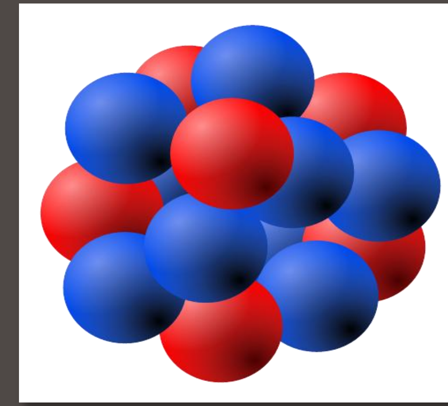
January 26th , 2015

a place of mind
THE UNIVERSITY OF BRITISH COLUMBIA



Nuclear Reactions



$^{16,22}O, ^{40,48}Ca$

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Electromagnetic (EM) Reactions

Small coupling constant $\alpha \ll 1$ → Perturbative treatment

"*With the electro-magnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself* "

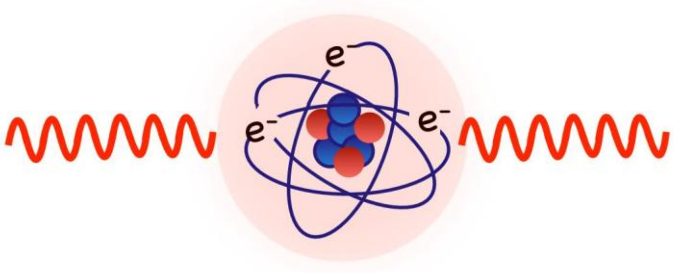
[De Forest-Walecka, Ann. Phys. 1966]

→ Photo-absorption reactions

→ Coulomb excitation reactions

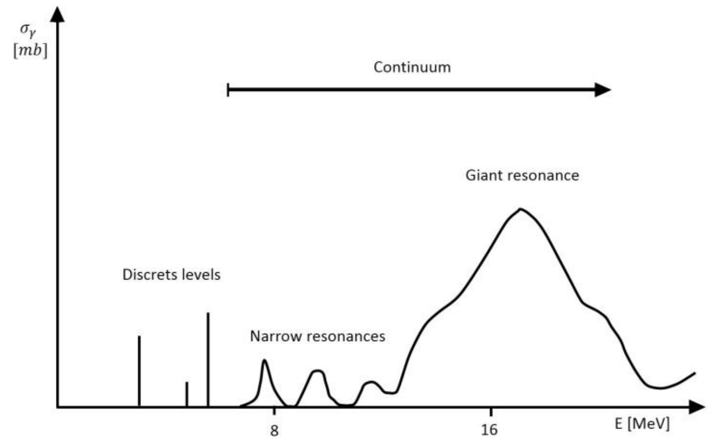
EM Reactions: Photo-absorption

Interaction of a (real, low-energy) photon with a nucleus.



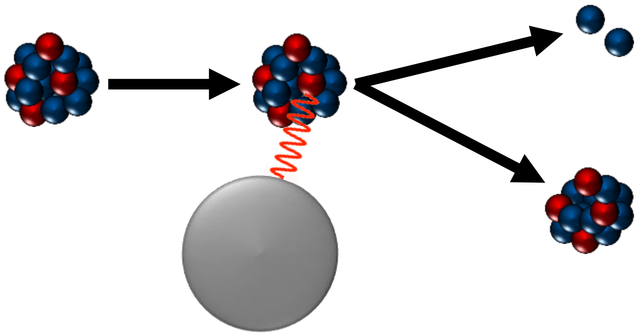
Giant Dipole Resonance (GDR)

- Observed across all the periodic table
- The peak is localized between 10-30MeV, the position changes with the mass number



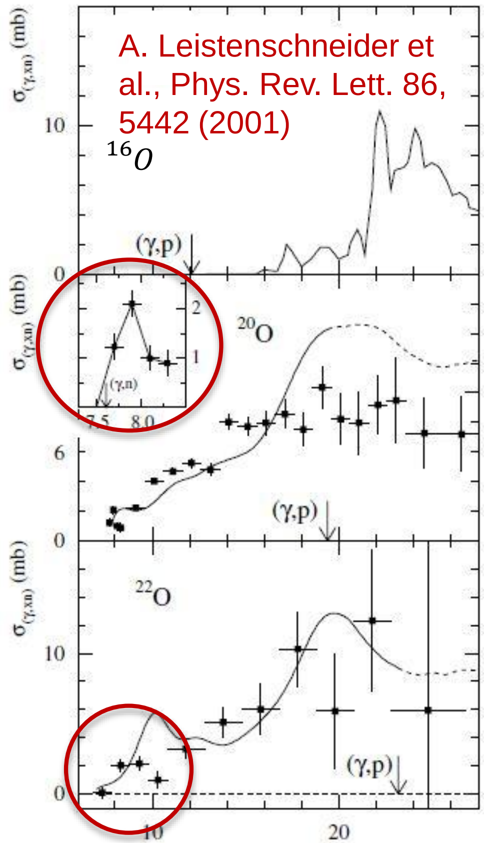
EM Reactions: Coulomb excitation

Inelastic scattering between two charged particles (exchange of a virtual photon).



Pigmy Dipole Resonance (PDR)

- Unstable nuclei can be used as projectiles
- Neutron-rich nuclei show fragmented low-lying strength (soft modes)

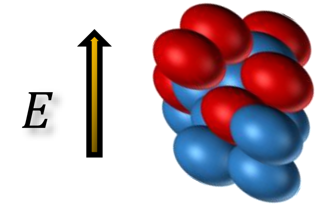


From the Resonances to the Polarizability

It is obtained from the dipole strength $R(\omega)$ as an inverse weighted sum-rule (via photo-absorption, Compton scattering, (p,p') reactions, Coulomb excitation, elastic scattering below Coulomb barrier, ...)

Interesting facts:

- Correlated to the neutron skin-radius
- Can be used to constraint the some of the parameters appearing in the EOS of neutron stars
- No *ab-initio* description for medium-mass nuclei (until now!)



$$D = \alpha_E E$$

$$\alpha_E = 2\alpha \int d\omega \frac{R(\omega)}{\omega}$$

Extremely interesting in neutron-rich nuclei: the soft modes at low energy enhance the electric dipole polarizability

Theoretical Approach

Current situation on the theoretical description:

- **Non-ab-initio**: via macroscopic models or mean field based methods
- **Ab-initio**: described via exact computations for light nuclei using the LIT+EIHH method (up to $A = 7$)

No ab-initio description of the GDR for $A > 7$
$\rightarrow$ need of a new approach for larger nuclei

What ingredients and tools do we need?

- **Continuum problem** $\rightarrow$ LIT
- **Many-body technique** $\rightarrow$ CC
- **Nuclear interactions** $\rightarrow$ ChPT

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LIT Method

The response function $R(\omega)$ is the key quantity

$$\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R(\omega)$$

$$\alpha_E = 2\alpha \int d\omega \frac{R(\omega)}{\omega}$$

$$R(\omega) = \sum\!\!\!\!\!\!\int_f |\langle f|\hat{\theta}|i\rangle|^2 \delta(E_f - E_i - \omega)$$

- Final states problem is tackled with the Lorentz Integral Transform (LIT) method

$$L(\omega_0, \Gamma) = \frac{\Gamma}{\pi} \int d\omega \frac{R(\omega)}{(\omega_0 - \omega)^2 + \Gamma^2}$$

where $(H - E_i + \sigma)|\tilde{\psi}\rangle = \theta|i\rangle$ and $\sigma = -\omega_0 - i\Gamma$

$$L(\sigma) = \frac{\Gamma}{\pi} \langle i|\theta^+(H - E_i + \sigma^*)^{-1}(H - E_i + \sigma)^{-1}\theta|i\rangle = \langle\tilde{\psi}|\tilde{\psi}\rangle < \infty$$

- The exact final state interaction is included in the continuum rigorously!

$$L(\sigma) \xRightarrow{Inversion} R(\omega)$$

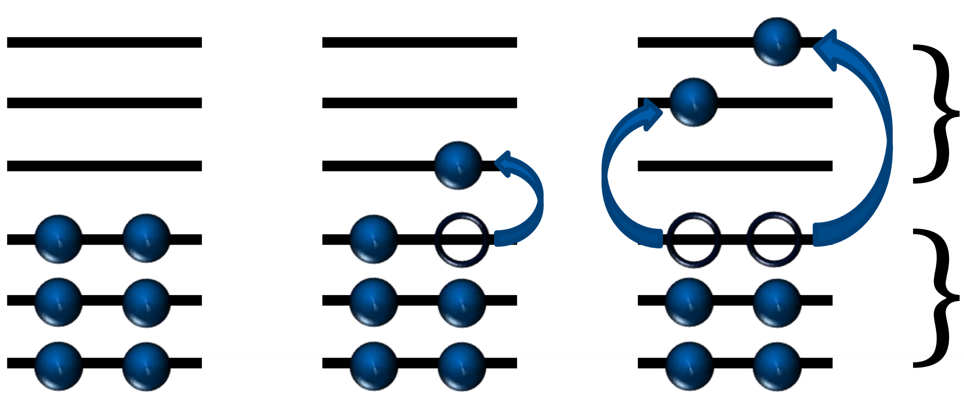
CC Theory

- Continuum problem → Bound state problem

$$\boxed{L(\sigma) = \frac{\Gamma}{\pi}\langle i|\theta^{+}(H - E_i + \sigma^{*})^{-1}(H - E_i + \sigma)^{-1}\theta|i\rangle}$$

- Computation of the ground state → **Coupled Cluster (CC) theory**

$$\boxed{|i\rangle = e^{T}|0\rangle} \qquad \boxed{\mathrm{T} = \sum_{i=1}^{A} T_i} \qquad \boxed{T_n = \frac{1}{(n!)^2}\sum_{\substack{a_1,a_2,\ldots,a_n \\ i_1,i_2,\ldots,i_n}} t_{i_1 i_2 \ldots i_n}^{a_1 a_2 \ldots, a_n}\{a_1^{+} i_1 a_2^{+} i_2 \ldots a_n^{+} i_n\}}$$



$N_{MAX} = model\ space\ size$

$|0\rangle$

CC Theory

- The energy is calculated via

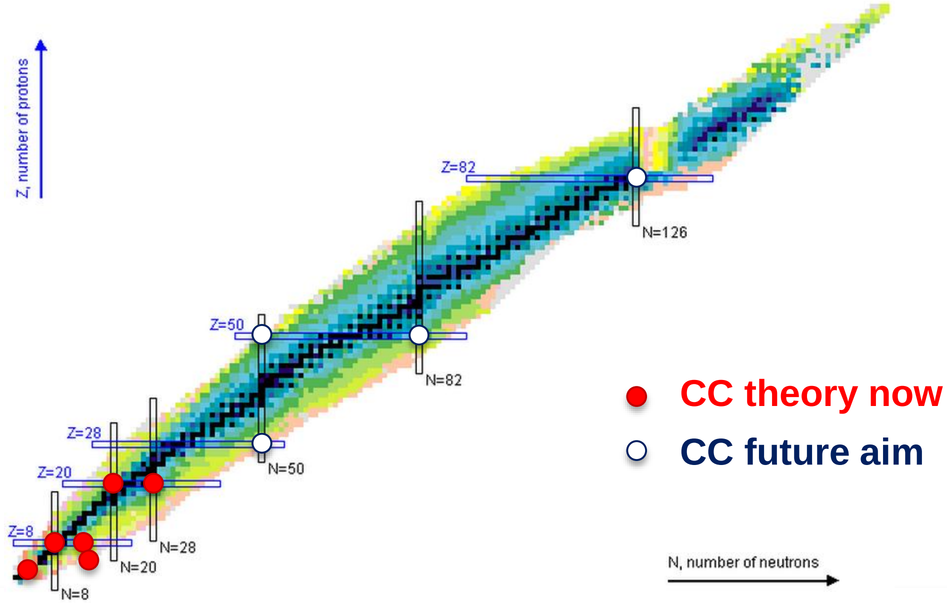
$$\boxed{E_i = \langle 0|\overline{H}|0\rangle}$$ Ground state energy

$$\boxed{\overline{H} = e^{-T}\, H_N e^{T}}$$ Symilarity transformed Hamiltonian

- One needs the T amplitudes which are found solving a set of non-linear coupled equations

- **Presently calculate only closed shell and closed sub-shell nuclei**

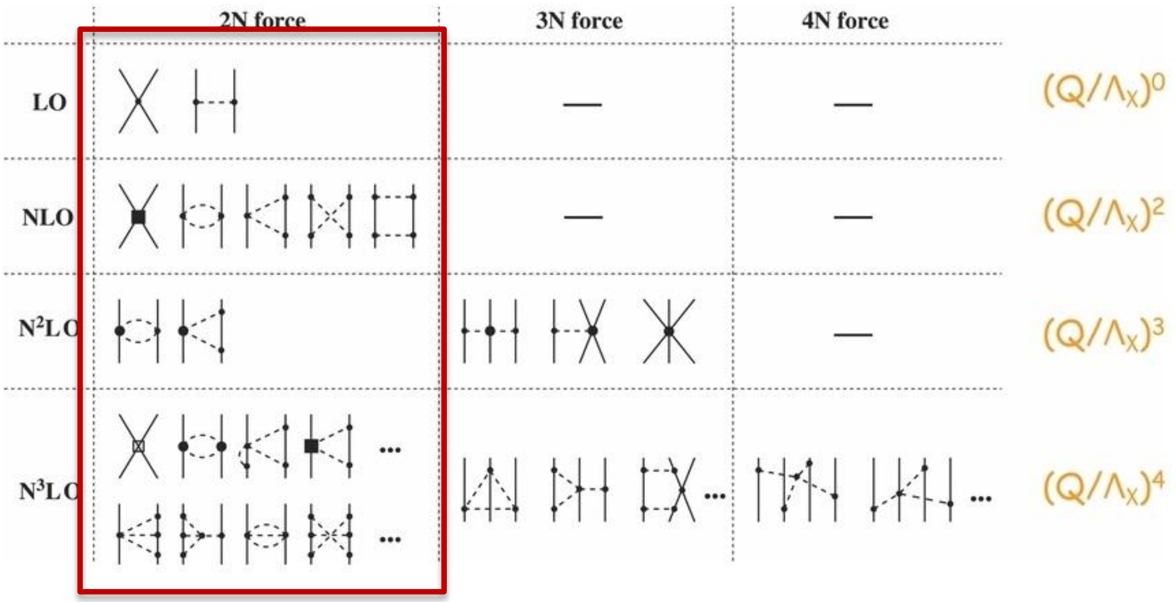
- **The aim is to extend to heavy nuclei (and open shell?)**



The TRIUMF header logo.

The LIT+CC method and χPT

R. Machleidt and D. Entem, Phys. Rep. 503, 1 (2011)



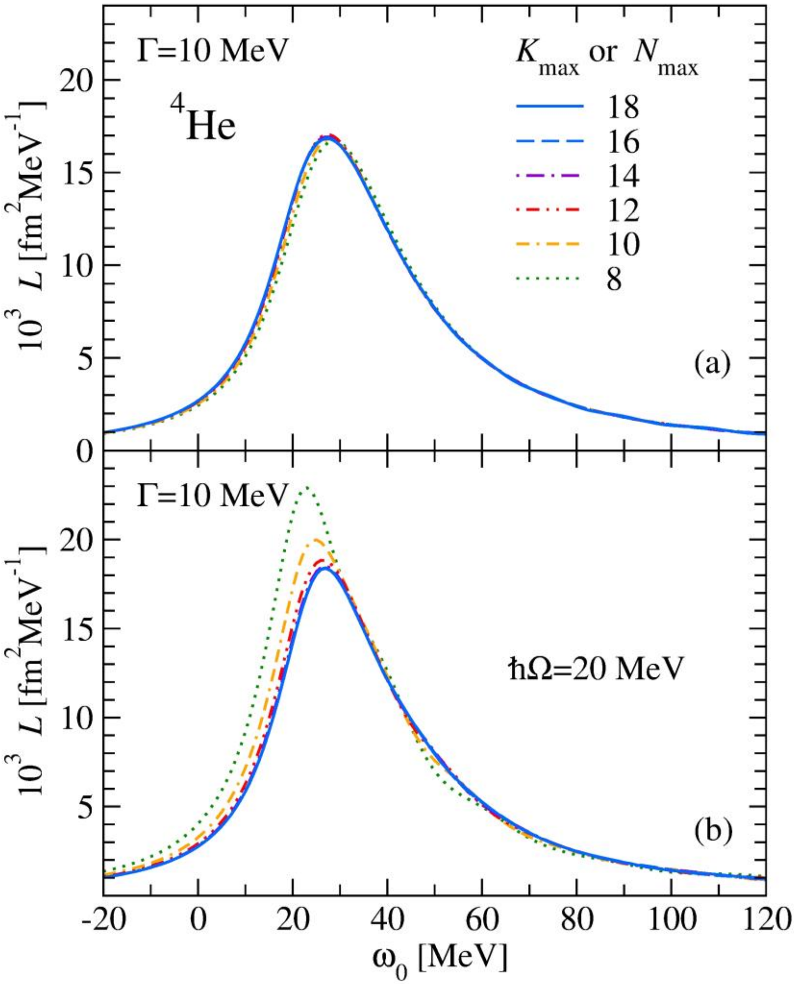
- **2NF up to N3LO**
- **CCSD approximation** $T = T_1 + T_2$

^{4}He: benchmarking the LIT+CC

Calculation of the LIT

EIHH

CC



- Calculation of the LIT curves for different model space sizes
- Convergence is approached increasing the model space

^{4}He: benchmarking the LIT+CC

Calculation of the LIT

EIHH

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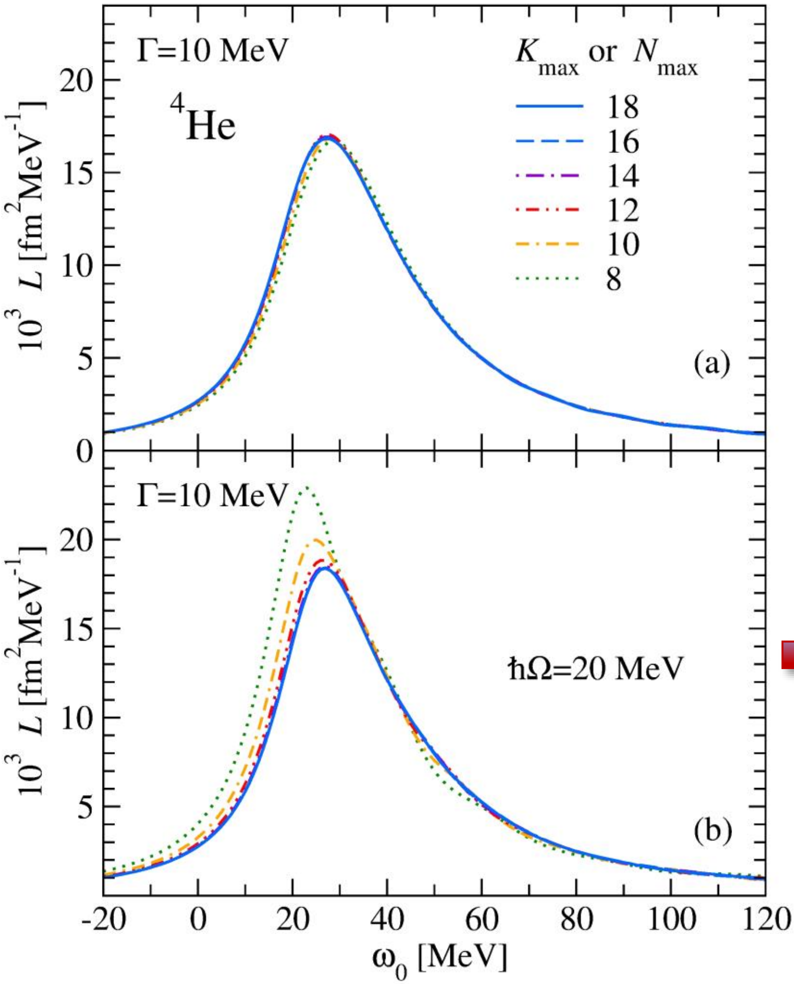

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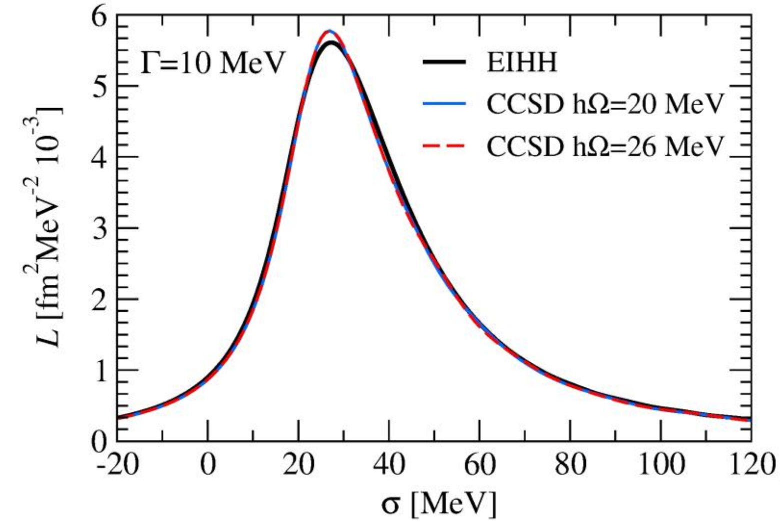


LIT Method

The inversion metod

$$L(\sigma) \xRightarrow{Inversion} R(\omega)$$

- Expand the response function on a set of known functions $\phi(\omega, \alpha)$

$$R(\omega) = \sum_{i=1}^{\nu} c_i \phi_i(\omega, \alpha)$$

- Apply the definition of LIT

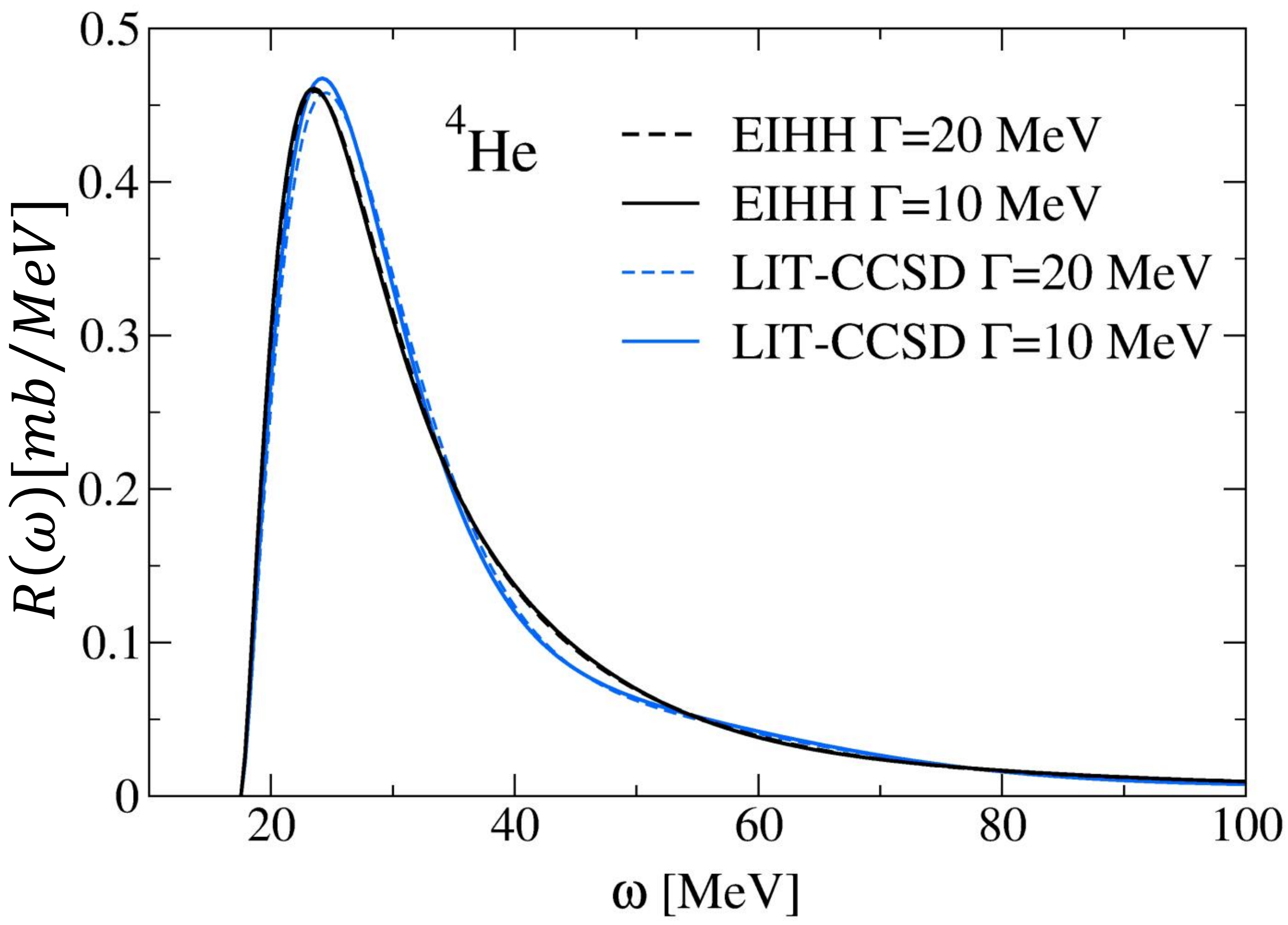
$$L(\omega_0, \Gamma) = \frac{\Gamma}{\pi} \sum_{i=1}^{\nu} c_i \int d\omega \frac{\phi_i(\omega, \alpha)}{(\omega_0 - \omega)^2 + \Gamma^2} = \sum_{i=1}^{\nu} c_i L_i(\sigma, \alpha)$$

- Evaluation of the coefficients minimizing the quantity

$$\sum_{j=1}^{\mu} \left| L(\sigma_j) - \sum_{i=1}^{\nu} c_i L_i(\sigma_j, \alpha) \right|^2$$

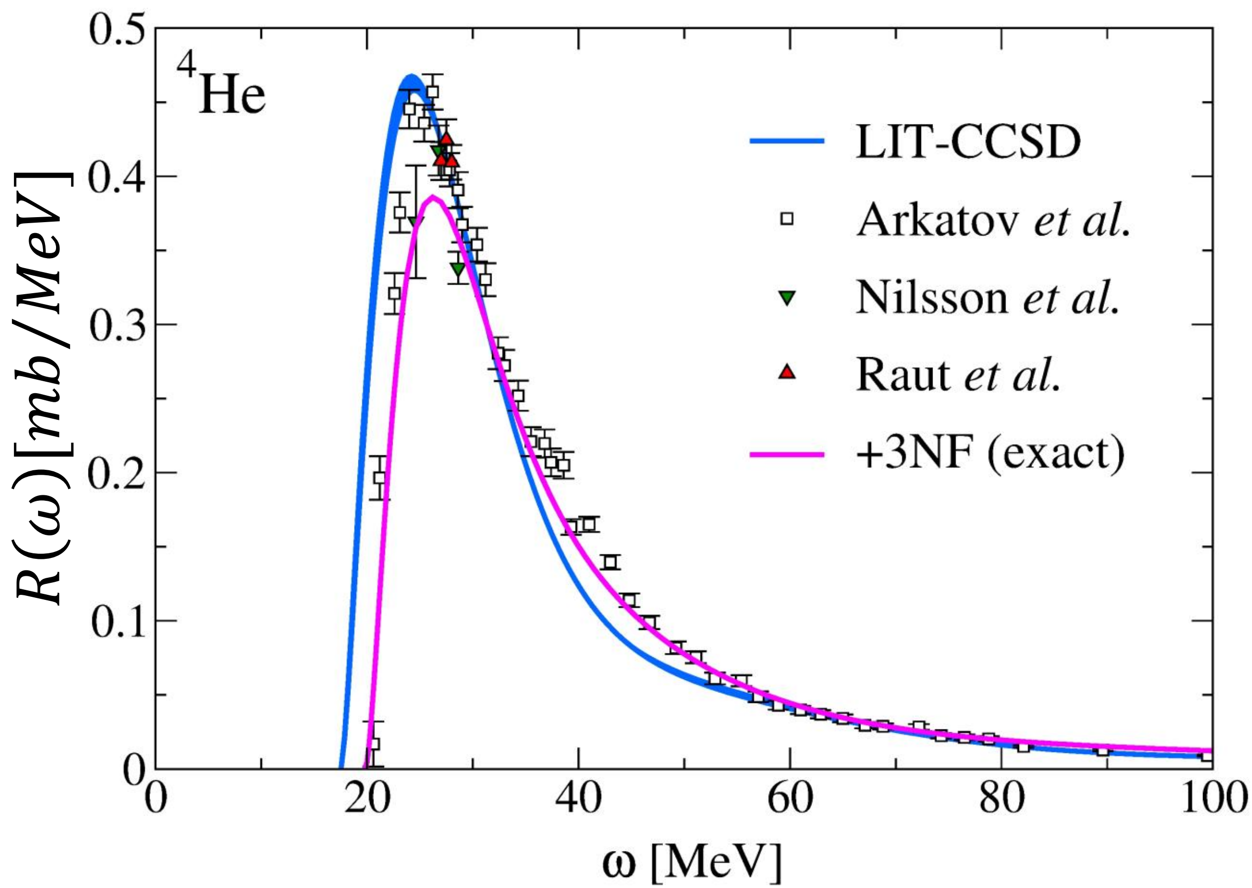
^{4}He: benchmarking the LIT+CC

Response function, comparison EIHH and CC



^{4}He: benchmarking the LIT+CC

2NF (CCSD) vs 3NF (EIHH)



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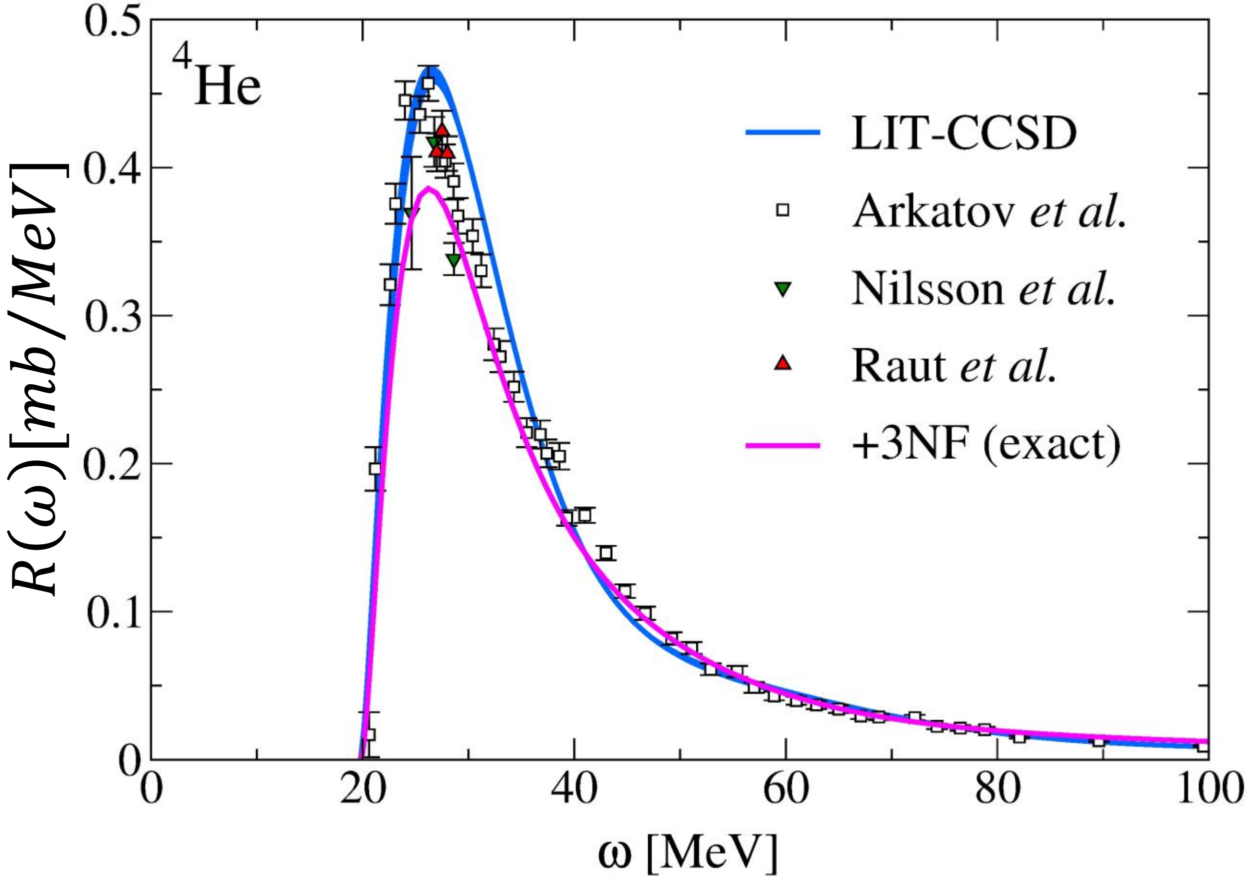
^{4}He: benchmarking the LIT+CC

Correcting for the trivial 3NF effects on the break-up threshold energy



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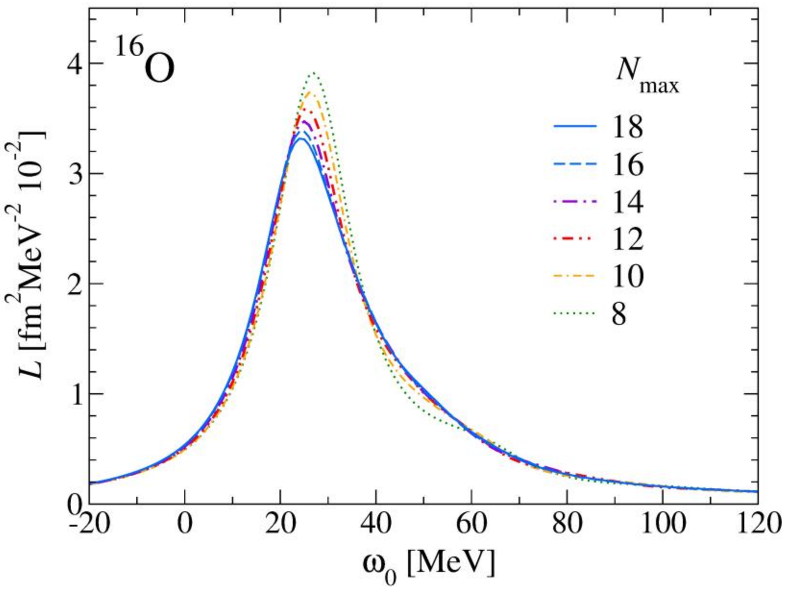
The Oxygen Isotopes - ^{16}O

LIT convergence check



The Oxygen Isotopes - ^{16}O

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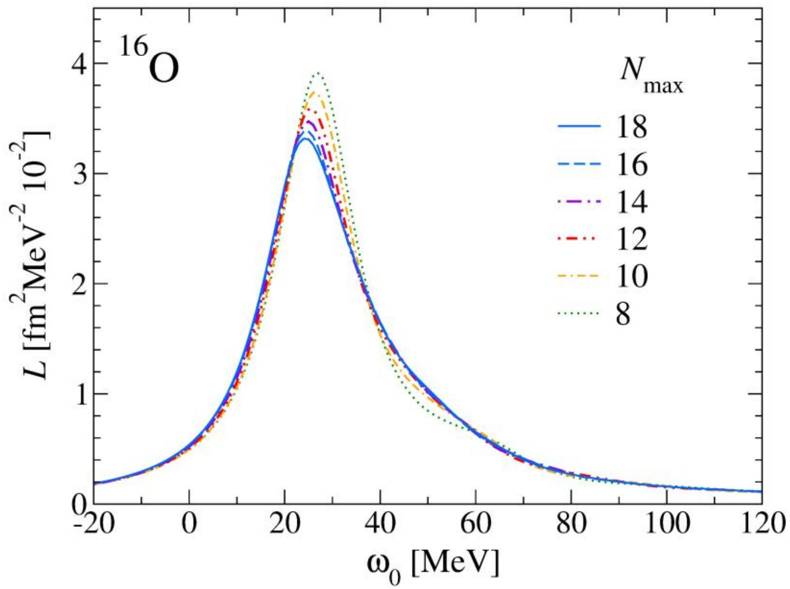
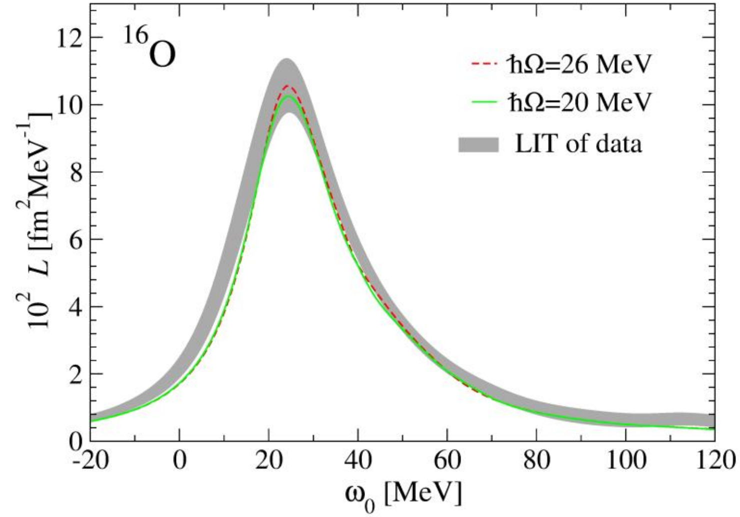


Comparison: calculated LIT vs LIT of the exp. data



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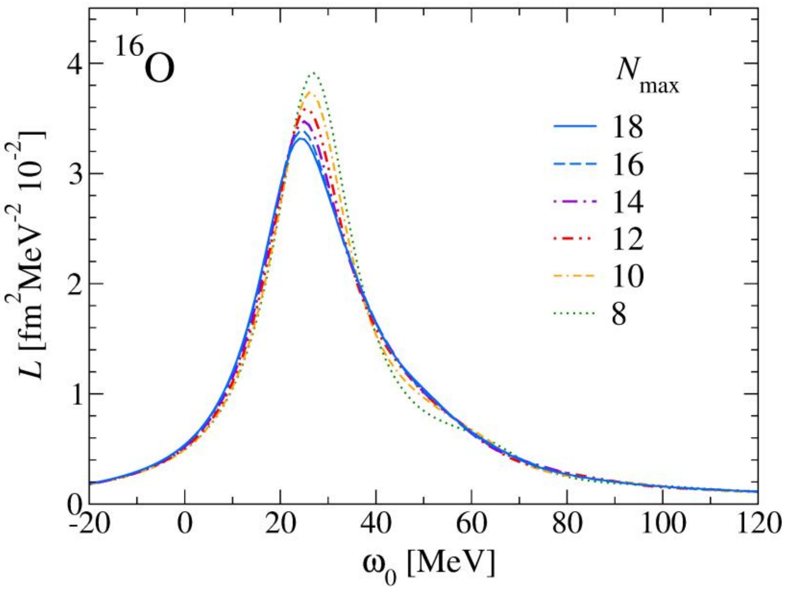

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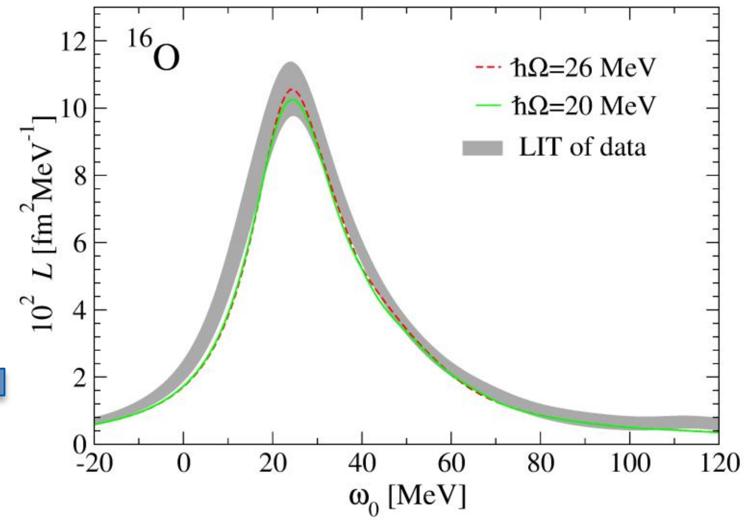
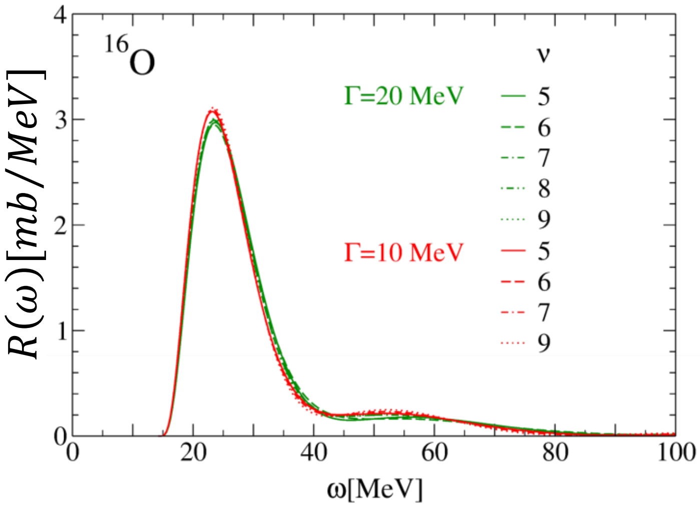
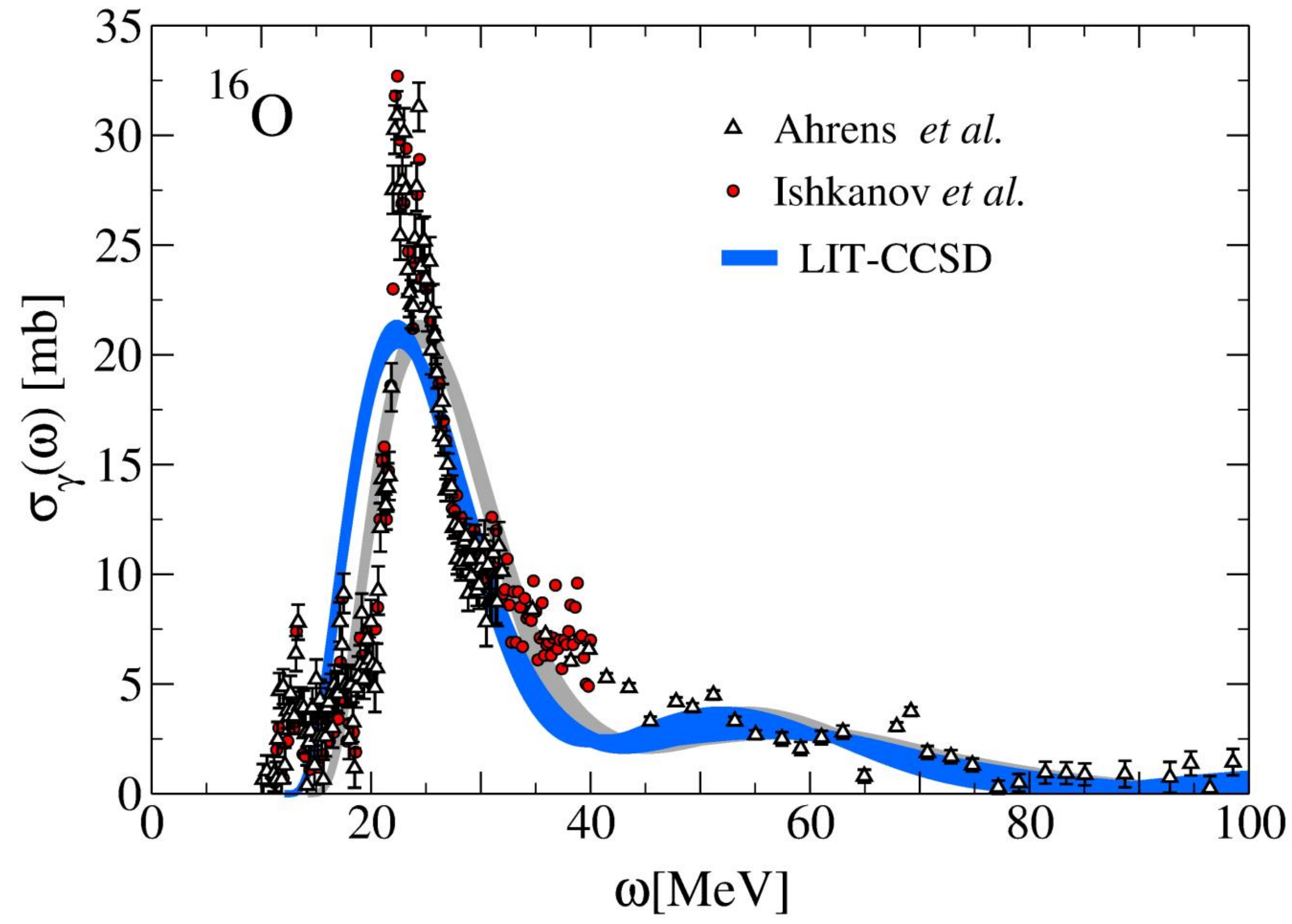


Stability check of the inversion



The Oxygen Isotopes - ^{16}O

Phys. Rev. Lett. 111, 122502 (2013)



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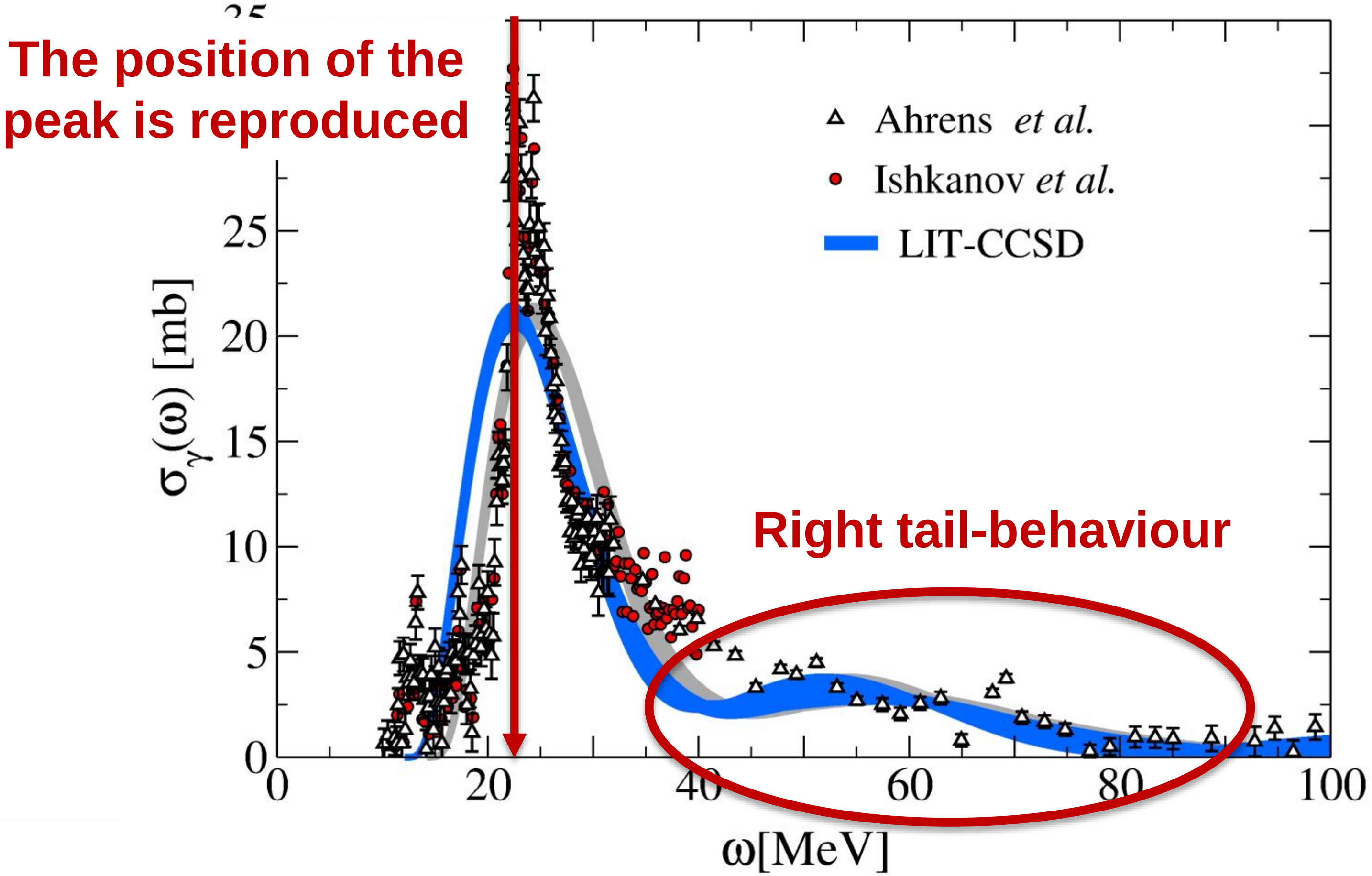
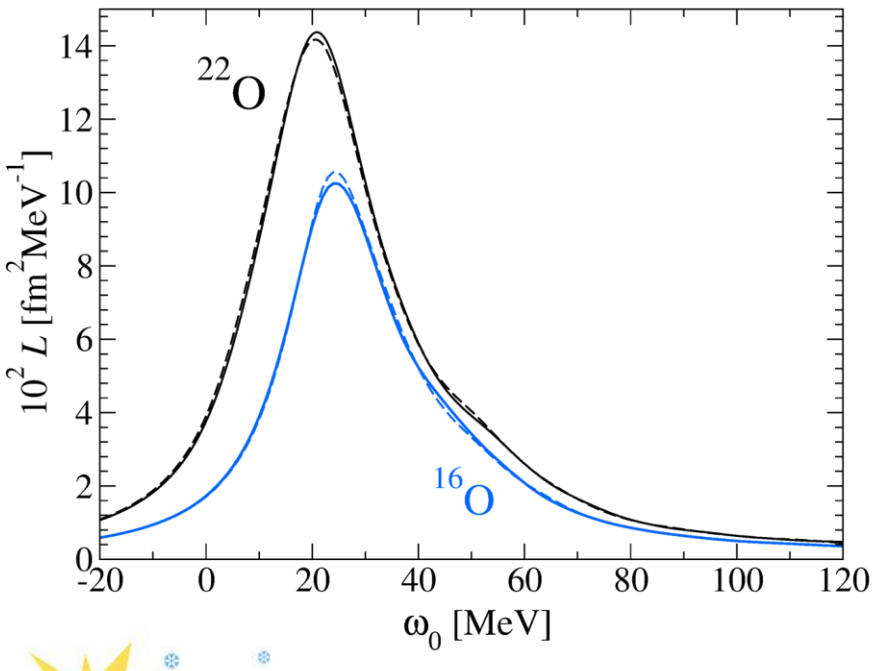


The Oxygen Isotopes - ^{22}O

LIT comparison
^{16}O **vs** ^{22}O

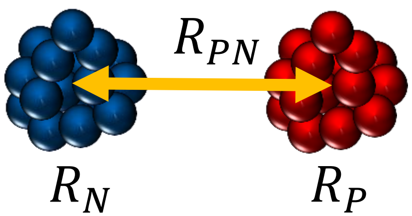


- **Different strength**

  Bremsstrahlung sumrule:

  Strength $\propto \left(\frac{NZ}{A}\right)^2 R_{PN}$

  (reproduces the ratio with an error of 10%)



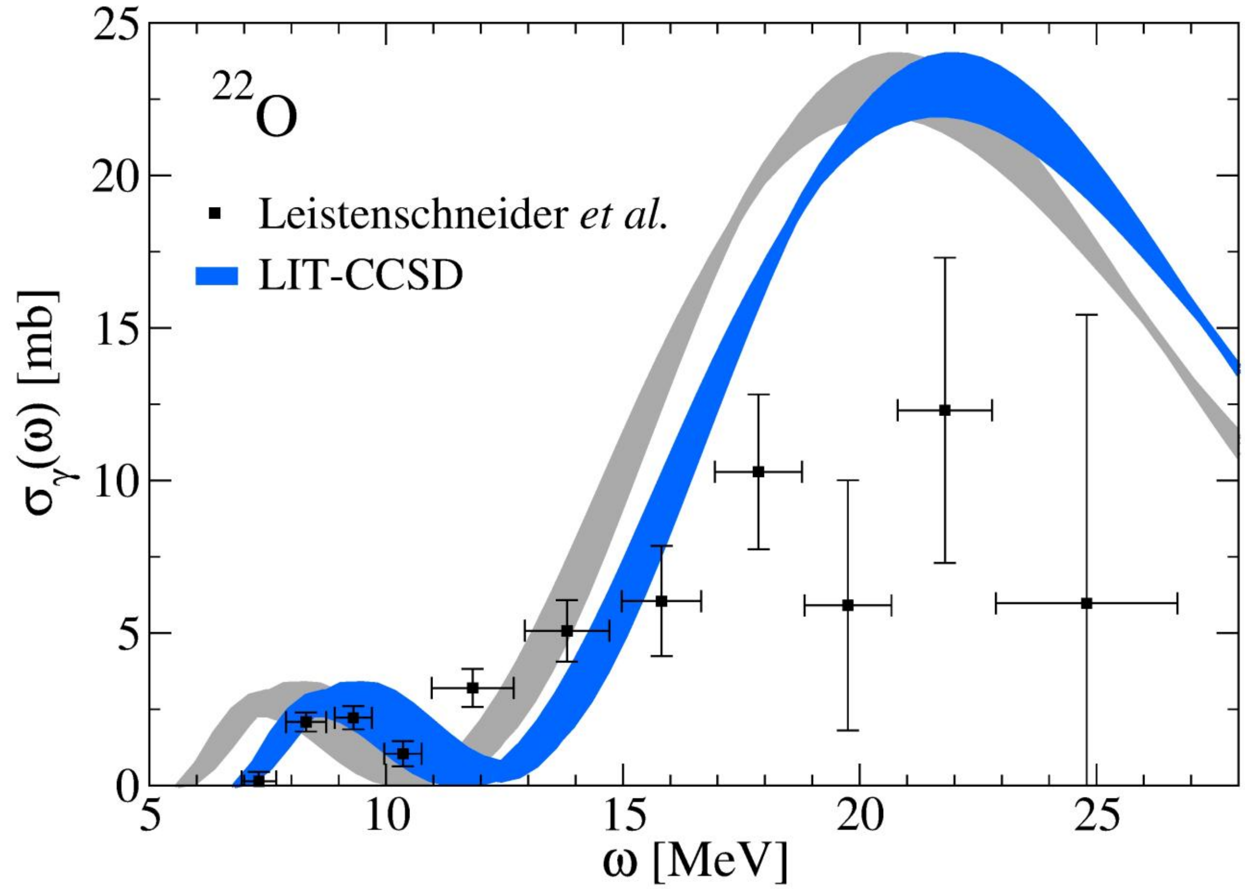
- **Peak shift**

  the shift suggests more strength in the low-energy region for ^{22}O

Phys. Rev. C 90, 064619
(December, 24th 2014)

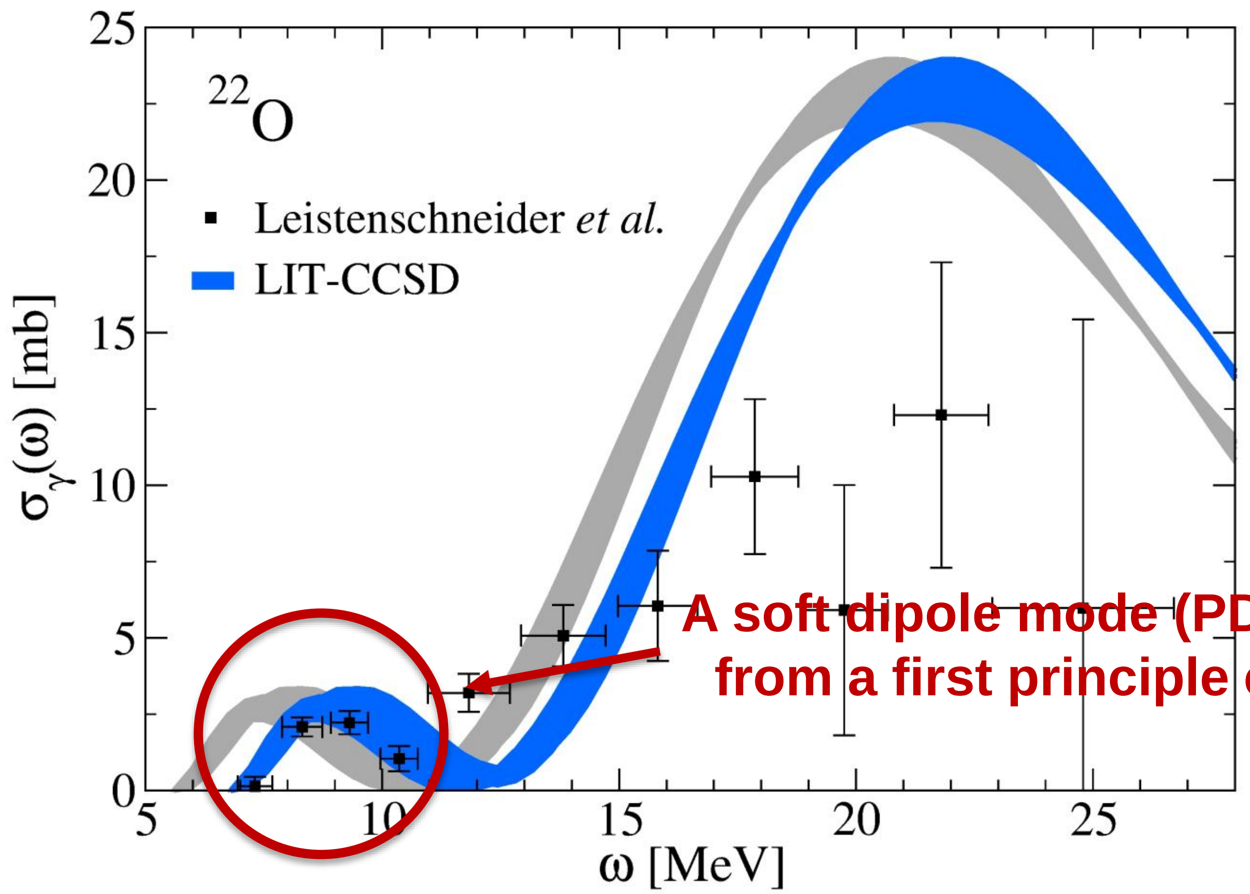
The Oxygen Isotopes - ^{22}O

Phys. Rev. C 90, 064619 (2014)



The Oxygen Isotopes - ^{22}O

Phys. Rev. C 90, 064619 (2014)



A soft dipole mode (PDR) emerges from a first principle calculation

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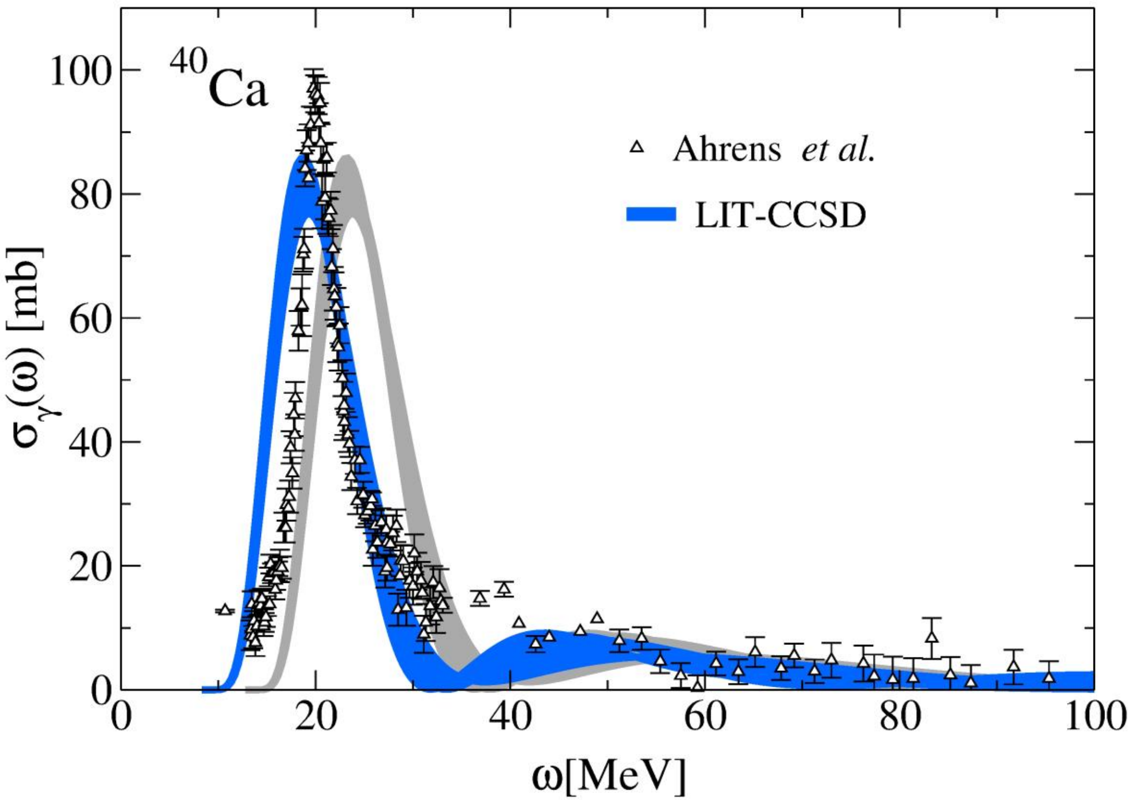
^{40}Ca - The Response Function

The presence of a GDR is predicted theoretically from first principles!



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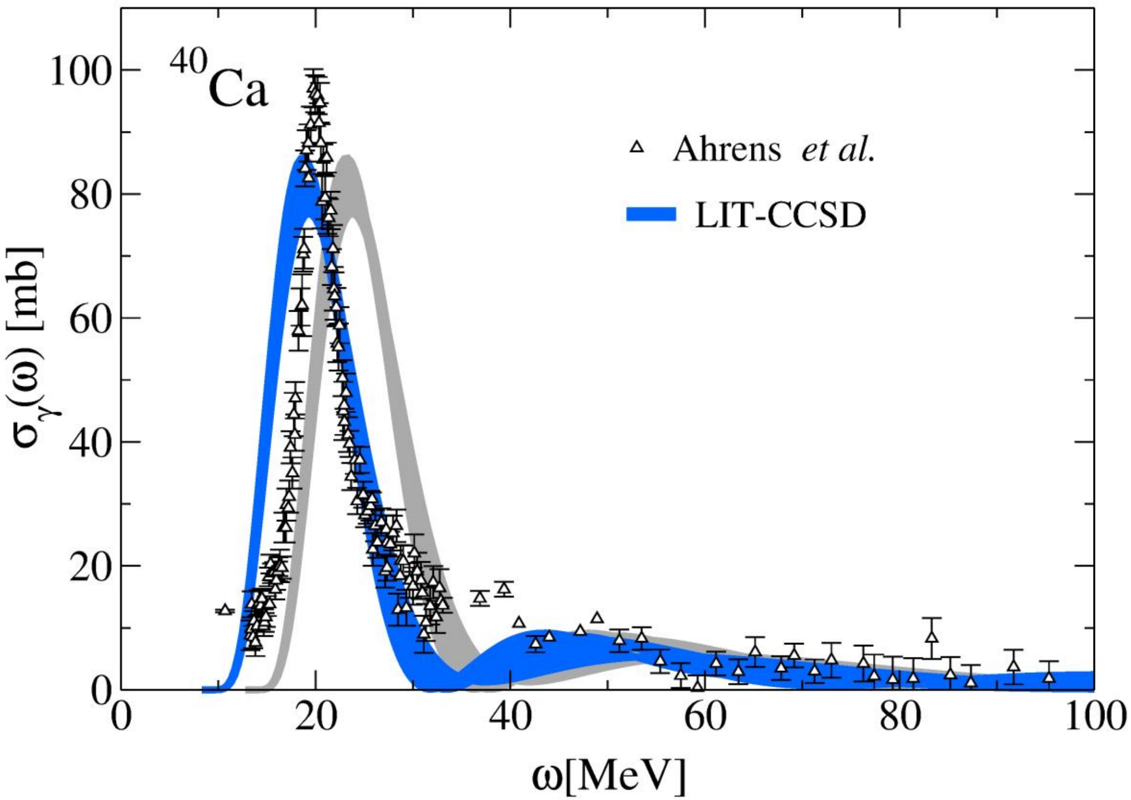


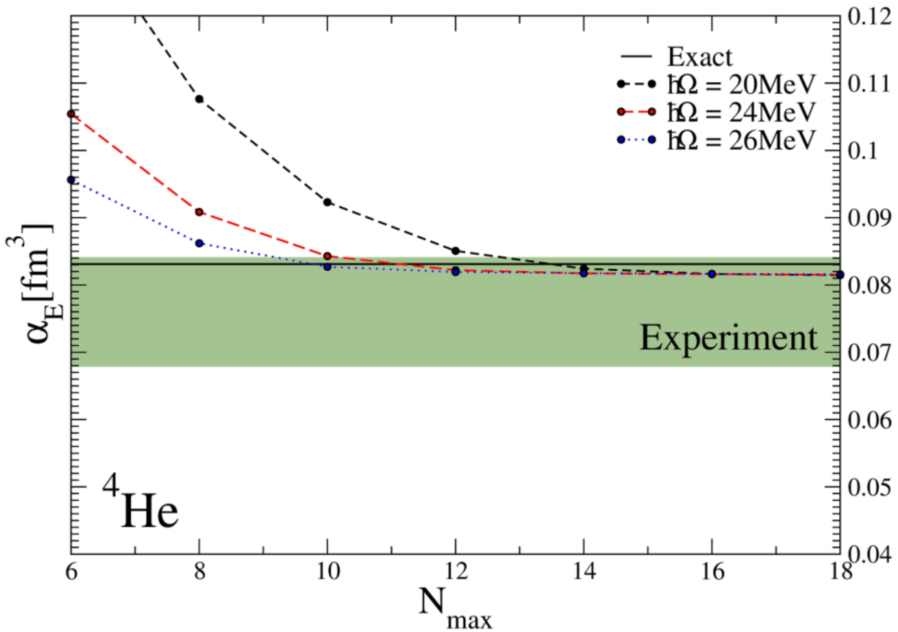
Phys. Rev. C 90, 064619 (2014)

The polarizability can be obtained integrating the cross-section:

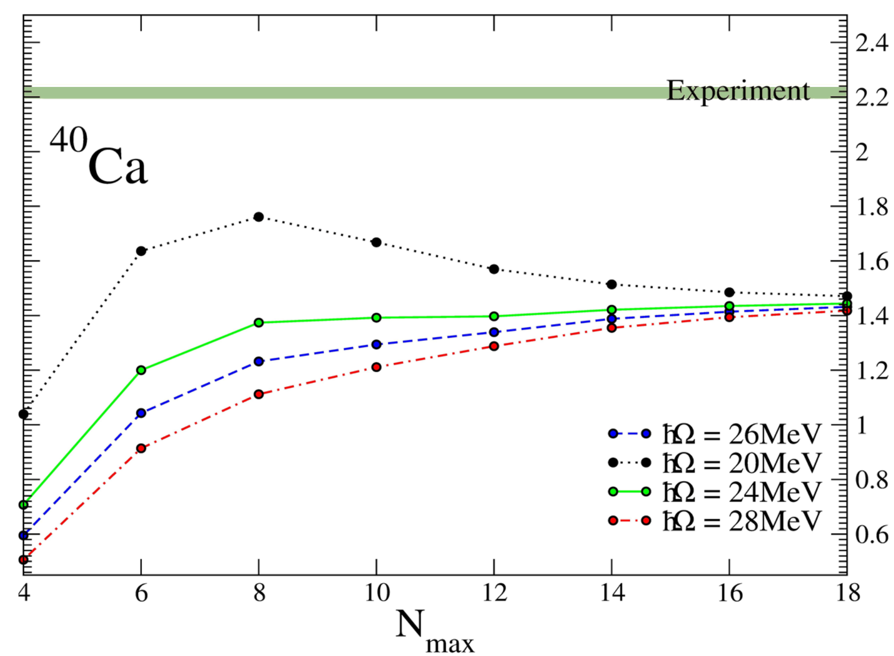
$$\alpha_E = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty} \frac{\sigma_\gamma(\omega)}{\omega^2}\, d\omega$$

^{40}Ca - The Electric Dipole Polarizability

^{4}He benchmark with LIT+EIHH



^{40}Ca polarizability from LIT+CC



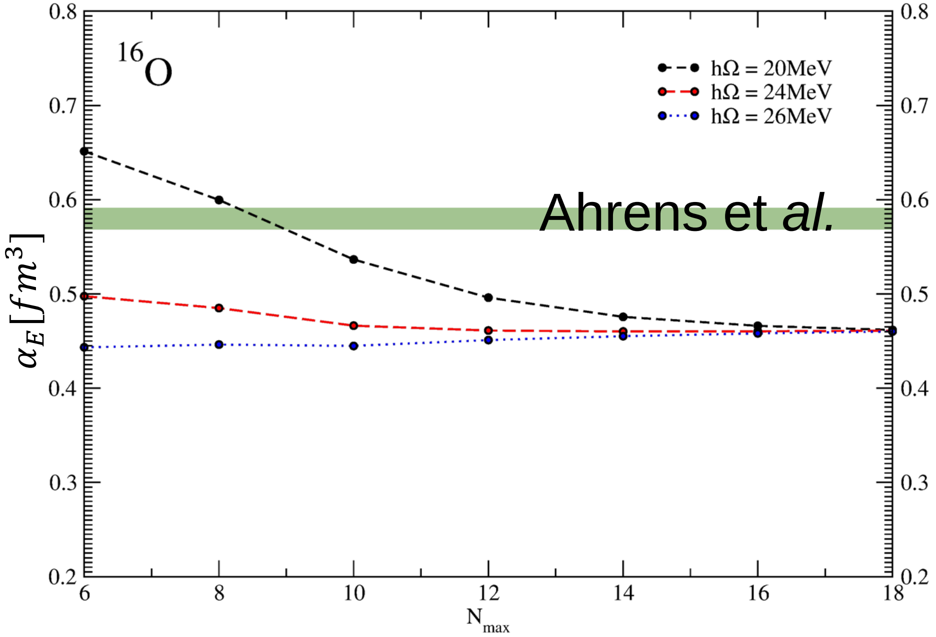
- **Good agreement in ^{4}He!**
- **Largely underestimated in ^{40}Ca!**

$$\alpha_E = 1.47 fm^3$$

$$\alpha_E^{exp} = 2.23(3) fm^3$$

M. Miorelli et *al.*, in preparation (2015)

Polarizability of ^{16}O and ^{48}Ca



M. Miorelli et *al.*, in preparation (2015)

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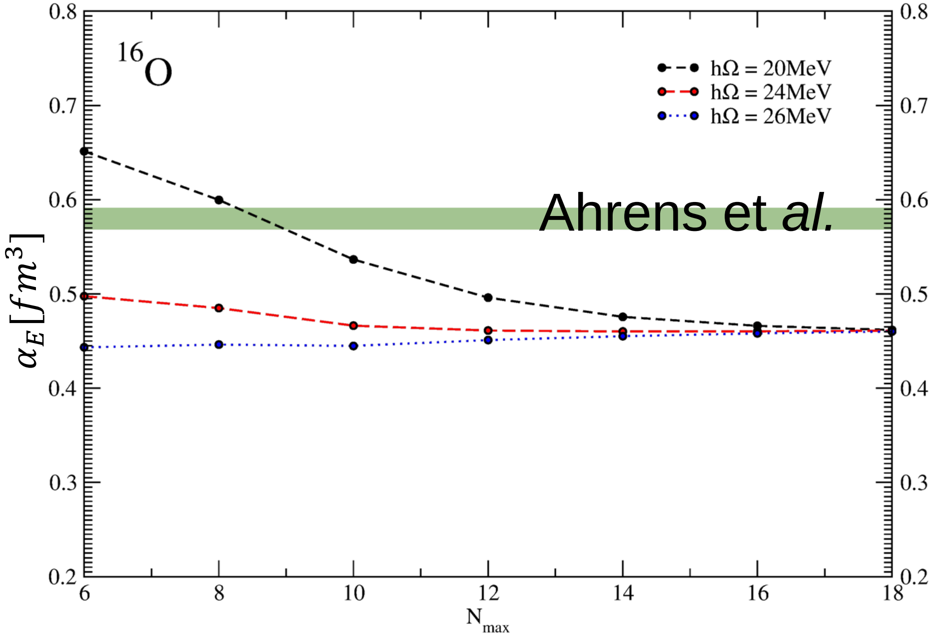
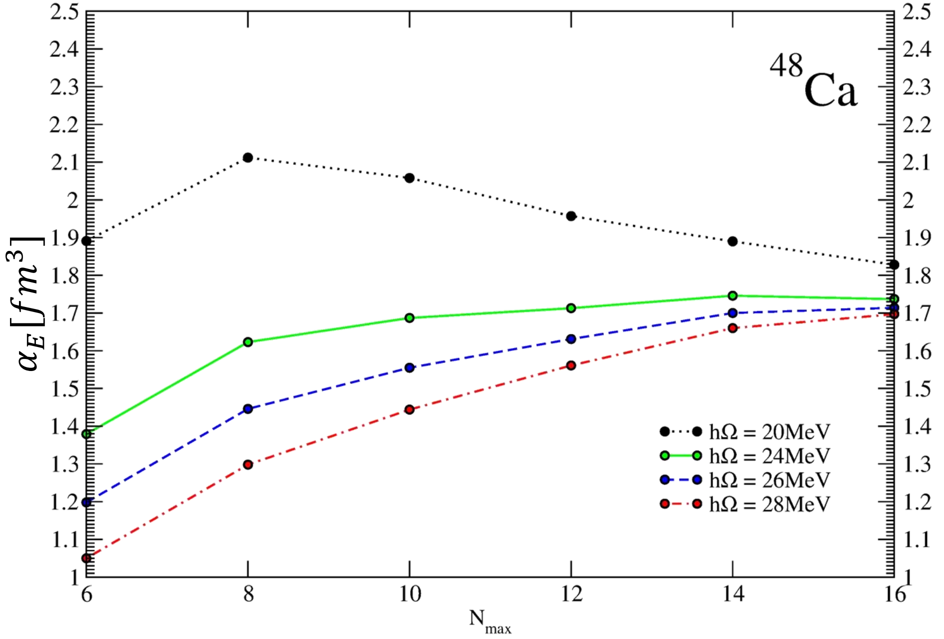
No experimental data for the polarizability of ^{48}Ca! (Ongoing experiment at RCNP(Japan) with (p,p') reactions)





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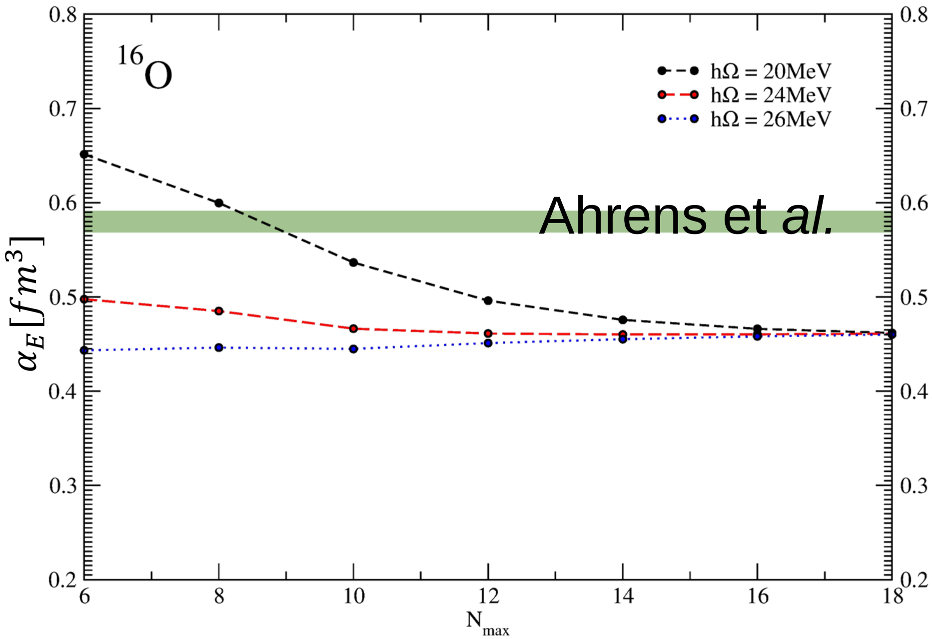
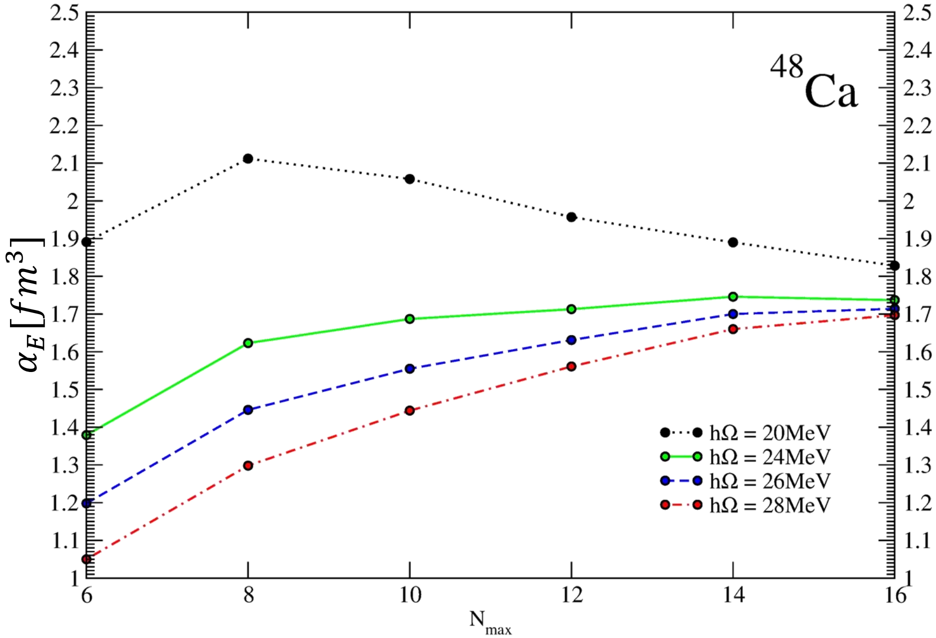




The current chiral Hamiltonians fitted on light nuclei predict too-compact medium-mass nuclei!

As a consequence we have higher dipole excitation energies, smaller radii and polarizabilities!

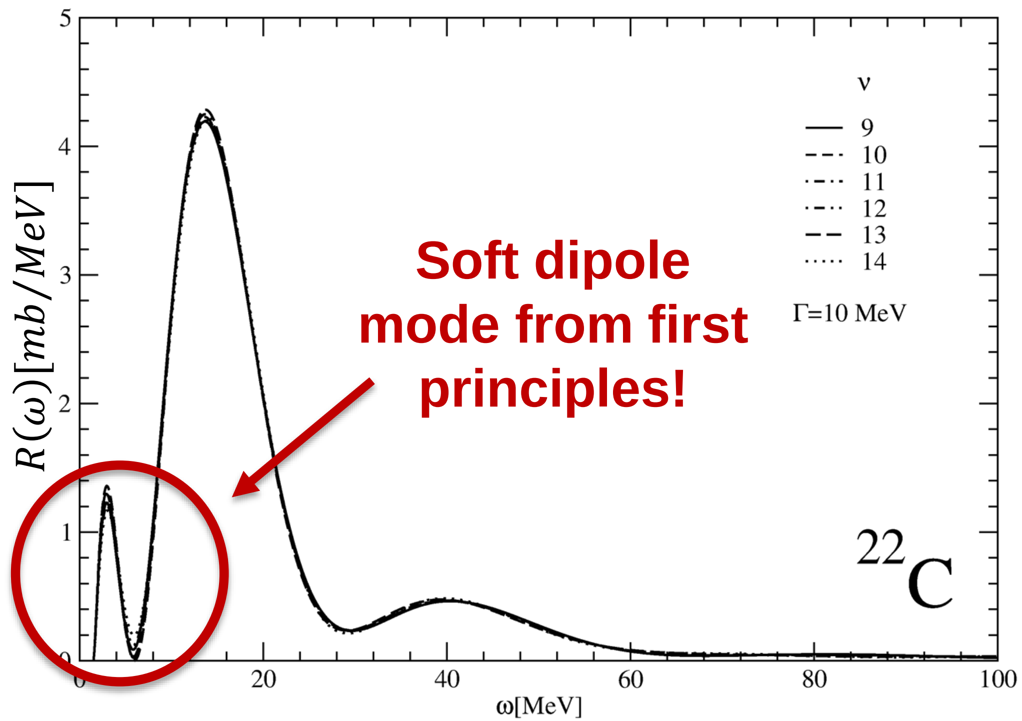
M. Miorelli et *al.*, in preparation (2015)

Outlook – PDR in other neutron-rich nuclei

Preliminary result

Very neutron-rich and unstable nuclei:

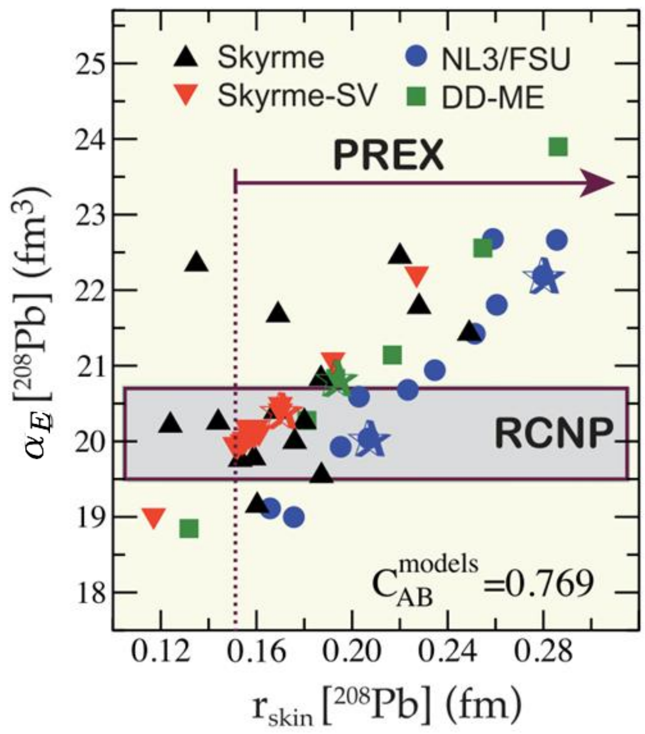
- $^{22}O: \frac{N}{Z} = 1.75$
- $^{22}C: \frac{N}{Z} = 2.67$



Can these observables be measured here at TRIUMF?

Outlook – Correlations in neutron-rich nuclei

Phys. Rev. C 85, 041302 (2012)
Energy Density Functional Theory



- α_E **is correlated to** r_{skin}

Outlook – Correlations in neutron-rich nuclei

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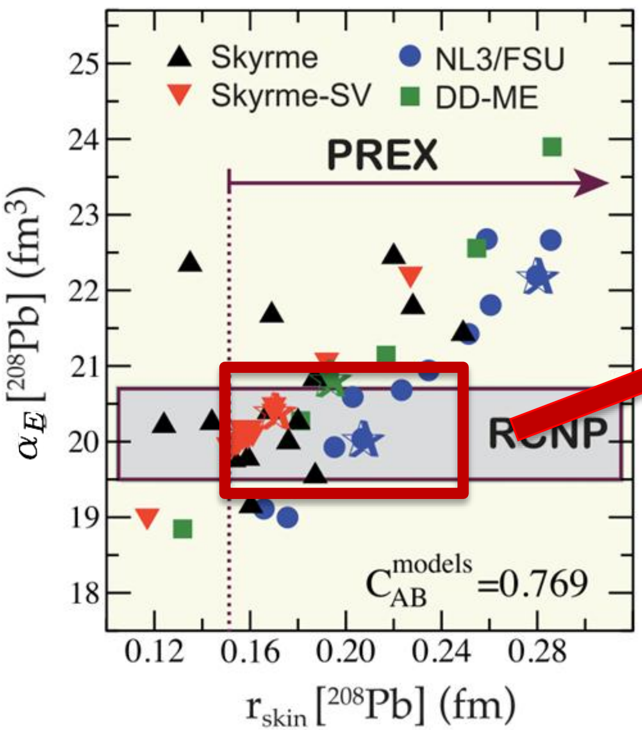
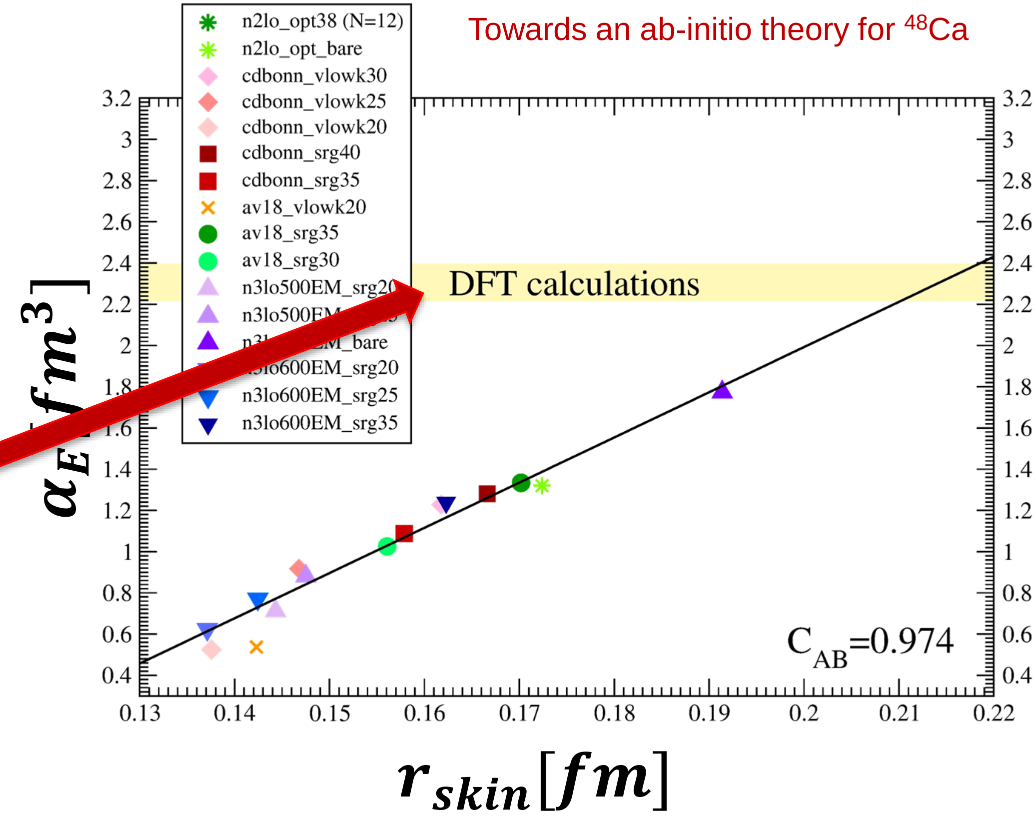


Towards an ab-initio theory for ^{48}Ca



- α_E is correlated to r_{skin}
- Future insight from RCNP and JLAB(CREX) experiments on ^{48}Ca

Outlook – Correlations in neutron-rich nuclei

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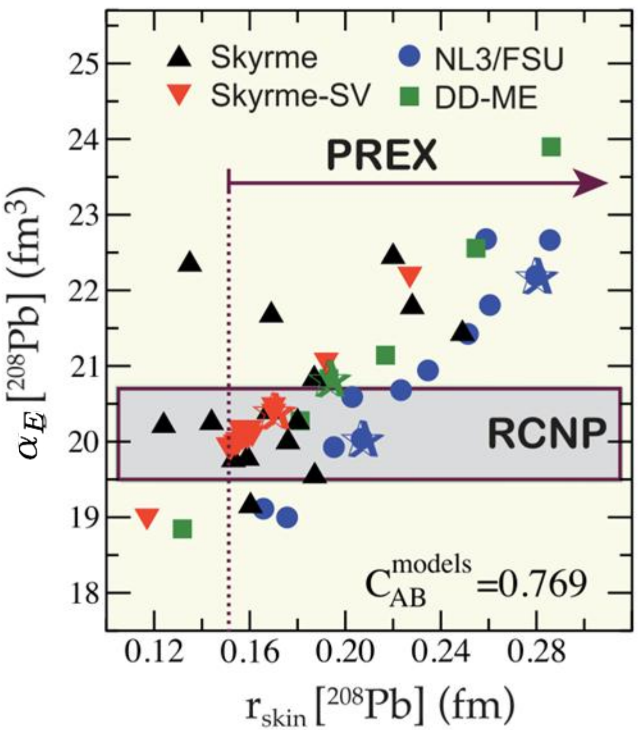

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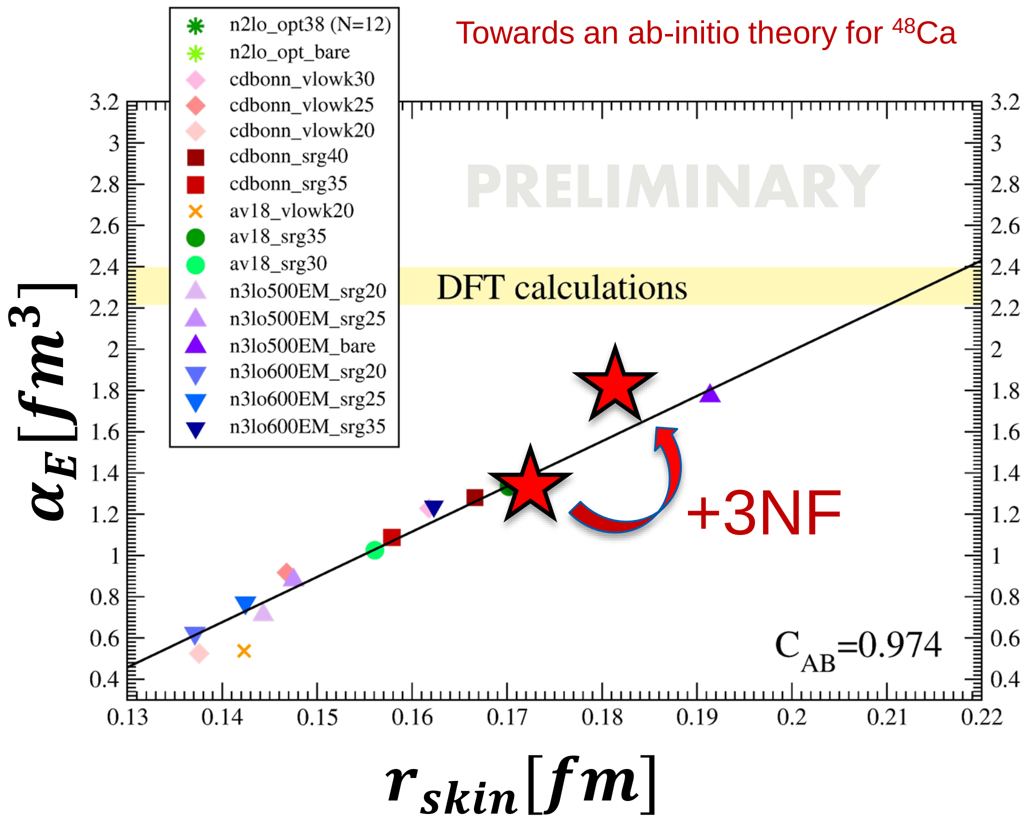

- α_E **is correlated to** r_{skin}
- **Future insight from RCNP and JLAB(CREX) experiments on** ^{48}**Ca**

Summary and Outlook

SUMMARY

- We extended the LIT method to heavier nuclei using the CC theory
- The dipole response functions of ^{16,22}O and ^{40}Ca have been computed with a first principles based method for the first time
- We observe a soft dipole mode in ^{22}O which describes quite well the GSI data
- The N3LO NN interaction overbinds medium-mass nuclei which in turn exhibit too-small charge radii and polarizabilities

OUTLOOK

- The method can be extended to other neutron-rich nuclei such as ^{22}C and ^{48}Ca
- With different interactions we can investigate the correlation between different observables (i.e. polarizability and radii)
- Future improvement of the calculations -> include 3-body correlations:
  - add 3NF
  - add triple excitations for the coupled cluster T operator

Thank you!
Merci

LIT Method + CC Theory

- **We use the exponential ansatz of CC theory with the response function from LIT method:**

LIT

$$L(\sigma) = -\frac{i}{2\Gamma}\left[\langle i|\theta^{+}|\tilde{\psi}(\sigma)\rangle - \langle i|\theta^{+}|\tilde{\psi}(\sigma^{*})\rangle\right]$$

$$(H - E_0 + \sigma)\left|\tilde{\psi}(\sigma)\right\rangle = \theta|i\rangle$$

$$H \rightarrow \bar{H} = e^{-\hat{T}}\, H_N e^{\hat{T}}$$

$$\theta \rightarrow \bar{\theta} = e^{-\hat{T}}\, \theta_N e^{\hat{T}}$$

LIT + CC

$$L(\sigma) = -\frac{i}{2\Gamma}\left[\langle 0_L|\bar{\theta}^{+}|\tilde{\psi}_R(\sigma)\rangle - \langle 0_L|\,\bar{\theta}^{+}|\,\tilde{\psi}_R(\sigma^{*})\rangle\right]$$

$$(\bar{H} - \Delta E_0 + \sigma)\left|\tilde{\psi}_R(\sigma)\right\rangle = \bar{\theta}|0_R\rangle$$

LIT Method + CC Theory

Coupled Cluster Equation of Motion (CC-EOM) method

$$\boxed{L(\sigma) = -\frac{i}{2\Gamma}\left[\langle 0_L|\bar{\theta}^{+}|\tilde{\psi}_R(\sigma)\rangle - \langle 0_L|\,\bar{\theta}^{+}|\,\tilde{\psi}_R(\sigma^{*})\rangle\right]}$$

$$\boxed{\left|\tilde{\psi}_R(\sigma)\right\rangle = R(\sigma)|0_R\rangle}$$

$$\boxed{R(\sigma) = r_0(\sigma) + \sum_{a,i} r_i^a(\sigma)\{c_a^{+}c_i\} + \frac{1}{4}\sum_{ab,ij} r_{ij}^{ab}(\sigma)\left\{c_a^{+}c_i c_b^{+}c_j\right\} + \cdots}$$

$$\boxed{L(\sigma) = -\frac{i}{2\Gamma}\langle 0_L\big|\bar{\theta}^{+}[R(\sigma) - R(\sigma^{*})]\big|0_R\rangle}$$

$$\boxed{R(\sigma)} \longleftarrow \boxed{(\bar{H}R(\sigma) - R(\sigma)\bar{H})|0_R\rangle = -\sigma R(\sigma)|0_R\rangle + \bar{\theta}|0_R\rangle}$$

Lanczos method

$$L(\sigma) = -\frac{i}{2\Gamma}\langle 0_L|\hat{\bar{\theta}}^\dagger(\hat{R}(\sigma) - \hat{R}(\sigma^*))|0_R\rangle$$

$$\boxed{L(\sigma) = -\frac{i}{2\Gamma}((\mathbf{S}^L)^\dagger \cdot \mathbf{S}^R)\mathbf{w}_0^T\left\{(\mathbb{M}+\mathbb{I}\sigma)^{-1} - (\mathbb{M}+\mathbb{I}\sigma^*)^{-1}\right\}\mathbf{v}_0}$$

$$\mathbf{v}_0 = \frac{\mathbf{S}^R}{\sqrt{(\mathbf{S}^L)^\dagger \cdot \mathbf{S}^R}}$$

$$\mathbf{w}_0^T = \frac{(\mathbf{S}^L)^\dagger}{\sqrt{(\mathbf{S}^L)^\dagger \cdot \mathbf{S}^R}}$$

$$\mathbb{M} = \begin{pmatrix} 0 & \langle 0_R|[\hat{\bar{H}}, N[a_d^\dagger a_k]]|0_R\rangle & \langle 0_R|[\hat{\bar{H}}, N[c_d^\dagger c_k c_e^\dagger c_l]]|0_R\rangle & \cdots \\ 0 & \langle \phi_i^a||[\hat{\bar{H}}, N[c_d^\dagger c_k]]|0_R\rangle & \langle \phi_i^a|[\hat{\bar{H}}, N[c_d^\dagger c_k c_e^\dagger c_l]]|0_R\rangle & \cdots \\ 0 & \langle \phi_{ij}^{ab}|[\hat{\bar{H}}, N[c_d^\dagger c_k]]|0_R\rangle & \langle \phi_{ij}^{ab}|[\hat{\bar{H}}, N[c_d^\dagger c_k c_e^\dagger c_l]]|0_R\rangle & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$\hat{R}(\sigma^*) = r_0(\sigma^*) + \sum_{i,a} r_i^a(\sigma^*)N[c_a^\dagger c_i] + \sum_{ij,ab} r_{ij}^{ab}(\sigma^*)N[c_a^\dagger c_i c_b^\dagger c_j] + \ldots$$

$$(\mathbf{S}^L)^\dagger = \left(\langle 0_L|\hat{\bar{\theta}}^\dagger|0_R\rangle, \langle 0_L|\hat{\bar{\theta}}^\dagger|\phi_i^a\rangle, \langle 0_L|\hat{\bar{\theta}}^\dagger|\phi_{ij}^{ab}\rangle, \ldots\right)$$

Lanczos method

- Use Lanczos do diagonalize (avoid full direct diagonalization)

$$L(\sigma) = -\frac{i}{2\Gamma}((\mathbf{S}^L)^\dagger \cdot \mathbf{S}^R)\,(X_0(\sigma) - X_0(\sigma^*))$$

$$X_0(\sigma) = \cfrac{1}{(a_0+\sigma) - \cfrac{b_1^2}{(a_1+\sigma) - \cfrac{b_2^2}{(a_2+\sigma) - \cfrac{b_3^2}{\cdots}}}}$$

- The Electric Dipole Polarizability is obtained in a similar way directly from the Lanczos coefficients of the Lanczos tridiagonal matrix

$$\begin{aligned}
\alpha_E &= 2\alpha \lim_{\sigma_R\to 0} \int \frac{R(\omega)d\omega}{\omega+\sigma_R} \\
&= 2\alpha \lim_{\sigma_R\to 0} \sum\!\!\!\!\!\!\int_f \frac{\langle 0_L|e^{-\hat{T}}\hat{\theta}_N^\dagger|f\rangle\langle f|\hat{\theta}_N e^{\hat{T}}|0_R\rangle}{\Delta E_f - \Delta E_0 + \sigma_R} \\
&= 2\alpha \lim_{\sigma_R\to 0} \langle 0_L|\hat{\bar{\theta}}^\dagger[\hat{\bar{H}} - \Delta E_0 + \sigma_R]^{-1}\hat{\bar{\theta}}|0_R\rangle .
\end{aligned}$$

$$\alpha_E = 2\alpha((\mathbf{S}^L)^\dagger \cdot \mathbf{S}^R) \lim_{\sigma_R\to 0} X_0(\sigma_R)$$