Brownian motion: From the reality of atoms to the inner life of cells

Einstein Centenary Series
Fall 2015

John Bechhoefer
Dept. of Physics

Albert Einstein
Bern Patent Office, 1905

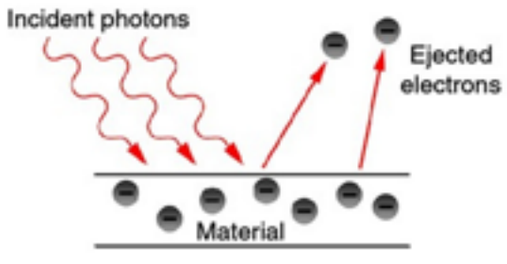
SFU SIMON FRASER UNIVERSITY
ENGAGING THE WORLD

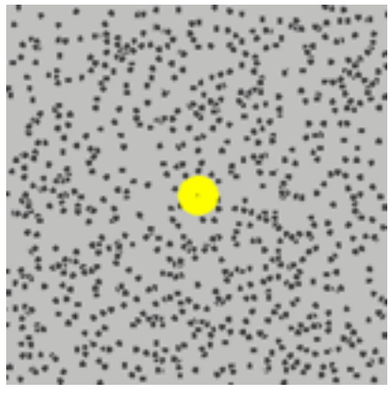
1905: Einstein's "Annus Mirabilis" *

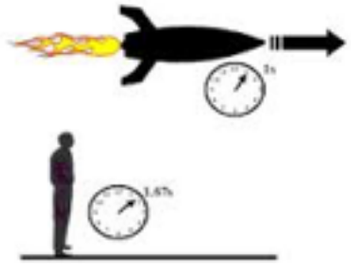
* Einstein's very good year

1905 → 1906

March	May	June	September
photons	Brownian motion	Special relativity	$E=mc^2$
"On a Heuristic Point of View about the Creation and Conversion of Light"	"On the Movement of Small Particles Required by the Molecular-Kinetic Theory of Heat"	"On the Electrodynamics of Moving Bodies"	"Does the Inertia of a Body Depend on Its Energy Content?"







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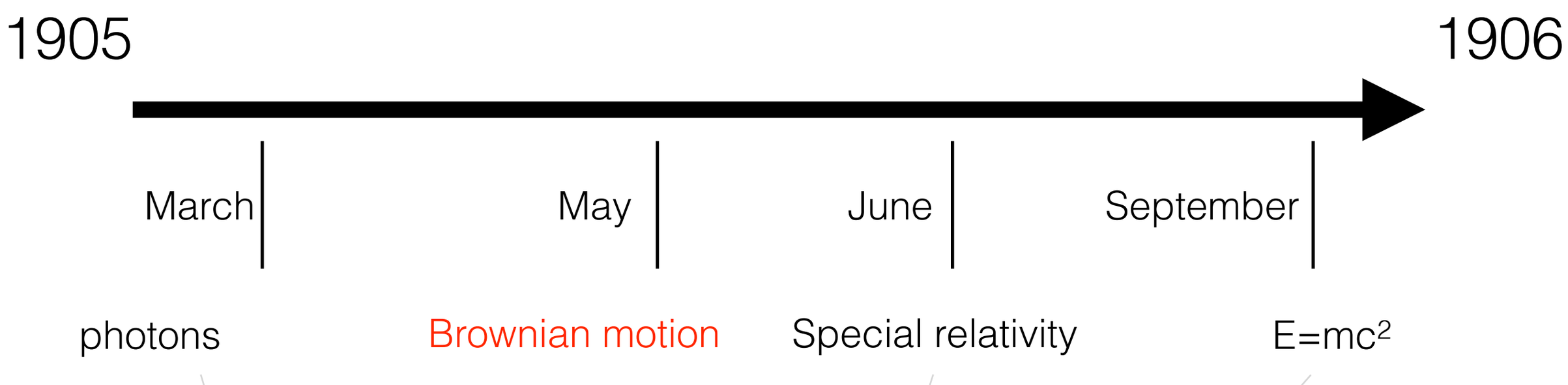

Revolutionary ideas!

1905: Einstein's "Annus Mirabilis" *

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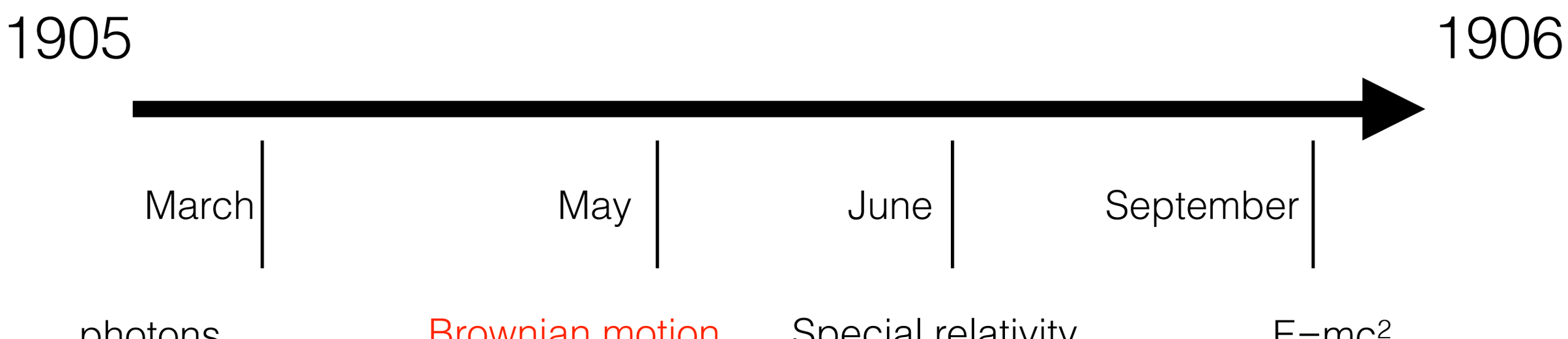

End of an old story: Do atoms exist?

1905: Einstein's "Annus Mirabilis" *

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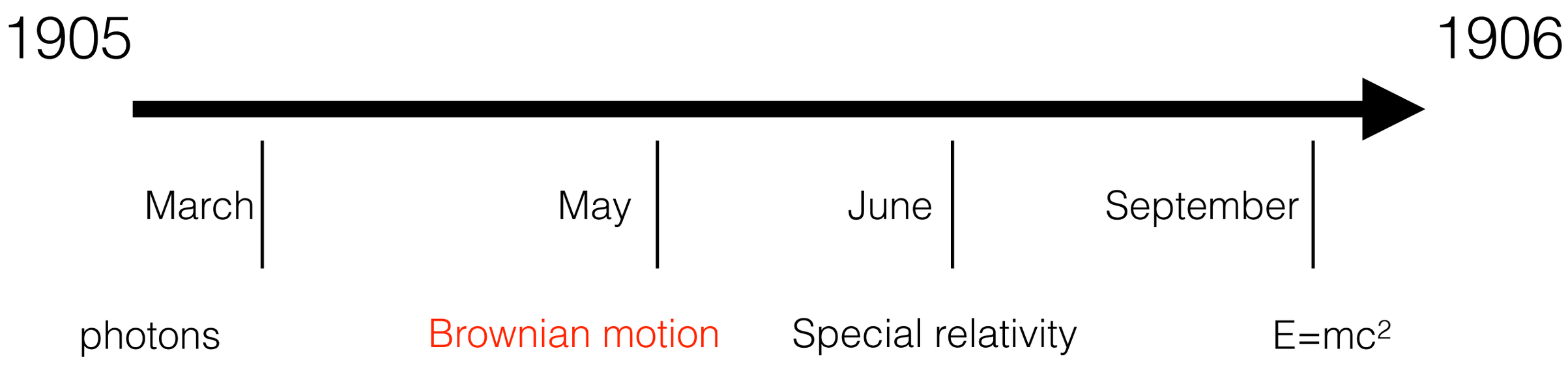


End of an old story: Do atoms exist?

Beginning of a new story: Fluctuations have meaning

1905: Einstein's "Annus Mirabilis" *

* Einstein's very good year

In April 1906, Einstein was promoted from **technischer Experte III** to **technischer Experte II**.

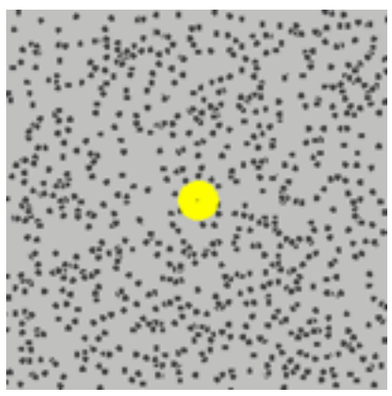
1905: Einstein's "Annus Mirabilis" *

* Einstein's very good year

1905 — March — May — June — September — 1906

Brownian motion

"On the Movement of Small Particles Required by the Molecular-Kinetic Theory of Heat"



This paper was important:

1) It convinced the skeptics that atoms really do exist.

2) It showed that noise can be useful.

Do atoms exist? The Classical "Atomists"

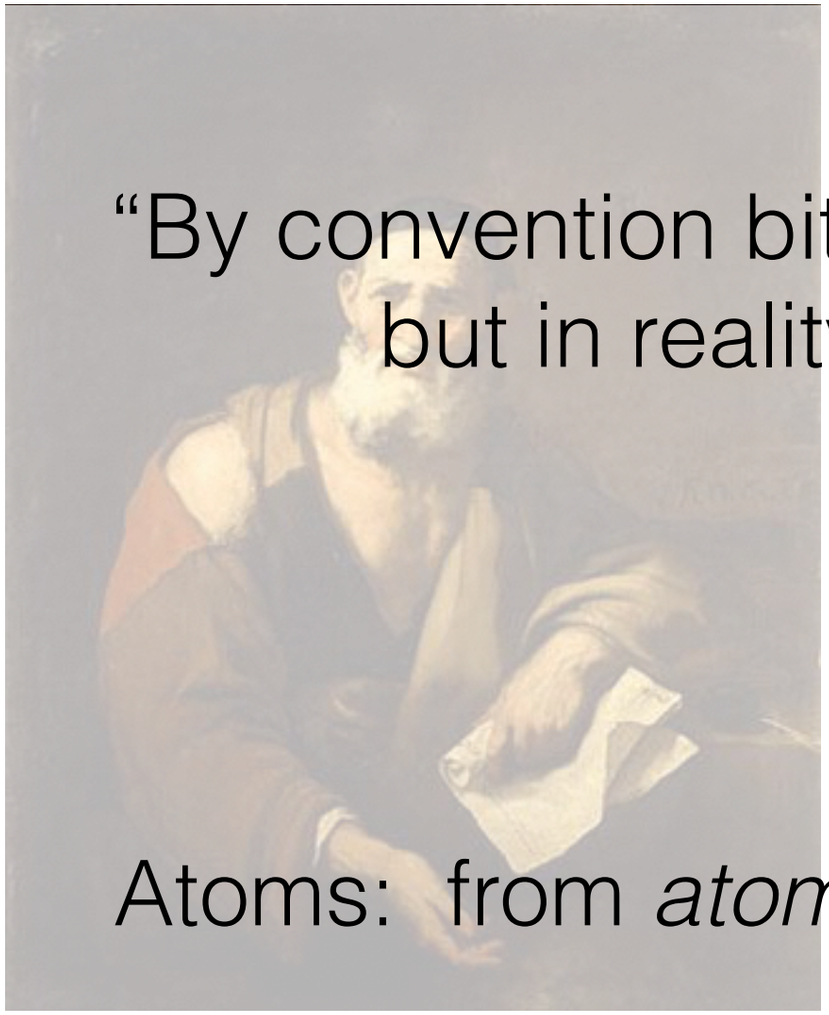
Leucippus, 5th c. BC

Hendrick ter Brugghen, 1628

Democritus, his student,
the laughing philosopher

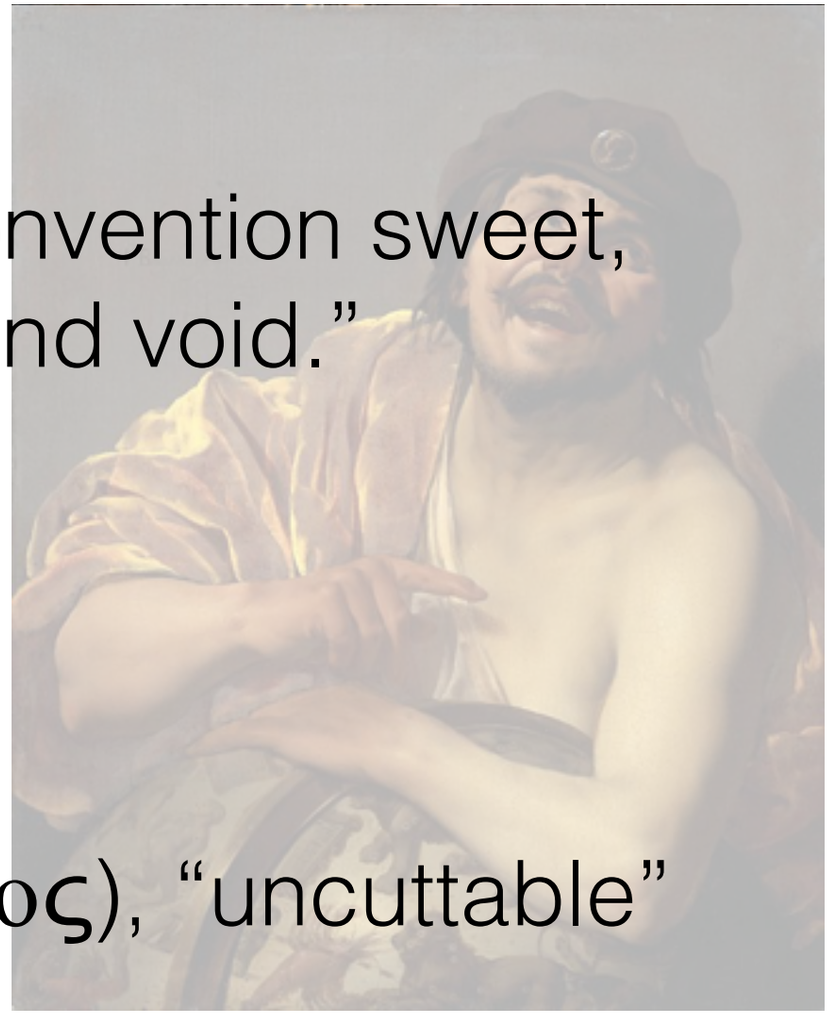
pre-Socratics (not from Athens)

Do atoms exist? The Classical "Atomists"



"By convention bitter, by convention sweet,
but in reality atoms and void."

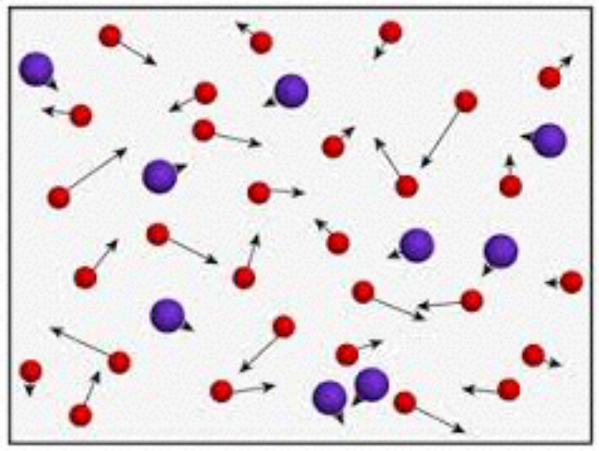
Atoms: from *atomos* (ἄτομος), "uncuttable"



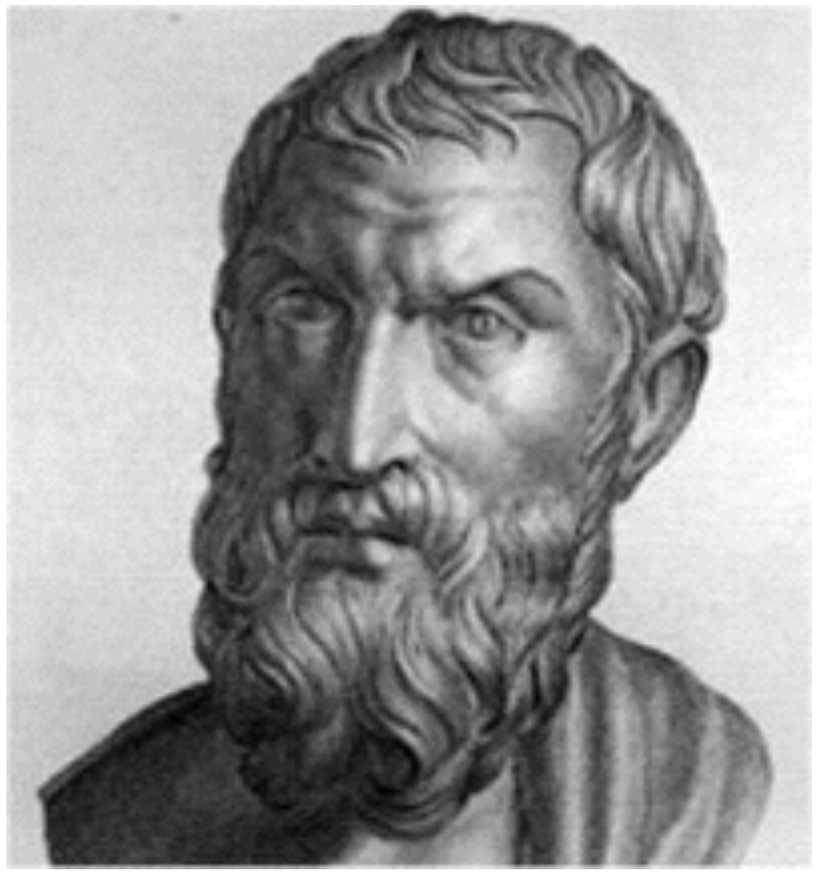
Hendrick ter Brugghen, 1628

Leucippus, 5th c. BC

Democritus, his student



Atoms in motion

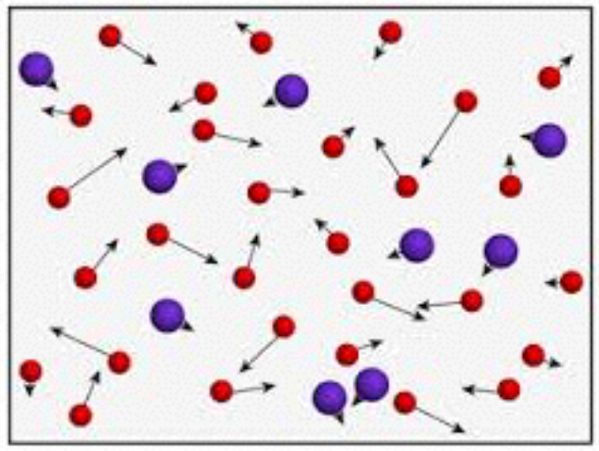


Lucretius, 99 BC - 55 BC

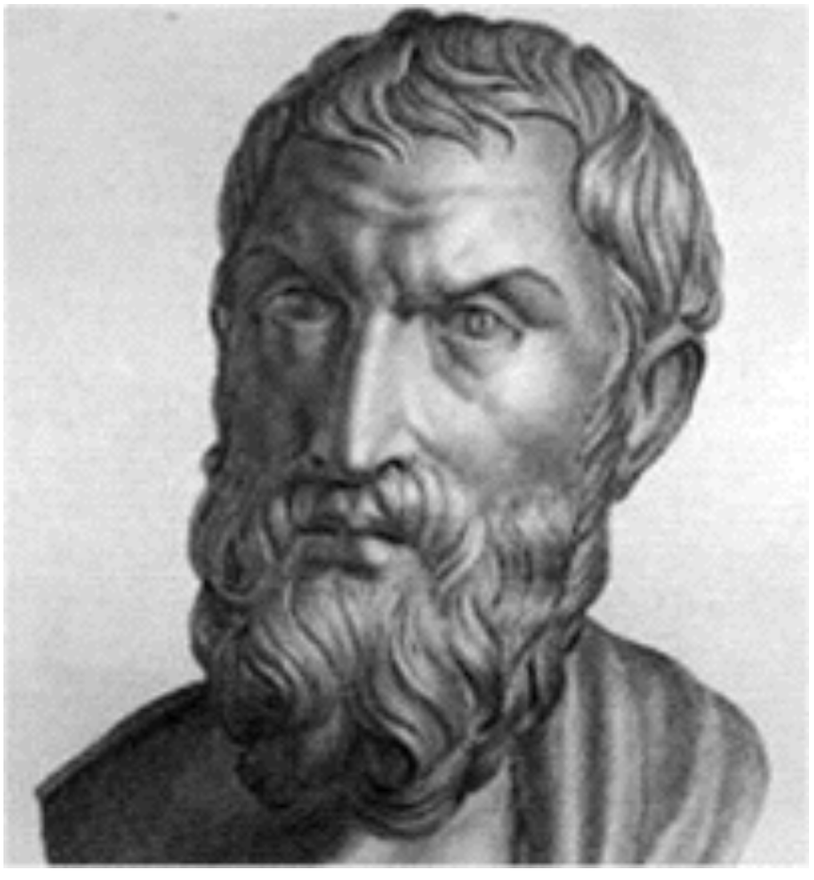
"If you think that the first-beginnings of things can stay still, and by staying still beget new movements in things, you stray very far away from true reasoning. For since they wander through the void, it must be that all the first-beginnings of things move either by their own weight or sometimes by the blow of another."

On the Nature of Things

Atomism: A philosophical / theological position backed up by qualitative observations that, in retrospect, were either off base or just lucky guesses → pre-scientific.



Atoms in motion



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On the Nature of Things

Why <u>accept</u> the atomic hypothesis in 1900?

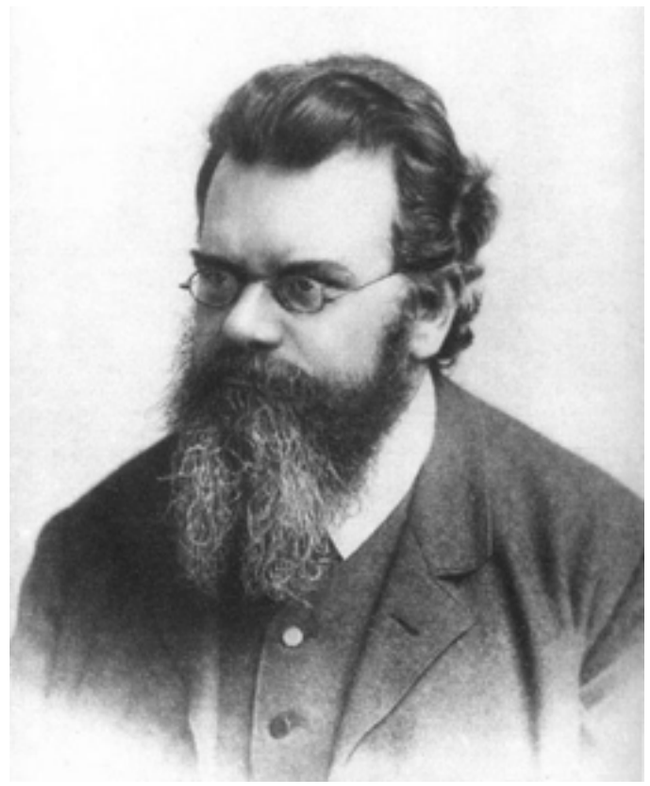
Chemistry

John Dalton, 1766-1844

chemical reactions
atomic weights

H=1, C=12, O=16, …

Physics



Ludwig Boltzmann, 1844-1906

kinetic theory
statistical physics

Why doubt the atomic hypothesis in 1900?

Ernst Mach, 1838-1916

a “phenomenalist”
(~ “logical positivist”)

who

denied the reality of all that is not directly observable

$$\frac{\text{object speed}}{\text{speed of sound}} = \text{Mach Number}$$

Subsonic
Mach < 1

Supersonic
Mach > 1

Hypersonic
Mach > 5

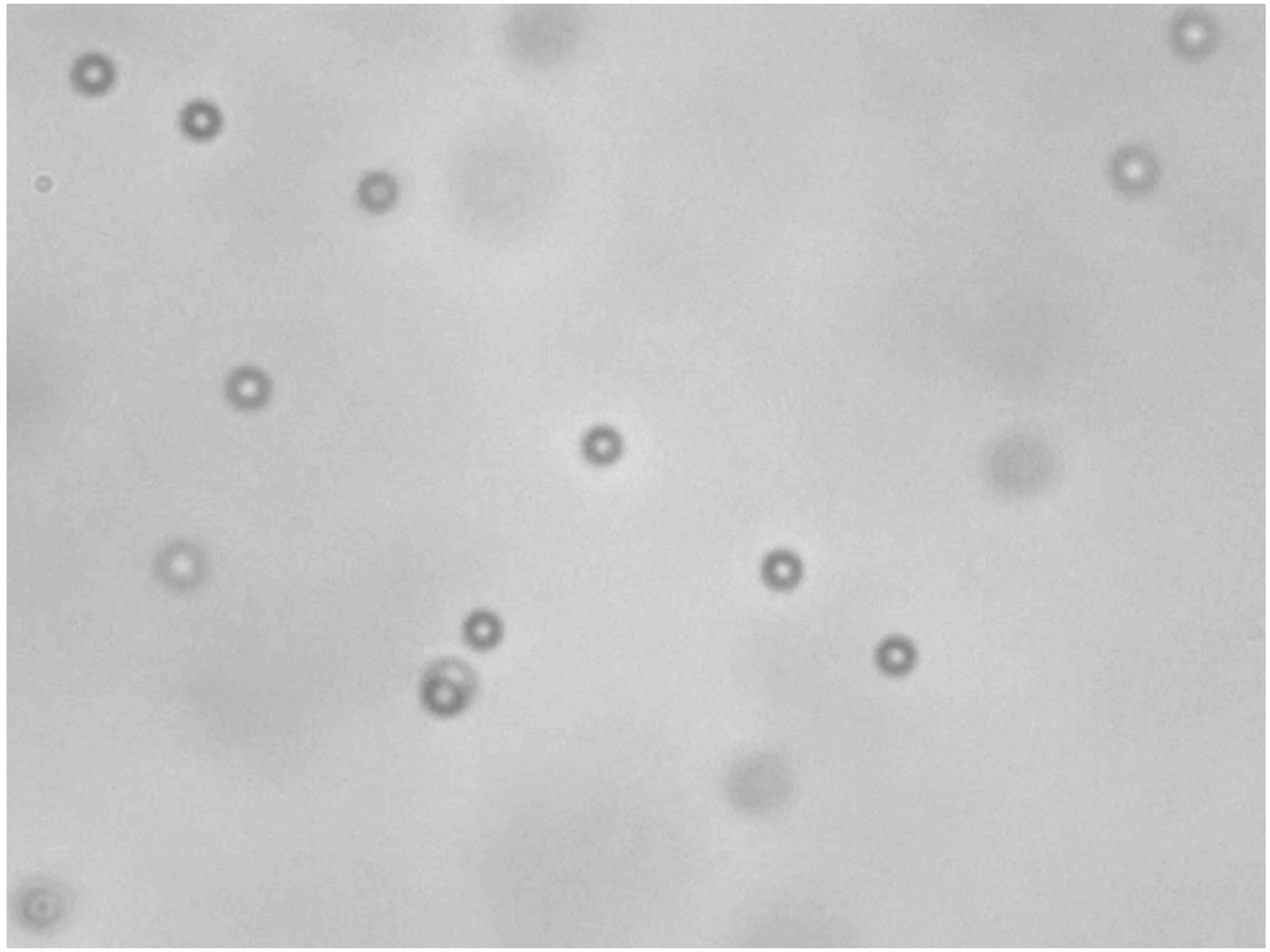
mechanics reversible
but not thermodynamics

Wilhelm Ostwald, 1853-1932
(Nobel in Chemistry, 1909)
“father of physical chemistry”

What is Brownian motion?

Robert Brown,
English botanist
1773-1858

discovered the nuclei of plant cells



Ricky Gill, Physics 433 (SFU)

observed grains of pollen
moved incessantly in water
(1827)

Early hypotheses:

1. Others (Gray, Gleichen, etc.) had seen Brownian motion but thought it had a biological origin.

2. Brown ruled out biology,
   showing that <u>any</u> small particle gave the same behaviour.

   even a piece of the Sphinx!

3. Evaporation and fluid flow.

4. Temperature variations due, e.g., to light.

5. Several people (Nägeli in 1879, Ramsay in 1882) considered molecular motion as an explanation and then ruled it out!

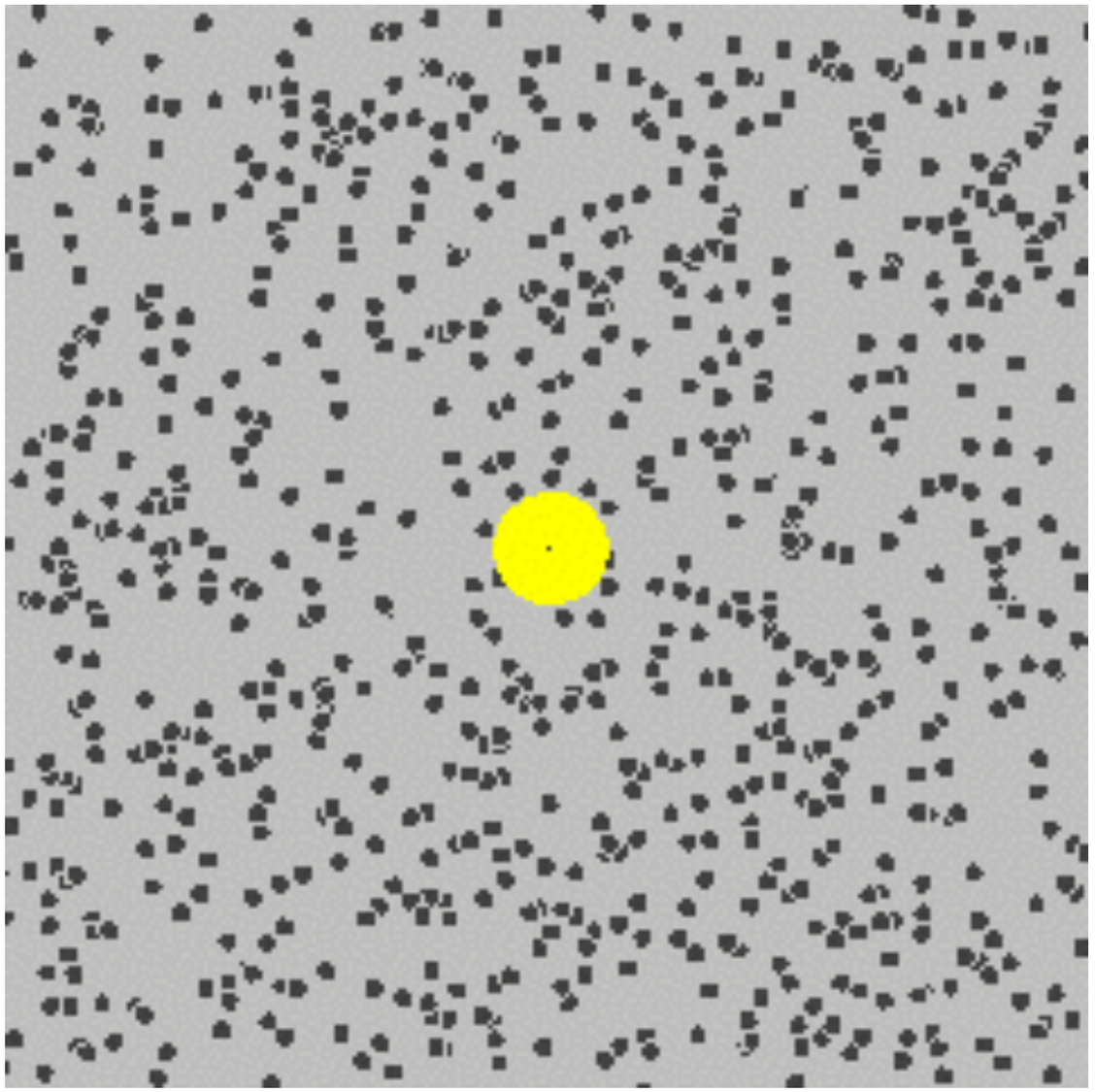
"I have some sea-mice—fine specimens—in spirits. And I will throw in Robert Brown's new thing—'Microscopic Observations on the Pollen of Plants'—if you don't happen to have it already." (Tertius Lydgate)

George Eliot, *Middlemarch*, 1872; set in 1829.

Einstein's view:

The particles we see are buffeted by the motion of many smaller, unseen molecules.

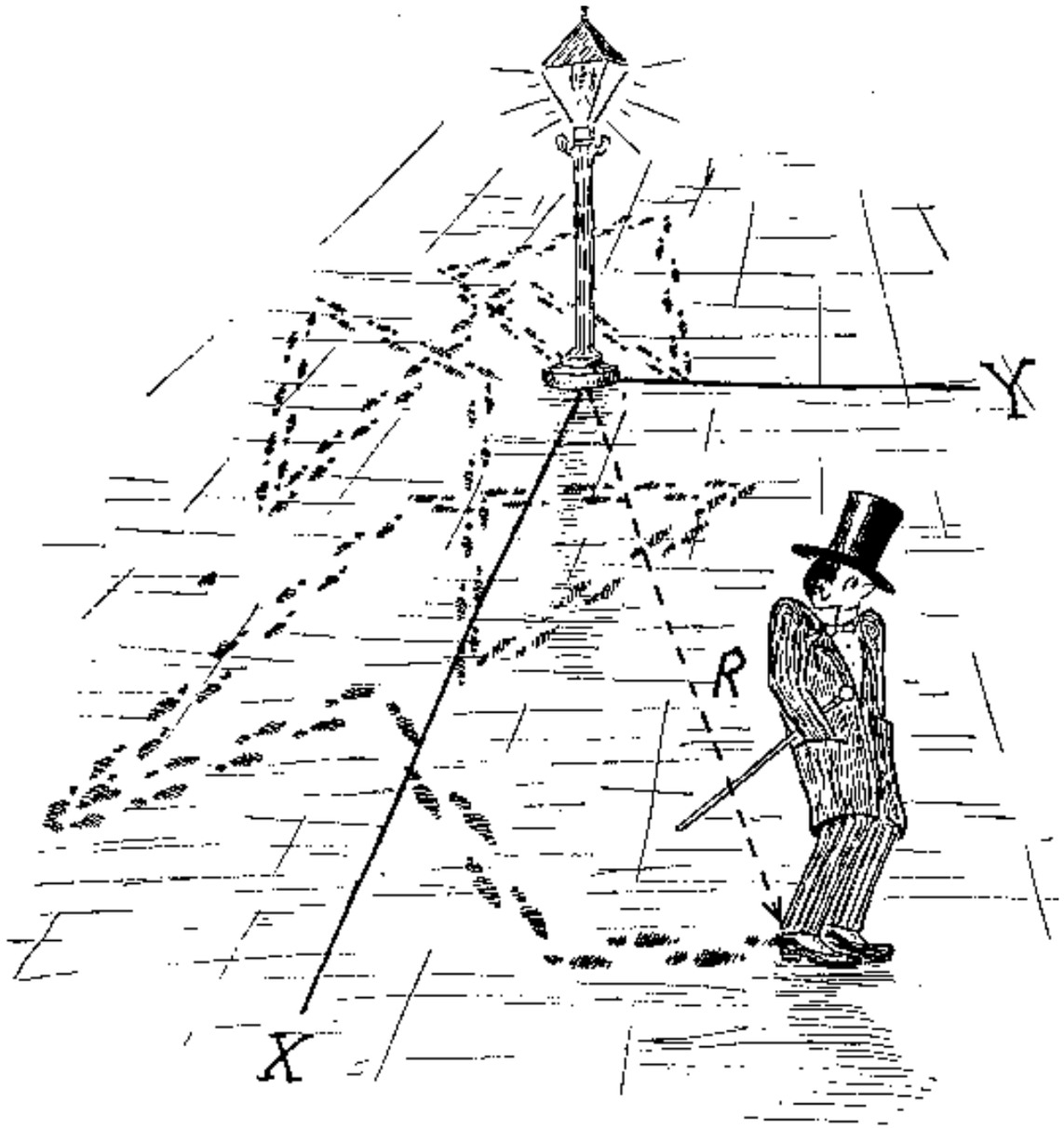
The irregular motion directly reflects matter's atomic nature



Loo Kang Wee

The big particle does a “random walk”

George Gamow, 1904-1968

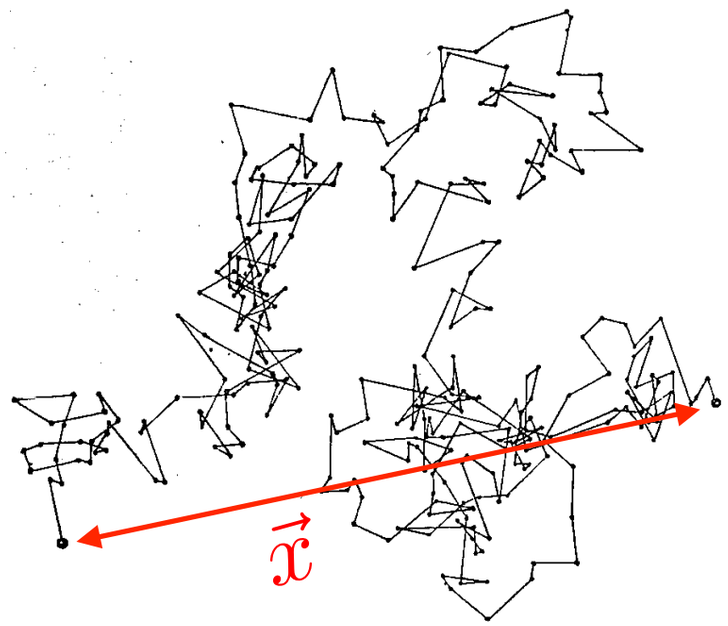


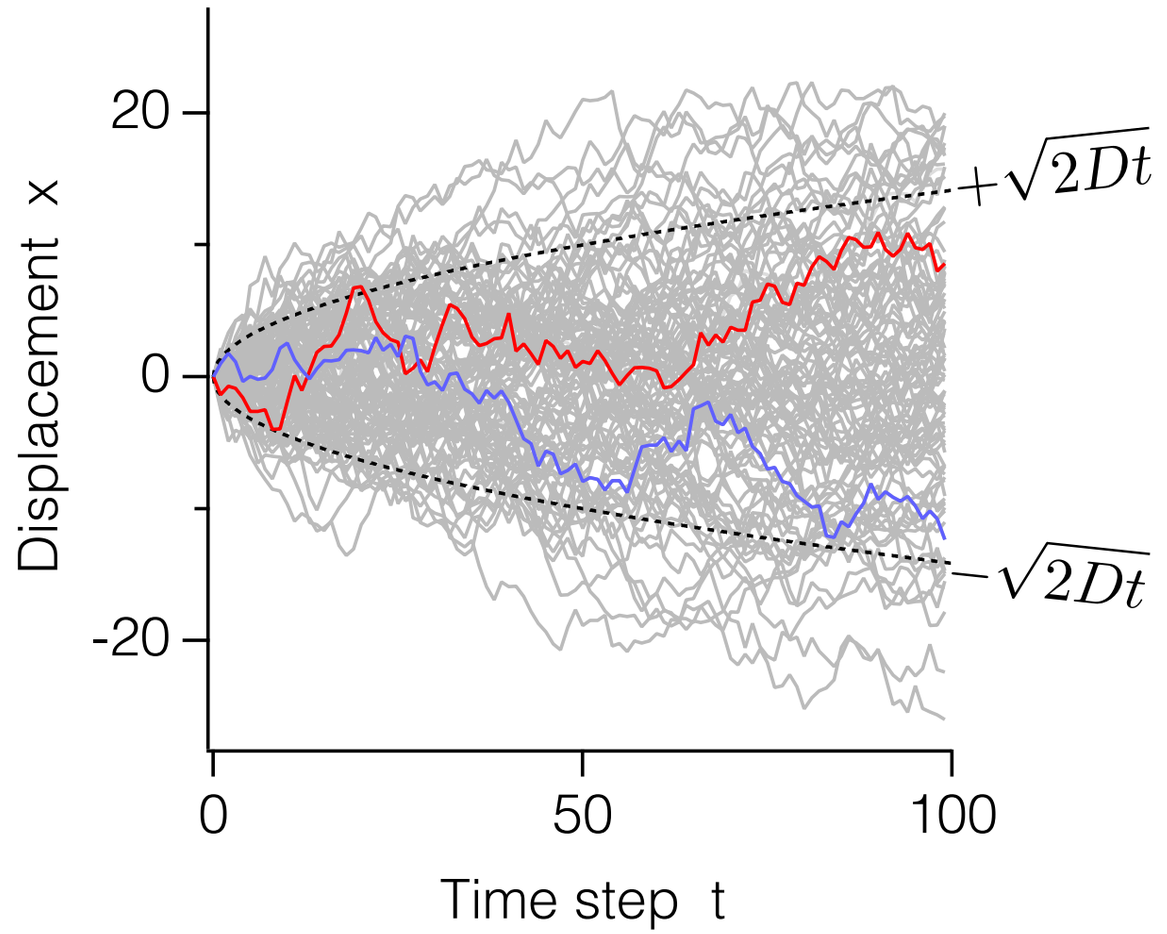
One Two Three … Infinity, p. 201
Rev. ed. Viking/Bantam, 1961.

Einstein's first nice result:

Order in random motion!

$$\langle x^2 \rangle = 2Dt$$



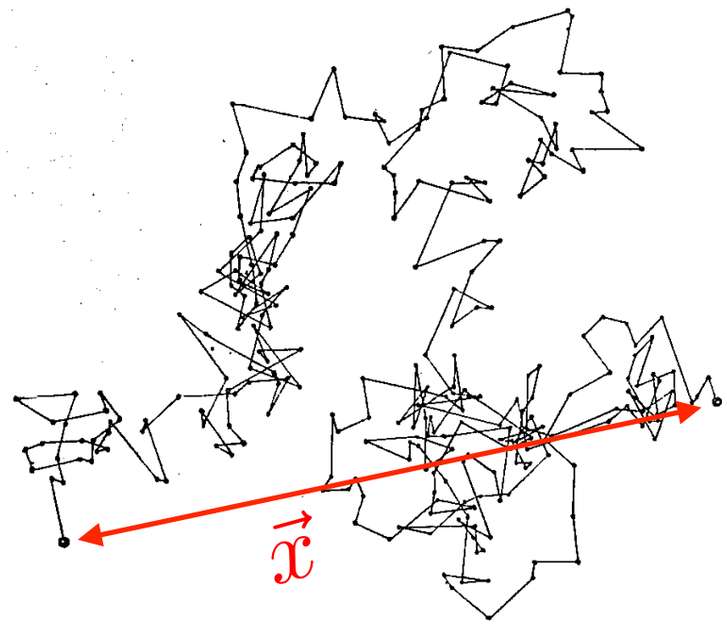


Einstein's second nice result:

$$D = \frac{k_B T}{6\pi\eta R}$$

temperature

smaller objects diffuse "faster"

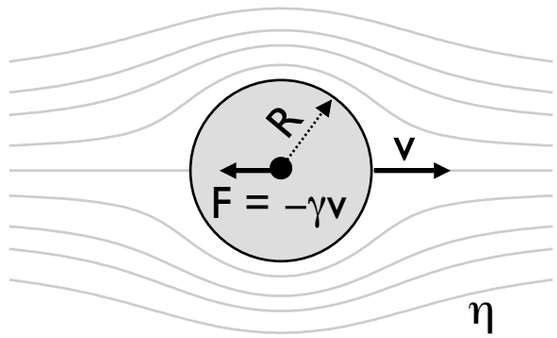


D ~ fluctuation

$$\langle x^2 \rangle = 2Dt$$

η ~ dissipation

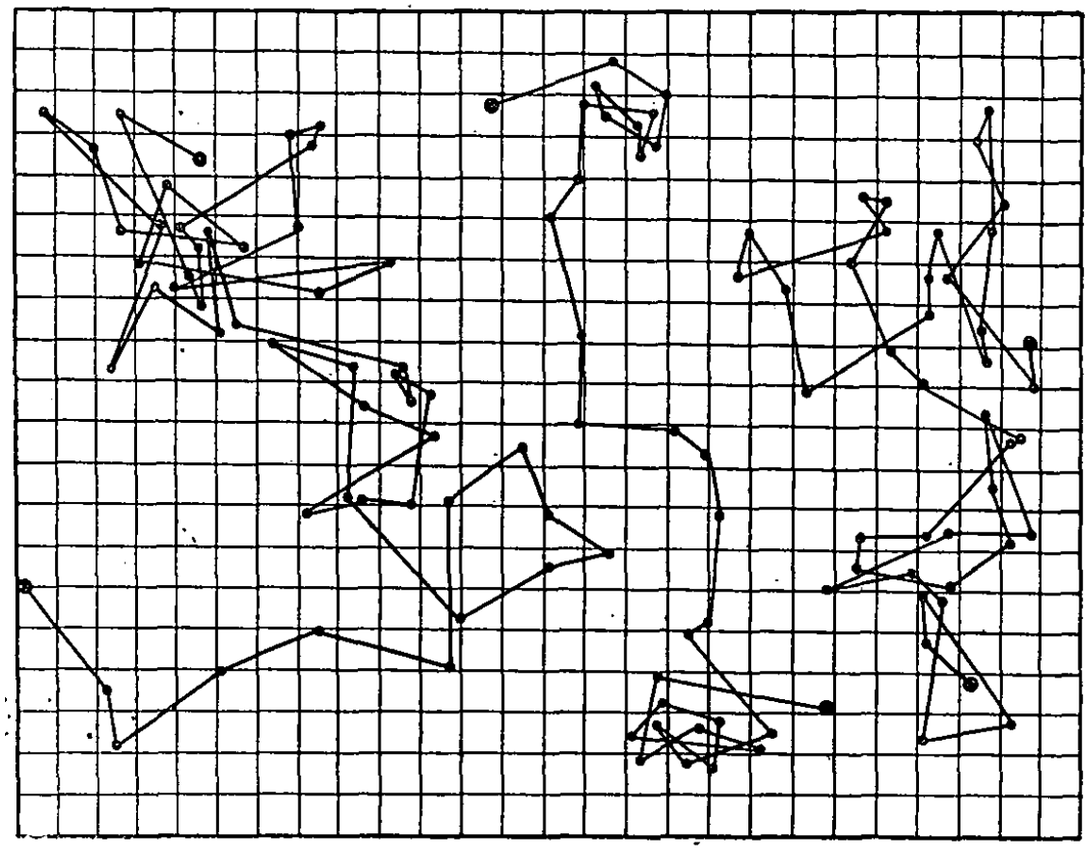
$$F = -\gamma v$$
$$= -(6\pi\eta R)\, v$$



Fluctuation-dissipation relation

Was Einstein right?

Jean Perrin, 1870-1942
(photo from 1908)

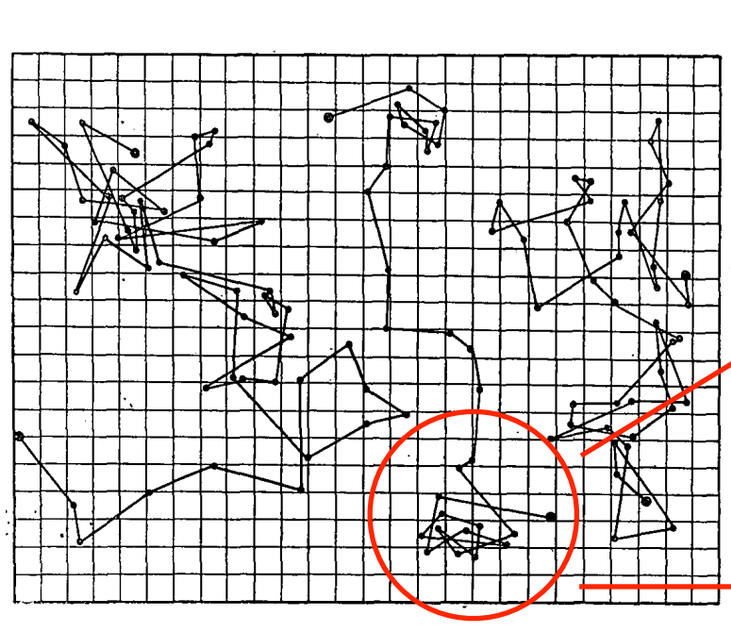


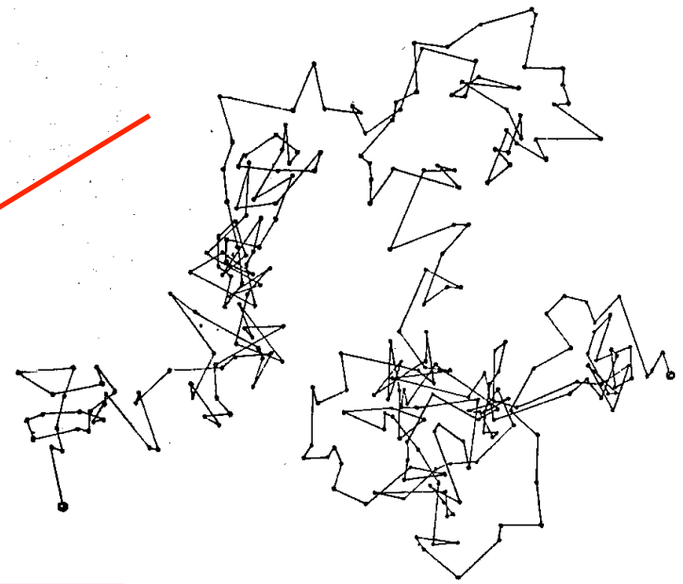
Jean Perrin,
"Mouvement brownien et réalité moléculaire,"
Annales de Chimie et Physique **8**, 1-114 (1909).

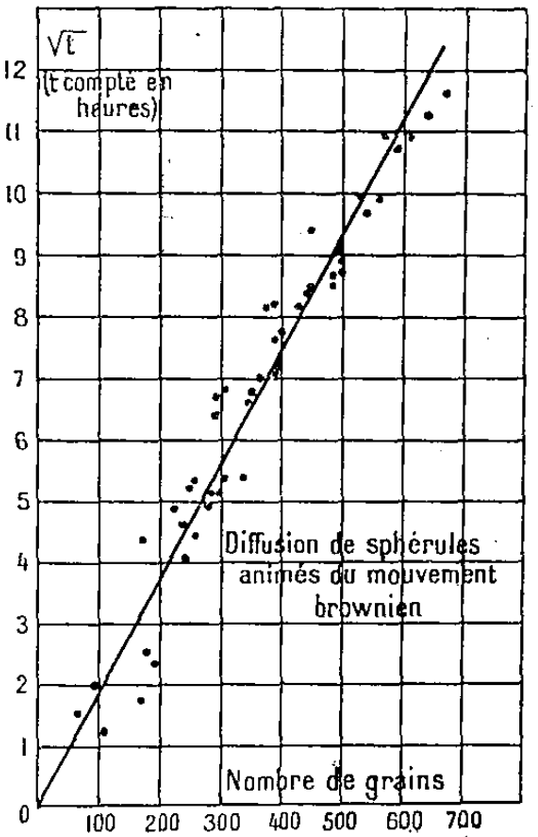
Nobel prize in Physics, 1926

Was Einstein right?

$$\langle x^2 \rangle = 2Dt$$





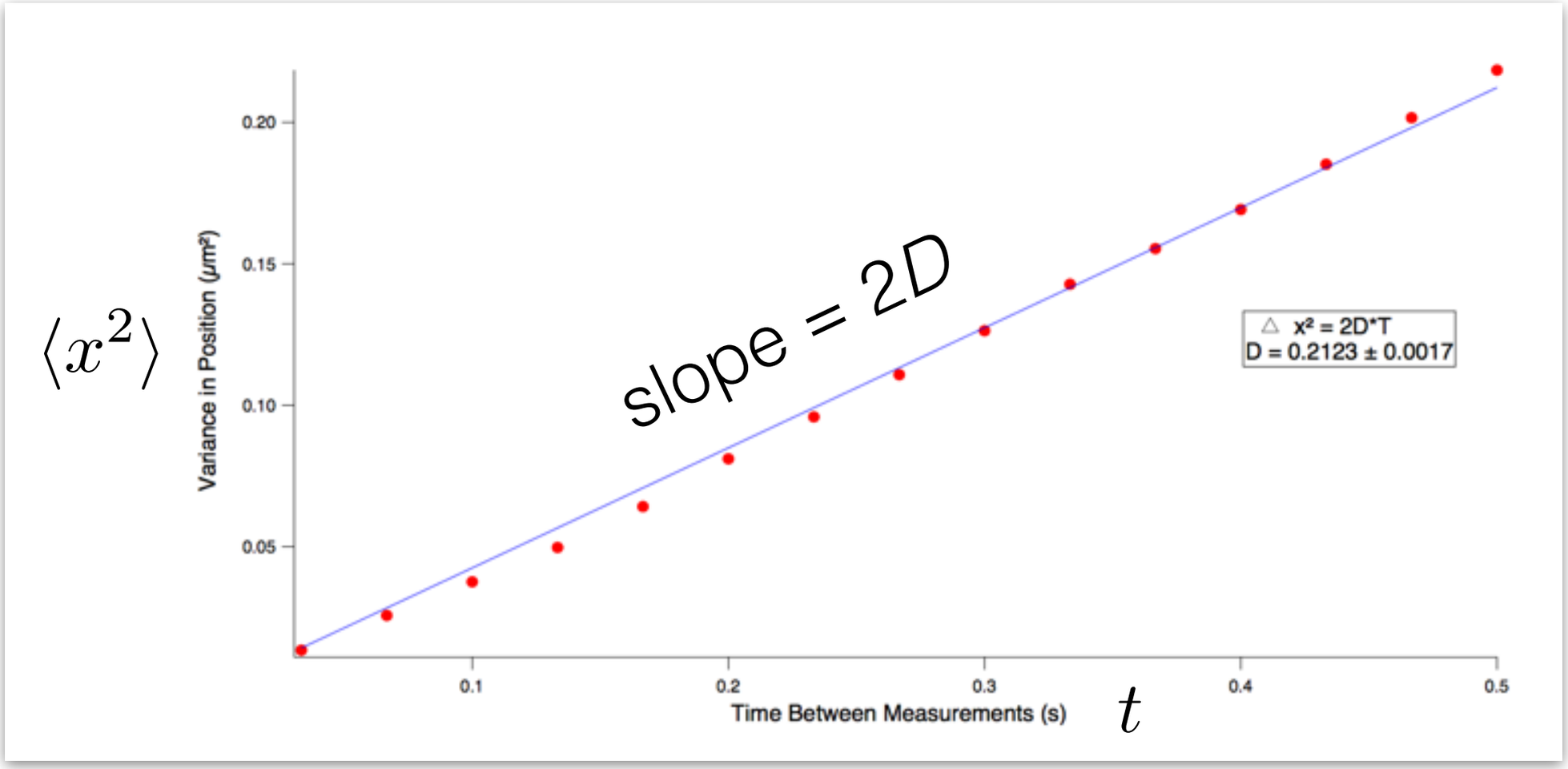


$\sqrt{t}$

$\sqrt{\langle x^2 \rangle}$

i.e., a fractal trajectory

Was Einstein right?



Karlton Scheu, 2013, Physics 433 (SFU)

Was Einstein right?

"*I have convinced myself that we have recently come into possession of experimental proof of the discrete or grainy nature of matter, for which the atomic hypothesis had vainly sought for centuries, even millennia.*

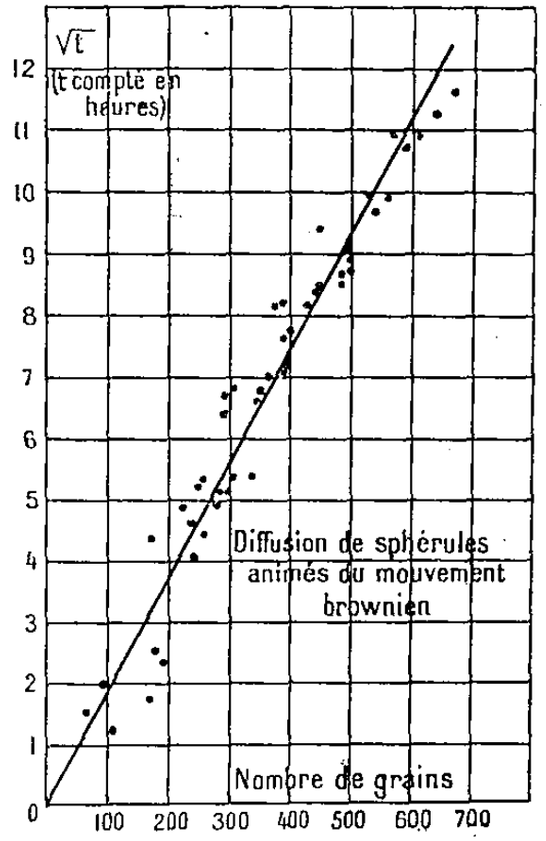
… the agreement of Brownian movements with the predictions of the kinetic hypothesis on the other hand, which has been shown by a series of researchers, most completely by J. PERRIN -- this evidence now justifies even the most cautious scientist in speaking of the *experimental* proof of the atomistic nature of space-filling matter. What has up to now been called the atomistic hypothesis is thereby raised to the level of a well-founded theory, which therefore deserves its place in any textbook intended as an introduction to the scientific subject of general chemistry."

Wilhelm Ostwald, 1909

Was Einstein right?

$$\langle x^2 \rangle = 2Dt$$

$\sqrt{t}$



$\sqrt{\langle x^2 \rangle}$

La théorie atomique a triumphé. Nombreux encore naguère, ses adversaires enfin conquis renoncent l'un après l'autre aux défiances qui longtemps furent légitimes et sans doute utiles.

Jean Perrin, epilogue to *Les Atomes* (1913)

Was Einstein right?

Ernst Mach, 1838-1916

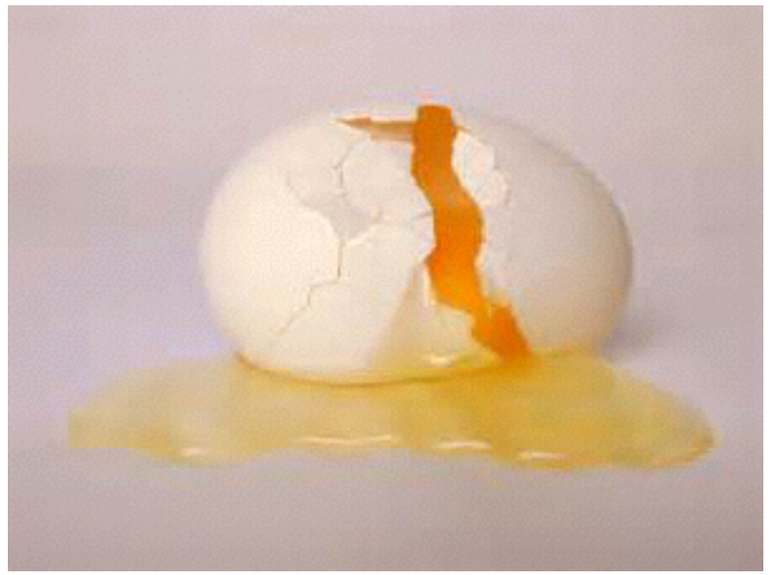
Mach never accepted the “atomic theory”.

The legacy:

entropy has a statistical interpretation

Boltzmann:

Laws of thermodynamics true only statistically.



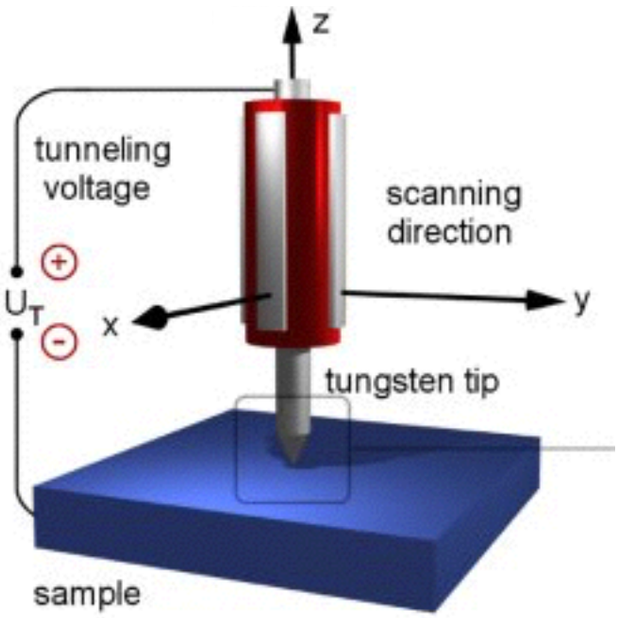
Fluctuations important in small systems.

Committed suicide in part over arguments with Ostwald.

The reality of atoms today

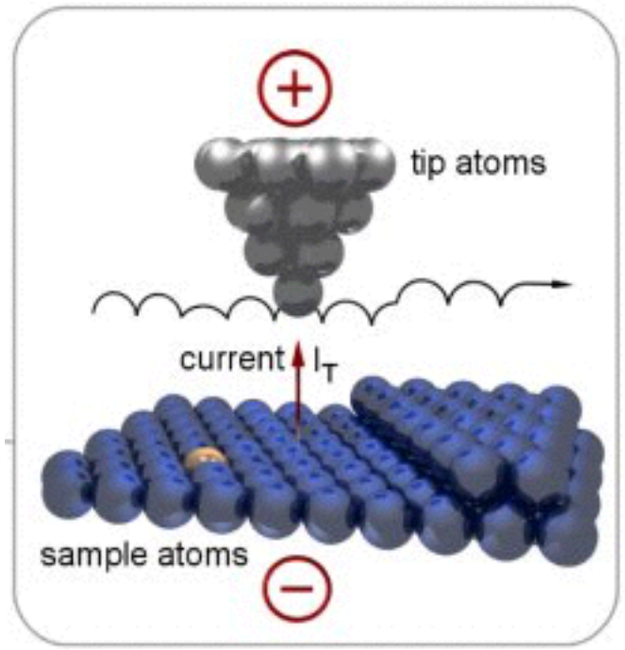
Scanning Tunneling Microscope (STM)

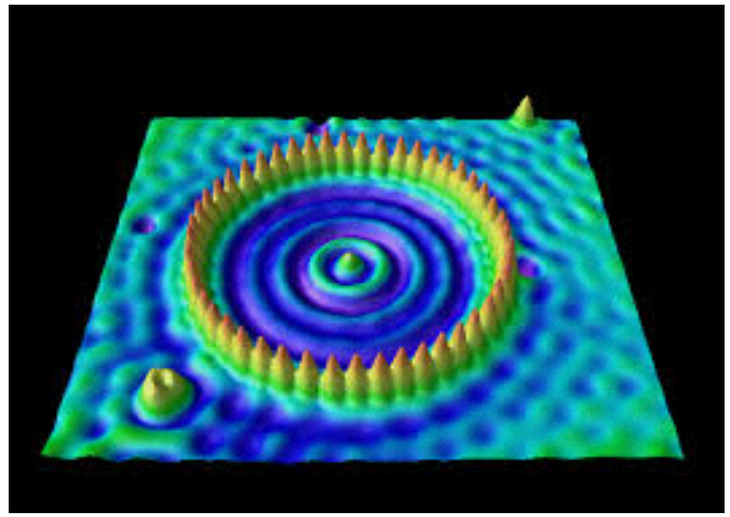
macroscopic scale

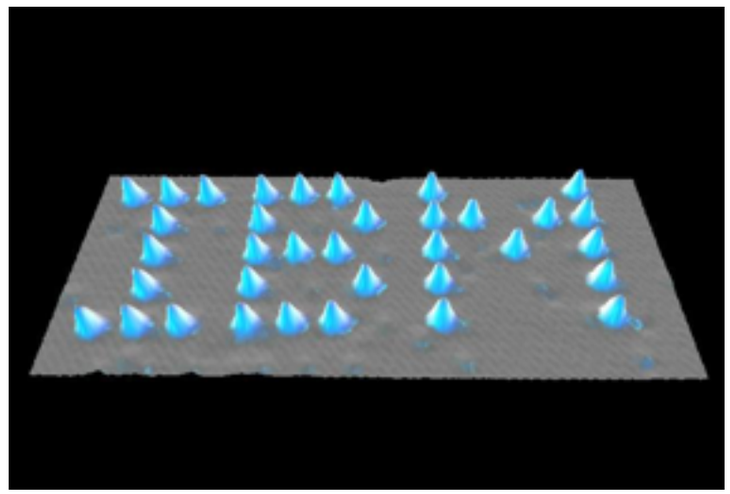


Lutz Kipp, Univ. Kiel
(Germany)

atomic scale







M.F. Crommie, C.P. Lutz, D.M. Eigler,
Science **262**, 218 (1993)

Diffusion today

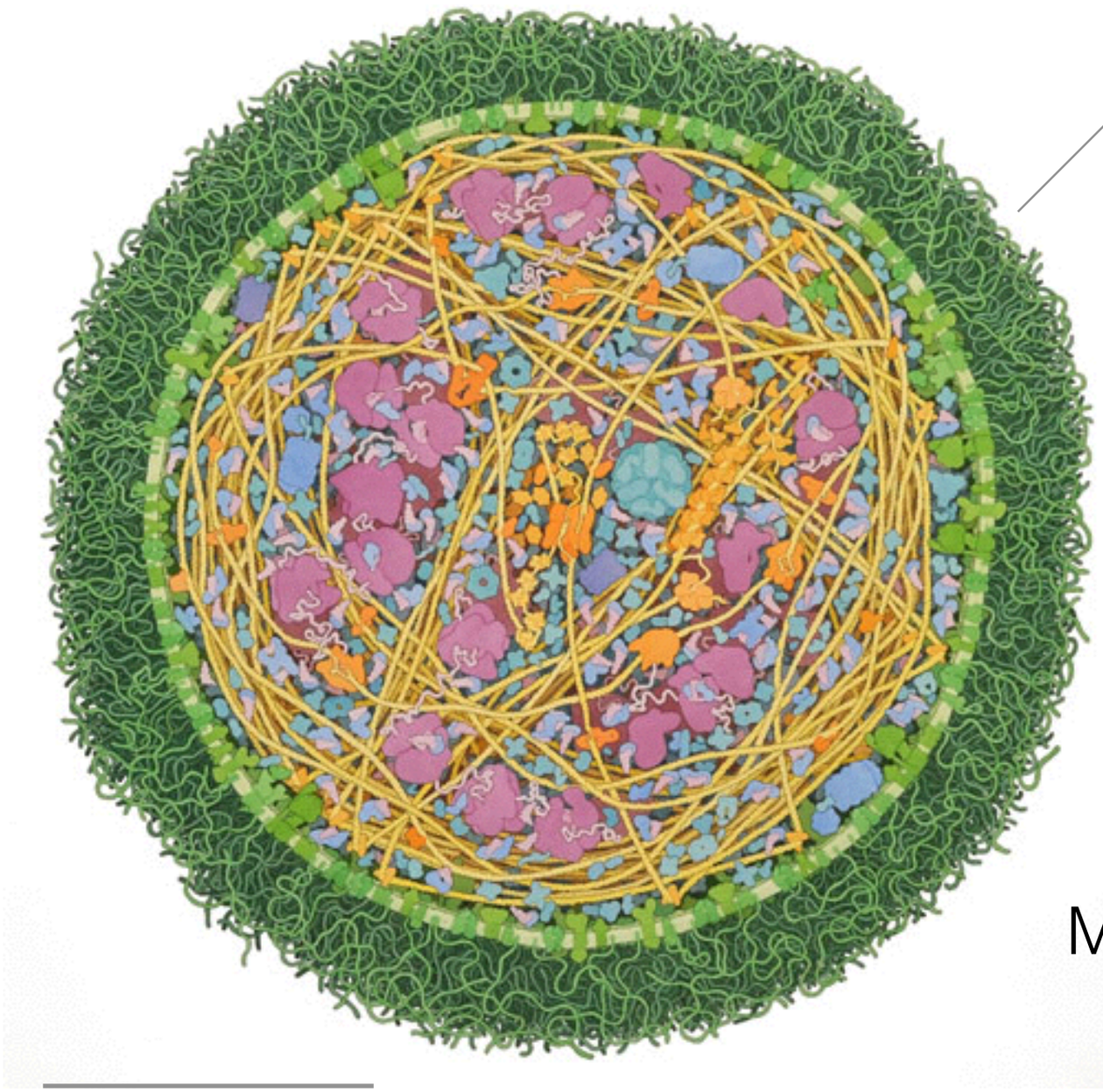
The lesson:

Noise,

rather than being useless or “bad”

can tell us something about the environment.

Diffusion today



100 nm

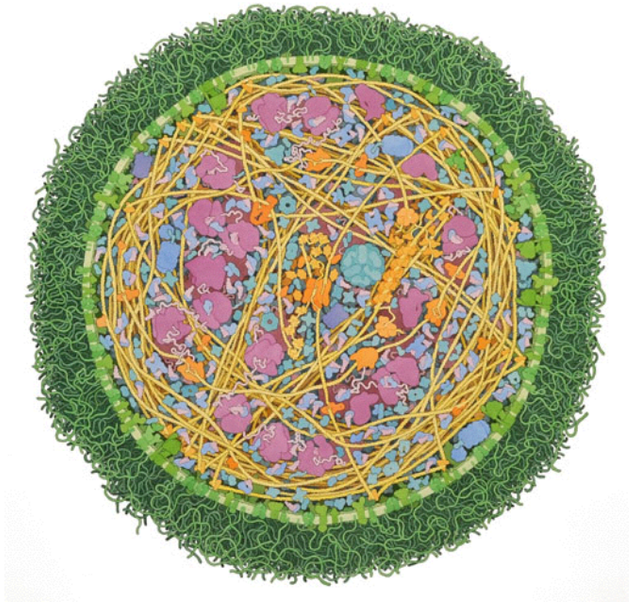
Lots of stuff inside!

Complex environments:

$D \rightarrow D(t)$

Mycoplasma bacterium:
one of the simplest!

David Goodsell, 2011
Scripps Research Institute



Diffusion today

Subdiffusion

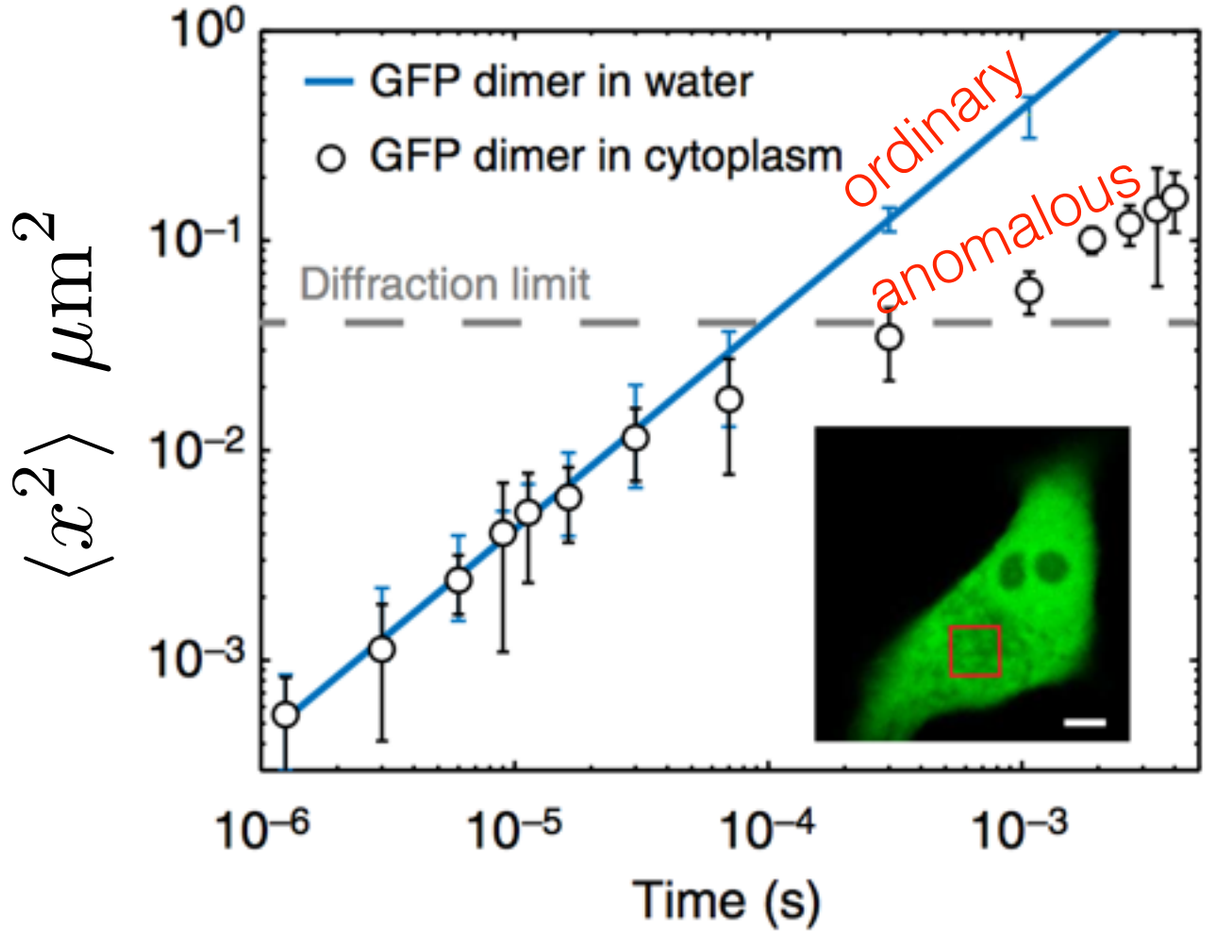
Recall: $D \sim \dfrac{a^2}{\Delta t}$ a ~ step size

Δt ~ step interval

traps ⇒ Δt is different

"each step"

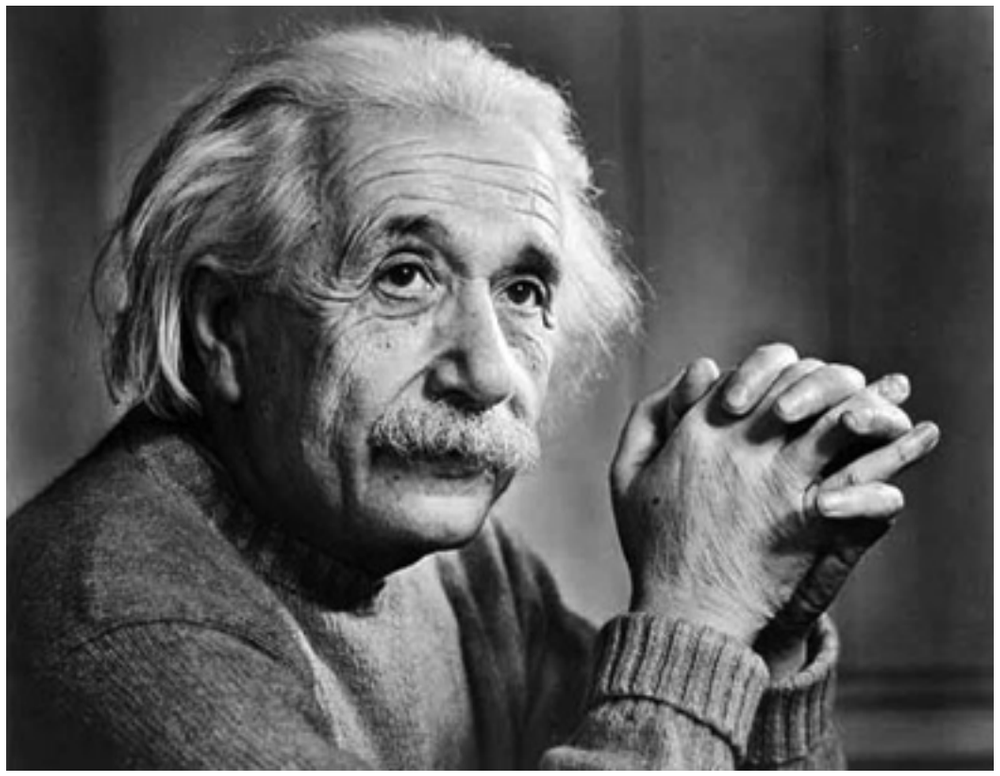
$$\langle x^2 \rangle \sim t^\alpha \qquad \alpha < 1$$



C. di Renzo et al., *Nature Comm.* **5**, 5891 (2014)

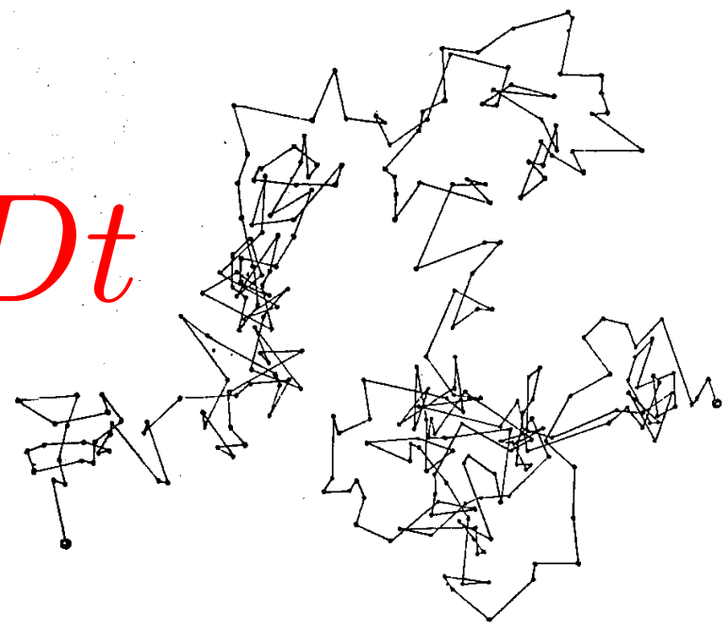
Conclusions:

- Brownian motion results from fluctuations due to collisions with numerous, unseen molecules in the fluid
- Einstein's explanation was the "tipping point" that led to universal acceptance of the atomic hypothesis
- Randomness is ruled by laws that are not "random"!
- Noise can be a tool to explore the local environment of a probe



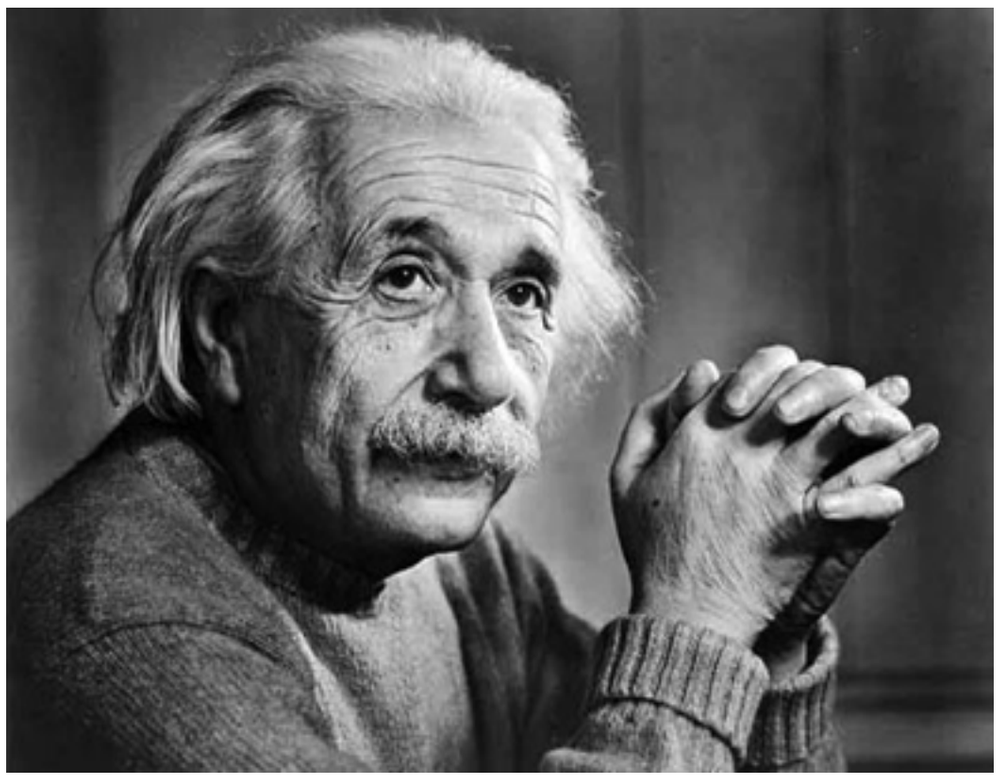
"Everything should be made as simple as possible, but not simpler."

$$\langle x^2 \rangle = 2Dt$$



References:

- John Rigden, *Einstein 1905: The Standard of Greatness*, Harvard Univ. Press, 2005.
- Abraham Pais, *'Subtle is the Lord…': The Science and Life of Albert Einstein*, Oxford Univ. Press, 1982.
- Stephen G. Brush, "A History of Random Processes," *Archives for History of Exact Sciences* **5**, 1-36 (1968).
- Albert Einstein, *Investigations on the Theory of the Brownian Movement*, Methuen & Co., 1926; Dover reprint, 1956.



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