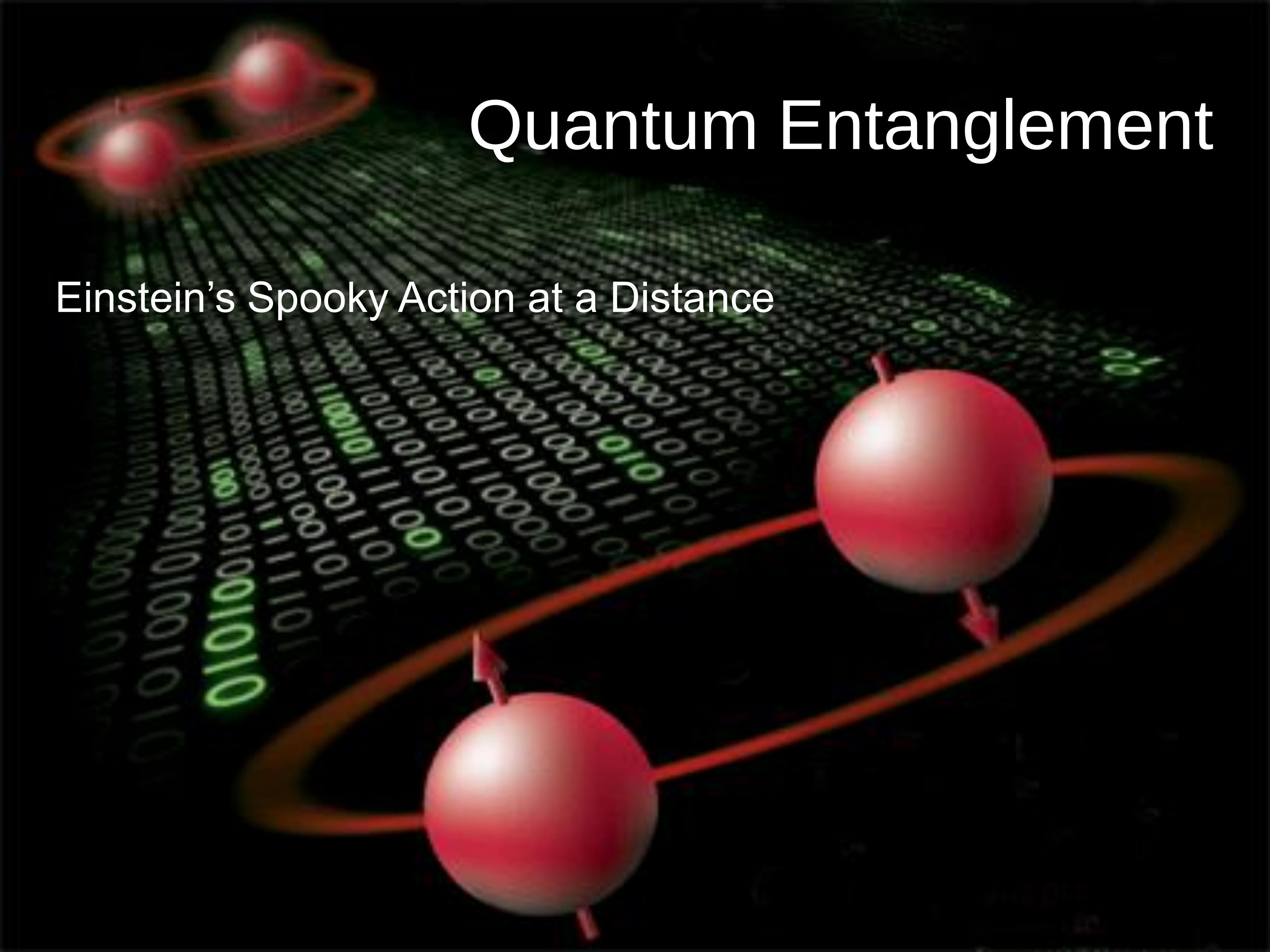
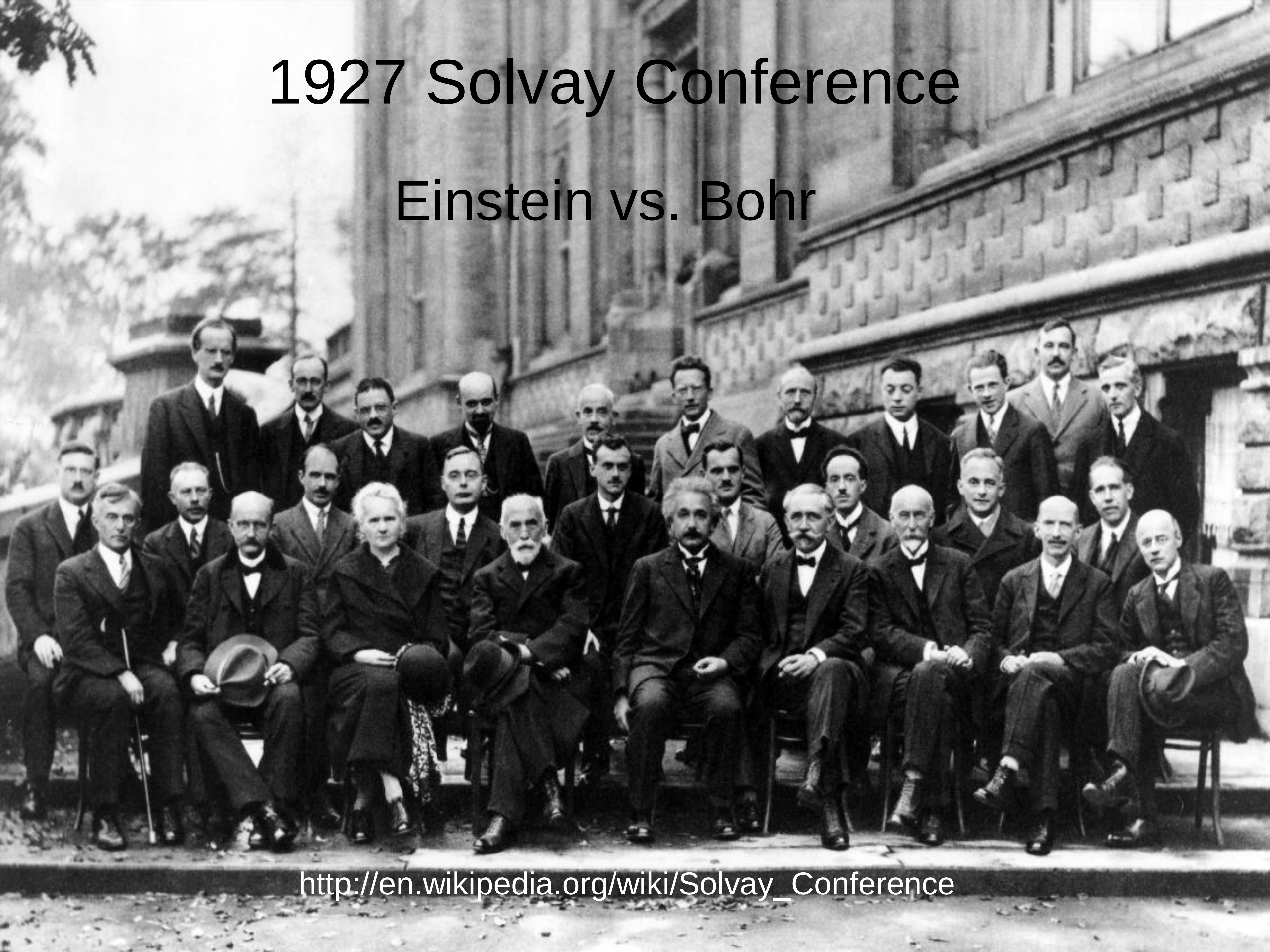
Quantum Entanglement

Einstein's Spooky Action at a Distance

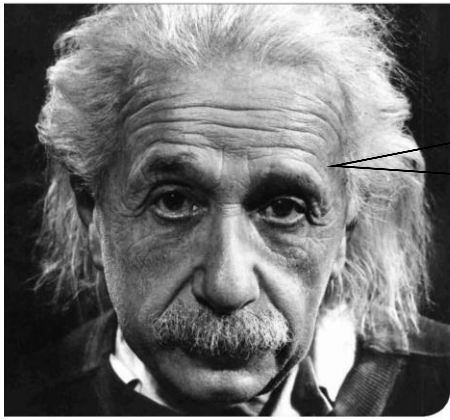


1927 Solvay Conference

Einstein vs. Bohr



http://en.wikipedia.org/wiki/Solvay_Conference



God does not play dice with the universe!

Einstein

$|\Psi\rangle$ The Wave function

Stop telling God what to do!

Bohr

Schrödinger Wave Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi = E\Psi$$

$$\text{Kinetic Energy} + \text{Potential Energy} = E$$

Classical

Conservation of Energy

Newton's Laws

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$$

Harmonic oscillator example.

$$F = ma = -kx$$

Quantum

Conservation of Energy

Schrodinger Equation

$$\frac{p^2}{2m} + \frac{1}{2}kx^2$$

The energy becomes the Hamiltonian operator

$$H\Psi = E\Psi$$

Wavefunction

Energy "eigenvalue" for the system.

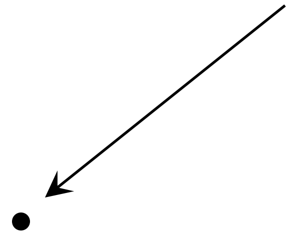
In making the transition to a wave equation, physical variables take the form of "operators".

$$p \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad x \rightarrow x$$

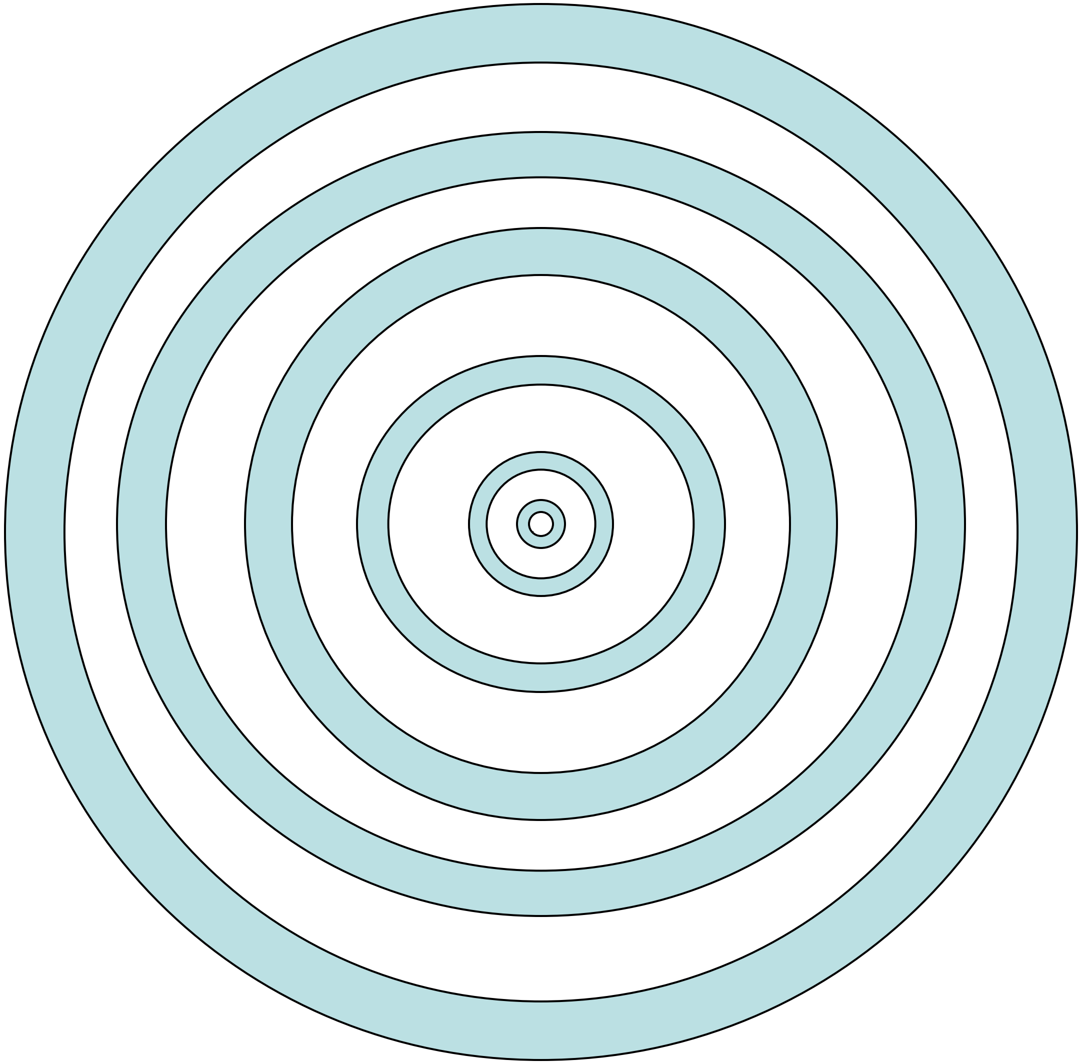
$$H \rightarrow \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}kx^2$$

The form of the Hamiltonian operator for a quantum harmonic oscillator.

This is an electron

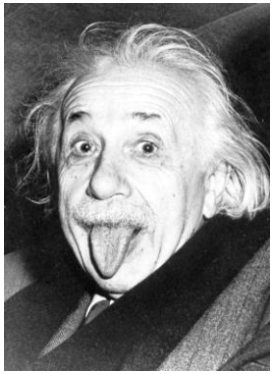


Where is the electron?

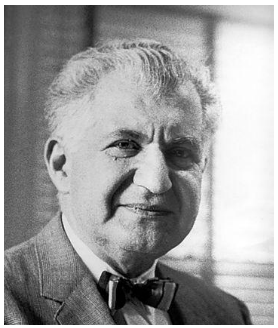


Here it is!

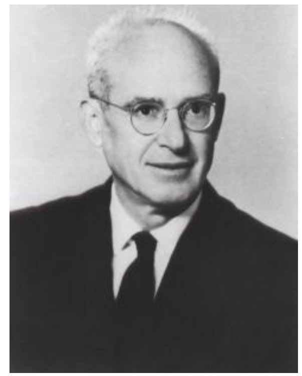
In 1935
Einstein strikes back
The EPR paradox



Albert Einstein



Boris Poldolsky



Nathan Rosen

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

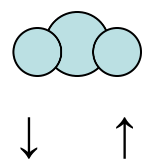
A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

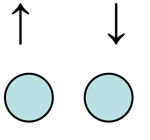
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

What is
The EPR Paradox?

This is nonsense!
What’s to prevent this?



or this?



What did the Physicists say?

Bohr, "I must sleep on it."
Next day; "The trend of their argumentation does not seem to me to adequately meet the actual situation with which we are faced in atomic physics, etc. etc."

Dirac, "Now we have to start all over again, because Einstein Proved that it does not work".

Pauli, "Einstein has once again expressed himself publicly on quantum mechanics, namely in the issue of Physical Review of 15 May (with Podolsky and Rosen – no good company by the way). As is well known, this is a catastrophe every time it happens.
'His mishap was an illusion,
And so he reasons pointedly:
That cannot be which should not be'." (Morgenstern)

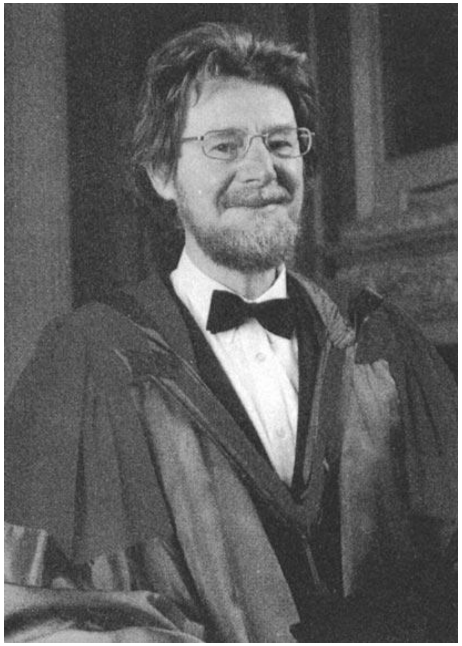
Schrödinger: “I was very happy that in the paper just published in P.R. you have evidently caught dogmatic q.m. by the Coattails with those things that we used to discuss so much in Berlin.”

Time passed. A theory to replace quantum mechanics which was deterministic, had hidden variables, and was local (i.e., no spooky-action-at-a-distance) did not appear

In 1952, physicist David Bohm came up with a quantum mechanics that was deterministic, had hidden variables, but was non-local (i.e., pilot waves with super luminal velocities to account for spooky-action-at-a-distance).

Gave same answers as regular Quantum Mechanics. Was more complicated and was promptly ignored.

Meanwhile Quantum Mechanics kept giving the right answers, so Q.M.’s dirty little secret like Bohm’s Q.M. was forgotten and ignored.



Except by John Stewart Bell (1928 – 1990)

III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

JOHN S. BELL†

Physics, *1*, 195-200 (1964).

Is there a way to distinguish between QM's spooky action
At a distance, and Einstein's pre-arranged local spin up
with spin down combinations, in any measured direction?

Yes there is!

And this paper showed how with the Bell inequality
But nobody really cared until…

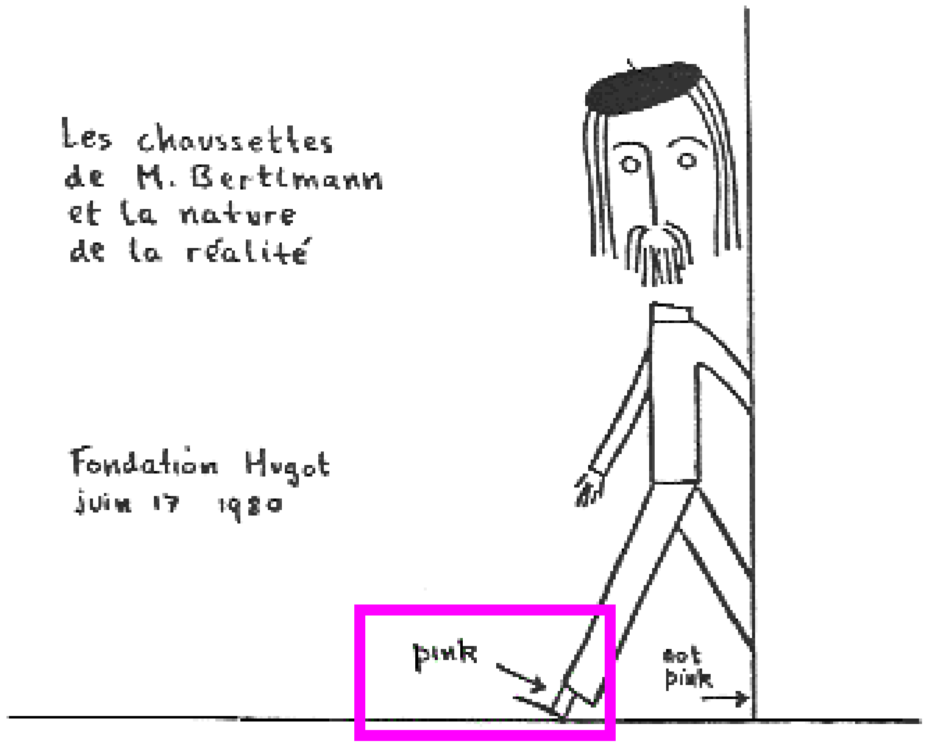
Until J.S. Bell published
In 1980

BERTLMANN'S SOCKS AND THE NATURE OF REALITY

J.S. Bell

CERN - Geneva

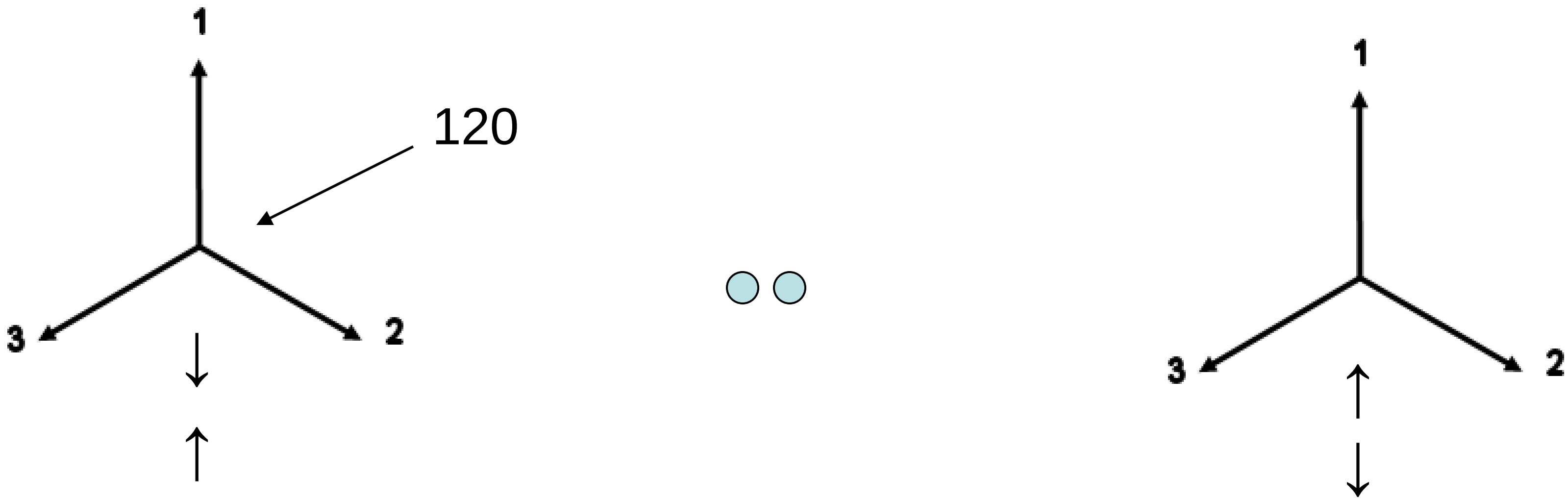
Who is Bertlmann and who cares about his socks?



from: J.S. Bell, "Bertlmann's socks and the nature of reality", Journal de Physique, Tome 42, Colloque C-2, supplément au No. 3, mars 1981.

reprinted in: J.S. Bell, "Speakable and unspeakable in quantum mechanics", Cambridge University Press, p. 139, 1987.

The Bell Apparatus

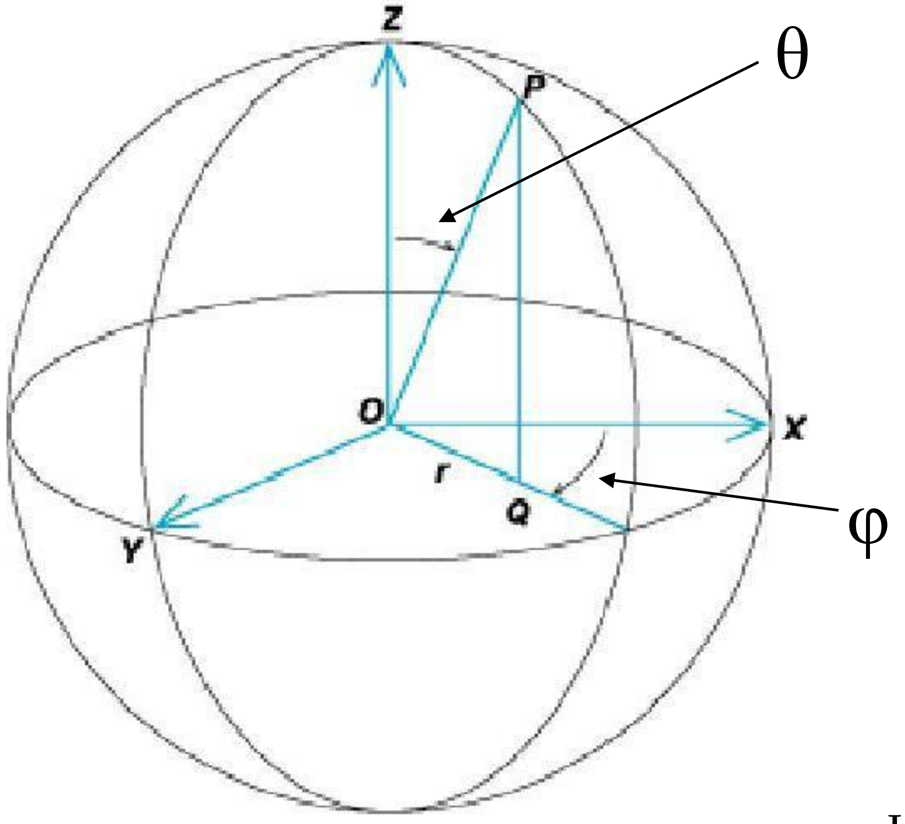


The wave function of the electron

$$\cos(\theta/2)\,|\uparrow\rangle + \sin(\theta/2)e^{i\varphi}\,|\downarrow\rangle$$

Probability up = $|\cos(\theta/2)|^2$ Probability down = $|\sin(\theta/2)|^2$

Total probability $|\cos(\theta/2)|^2 + |\sin(\theta/2)|^2 = 1$



The electron points up $\theta = 0$
Analyzer points up
$\cos(\theta/2) = 1$
$\sin(\theta/2) = 0$

$\cos(\theta/2)\,|\uparrow\rangle + \sin(\theta/2)e^{i\varphi}\,|\downarrow\rangle$

$|\uparrow\rangle$

If analyzer points 60 away from up
$\cos(\theta/2) = \sqrt{3}/2$
$\sin(\theta/2) = 1/2$
$\varphi = 0$

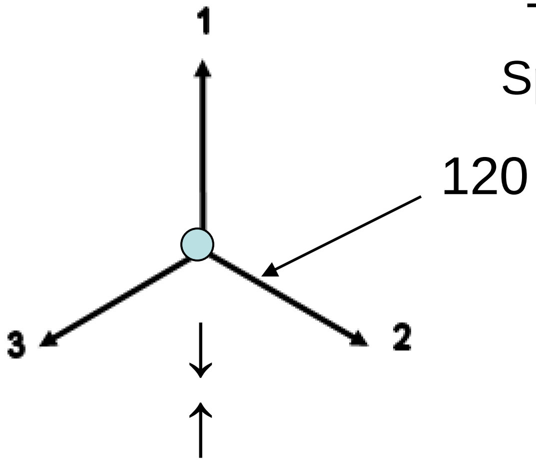
$\mathbf{1/2(\sqrt{3}\,|\uparrow\rangle + |\downarrow\rangle)}$

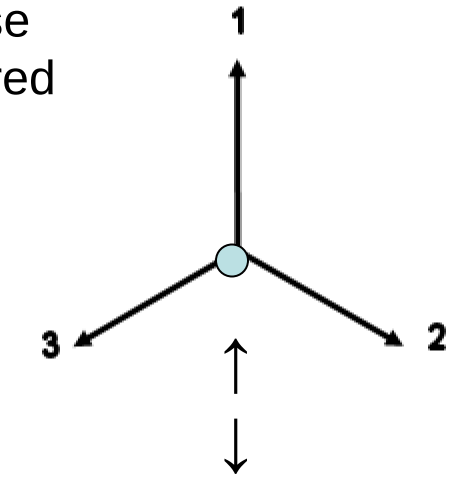
If analyzer points 120° away from up
$\cos(\theta/2) = 1/2$
$\sin(\theta/2) = \sqrt{3}/2$
$\varphi = 0$

$\mathbf{1/2(|\uparrow\rangle + \sqrt{3}\,|\downarrow\rangle)}$

The Bell Apparatus

The quantum mechanical case
Spin determined when measured





Case 1: right analyzer measures up
- left analyzer
  - If 1, it measures down, no agreement
  - If 2 or 3 agreement 3/4 of the time

Probability of agreement (0 + 3/4 + 3/4)/3 = 1/2

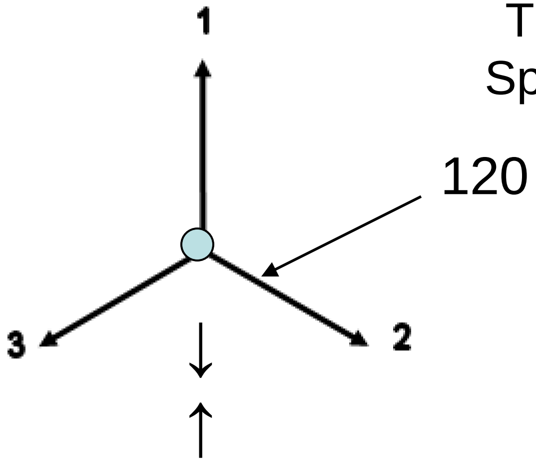
Quantum Mechanics predicts agreement one half (i.e., 50%) of the time

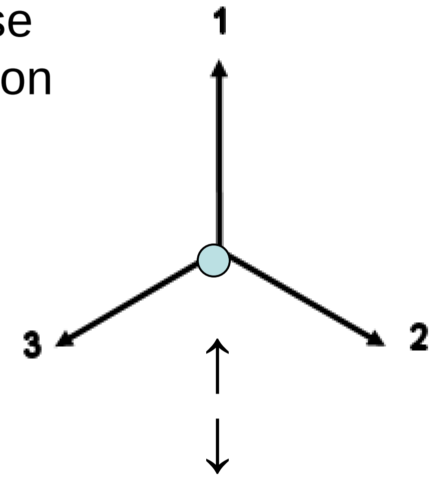
Case 2: right analyzer measures down
- left analyzer
  - If 1, it measures up, no agreement
  - If 2 or 3 agreement 3/4 of the time

Probability of agreement (0 + 3/4 + 3/4)/3 = 1/2

The Bell Apparatus

The pre-determined locality case
Spin determined before separation





Case 1: right analyzer measures up
• left analyzer
  •If 1, it measures down, no agreement
  •If 2 or 3 agreement 1/2 of the time
Probability of agreement (0 + 1/2 + 1/2)/3 = 1/3

Pre-determined locality predicts agreement only one third (i.e., 33.33%) of the time

Case 2: right analyzer measures down
• left analyzer
  •If 1, it measures up, no agreement
  •If 2 or 3 agreement 1/2 of the time
Probability of agreement (0 + 1/2 + 1/2)/3 = 1/3

Results of Experiment

- Confirms the prediction of quantum mechanics
- There is no theory that is deterministic, has hidden variables, and is local
- The current quantum mechanics which is un-deterministic and non-local describes nature as we see it.
- It is the most accurate theory we have
- **Spooky action-at-a-distance is real!**

Three particle entanglement

The Greenberger, Horne, and Zeilinger state

$$1/\sqrt{2}\,(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1\,|\downarrow\rangle_2\,|\downarrow\rangle_3)$$

These are states along the z axis. Evaluate this state in terms of spin up spin down states along the x direction for the no. 1 state and along the y direction for the no. 2 and 3 states. What do we get?

With a little calculation….

From Page No. ____

$$\frac{1}{4}\Big[\left(|+1\rangle^1_x + |-1\rangle^1_x\right)\left(|+1\rangle^2_y + |-1\rangle^2_y\right)\left(|+1\rangle^3_y + |-1\rangle^3_y\right) + \left(|+1\rangle^1_x - |-1\rangle^1_x\right)\left(|+1\rangle^2_y - |-1\rangle^2_y\right)\left(|+1\rangle^3_y - |-1\rangle^3_y\right)$$

$$= \frac{1}{4}\Big[\,|+\rangle^1_x|+1\rangle^2_y|+1\rangle^3_y + |+\rangle^1_x|+\rangle^2_y|-1\rangle^3_y + |+\rangle^1_x|-\rangle^2_y|+\rangle^3_y + |+\rangle^1_x|-\rangle^2_y|-\rangle^3_y$$

$$+\,|-\rangle^1_x|+\rangle^2_y|+\rangle^3_y + |-\rangle^1_x|+\rangle^2_y|-\rangle^3_y + |-\rangle^1_x|-\rangle^2_y|+\rangle^3_y + |-\rangle^1_x|-\rangle^2_y|-\rangle^3_y$$

$$+\,|+\rangle^1_x|+\rangle^2_y|+\rangle^3_y - |+\rangle^1_x|+\rangle^2_y|-\rangle^3_y - |+\rangle^1_x|-\rangle^2_y|+\rangle^3_y + |+\rangle^1_x|-\rangle^2_y|-\rangle^3_y$$

$$-\,|-\rangle^1_x|+\rangle^2_y|+\rangle^3_y + |-\rangle^1_x|+\rangle^2_y|-\rangle^3_y + |-\rangle^1_x|-\rangle^2_y|+\rangle^3_y - |-\rangle^1_x|-\rangle^2_y|-\rangle^3_y\,\Big]$$

$$= \frac{1}{4}\Big[2\,|+\rangle^1_x|+\rangle^2_y|+\rangle^3_y + 2\,|+\rangle^1_x|-\rangle^2_y|-\rangle^3_y + 2\,|-\rangle^1_x|+\rangle^2_y|-\rangle^3_y + 2\,|-\rangle^1_x|-\rangle^2_y|+\rangle^3_y\Big]$$

$$= \frac{1}{2}\Big[\,|+\rangle^1_x|+\rangle^2_y|+\rangle^3_y + |+\rangle^1_x|-\rangle^2_y|-\rangle^3_y + |-\rangle^1_x|+\rangle^2_y|-\rangle^3_y + |-\rangle^1_x|-\rangle^2_y|+\rangle^3_y\Big]$$

We get

$$1/2(|\uparrow\rangle_x|\uparrow\rangle_y|\uparrow\rangle_y + |\uparrow\rangle_x\,|\downarrow\rangle_y\,|\downarrow\rangle_y + |\downarrow\rangle_x\,|\uparrow\rangle_y\,|\downarrow\rangle_y + |\downarrow\rangle_x\,|\downarrow\rangle_y\,|\uparrow\rangle_y)$$

Note, if we assign:

- +1 for a measurement of an up $|\uparrow\rangle$ spin
- -1 for a measurement a down $|\downarrow\rangle$ spin

and measure the x-spin for the 1st particle and the y spin for the 2nd and 3rd particles, the Multiplication of all 3 assignments for all cases is +1

+1   +1 × +1 = +1
+1 × -1 × -1 = +1
-1 × +1 × -1 = +1
-1 × -1 × +1 = +1

We would get the same result if we looked at the GHZ state in the yxy decomposition or the yyx decomposition.

If we measure the y assignments of any two of the three particles and the x assignment of the third, the multiplicative value of the assignments will always be +1 !

Can we determine the x spins?

In the classical reality local deterministic sense, yes!

	1	2	3
y	+	+	+
x	+	+	+
y	+	+	-
x	-	-	+
y	+	-	+
x	-	+	-
y	+	-	-
x	+	-	-

	1	2	3
y	-	+	+
x	+	-	-
y	-	+	-
x	-	+	-
y	-	-	+
x	-	-	+
y	-	-	-
x	+	+	+

Now if you measure the x spin assignments of all 3 particles and multiply the results together we get +1.

However let us decompose the GHZ state according to QM w.r.t. the xxx states

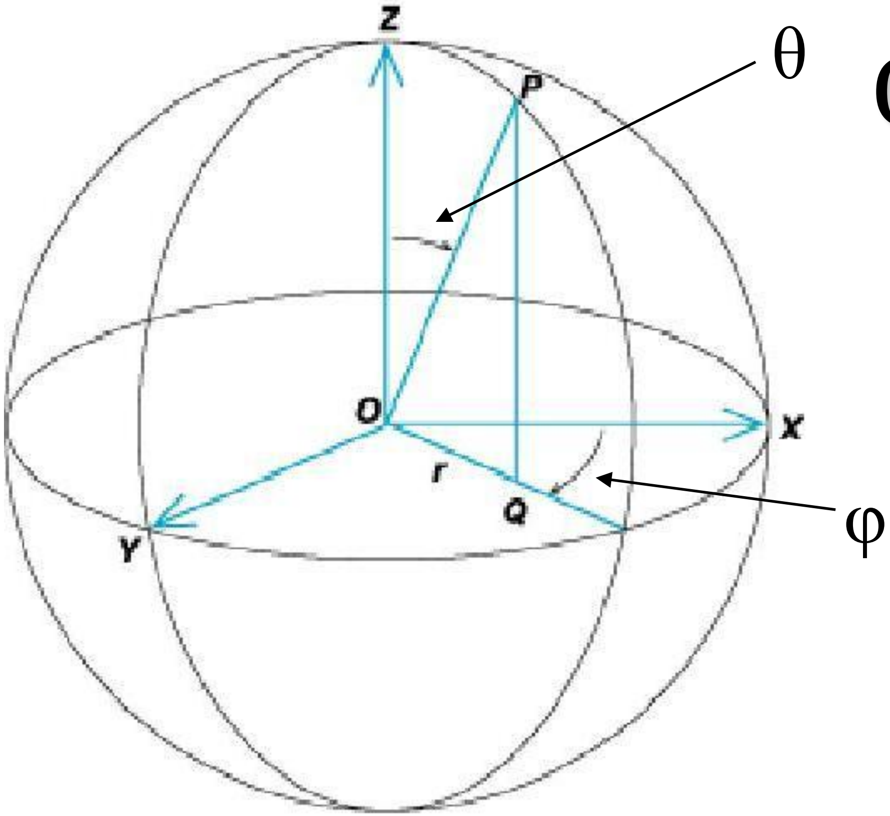
We get

$$1/2(|\uparrow\rangle_x|\uparrow\rangle_x|\downarrow\rangle_x + |\uparrow\rangle_x\, |\downarrow\rangle_x\, |\uparrow\rangle_x + |\downarrow\rangle_x\, |\uparrow\rangle_x\, |\uparrow\rangle_x + |\downarrow\rangle_x\, |\downarrow\rangle_x\, |\downarrow\rangle_x)$$

The multiplicative result for all 3 x spin assignments, for any result is

-1 NOT +1

- One result destroys the expectation of reality in the local deterministic sense
- No statistics required
- The quantum state is not resolved until it has been measured
- **Spooky action-at-a-distance is real!**



Quantum Computing

$$\cos(\theta/2)\,|\uparrow\rangle + \sin(\theta/2)e^{i\varphi}\,|\downarrow\rangle$$

define $\quad |\uparrow\rangle = |1\rangle \qquad |\downarrow\rangle = |0\rangle$

If we have $|\uparrow\rangle$ and we rotate spin by 90

$\cos(\theta/2) = 1/\sqrt{2}$

$\sin(\theta/2) = 1/\sqrt{2}$

$\varphi = 0$

We get

$$1/\sqrt{2}\,(\,|\uparrow\rangle + |\downarrow\rangle\,)$$

or

$$1/\sqrt{2}\,(\,|1\rangle + |0\rangle\,)$$

Thus if we define an electron with spin up as a bit, 1, and we hit it with a short timed blast of magnetic field along the y direction we end up with a state that is a superposition of both bit 1 and bit 0 at the same time. It will stay this way as long as you do not look at it!

This is a Qubit

It can point in any direction in the sphere. (θφ) Has an infinite amount of information to a classical Computer's one bit of information

Hadamard Gate

- $H|0\rangle \rightarrow 1/\sqrt{2}\ (|0\rangle + |1\rangle)$
- $H|1\rangle \rightarrow 1/\sqrt{2}\ (|0\rangle - |1\rangle)$
- This is just a rotation of 90 . There is no equivalent logical operation on a classical computer.
- $H|0,0\rangle \rightarrow 1/2\ (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$
- $1/2\ (|0,0\rangle + |0,1\rangle + |1,0\rangle + |1,1\rangle)$
- Application of a Hadamard gate gives us the superposition of all possible 2-bit inputs. We can then compute all possible results at once.

- Apply the algorithm f and we have
- $1/2\ (f|0,0\rangle + f|0,1\rangle + f|1,0\rangle + f|1,1\rangle)$
- All the answers at the same time as long as you don’t look at it.
- Look at it and you get only one answer e.g. $f|0,1\rangle$. The rest disappeared when the quantum superposition collapsed!

So what good is it?

Question

- Have function f that acts on n-bits
- $f(x_1,x_2,\ldots,x_n)=0$ or 1 where x_i's = 0 or 1
- If f always equals 0 or 1 it is constant.
- If f gives 50% 1 and 50% 0, it is balanced.
- Determine if f is constant, balanced, or neither.
- On a classical computer there are 2^n inputs or possible 0,1 combinations to test to get an answer.
- If n=32, there are 4,290,000,000 inputs to test.

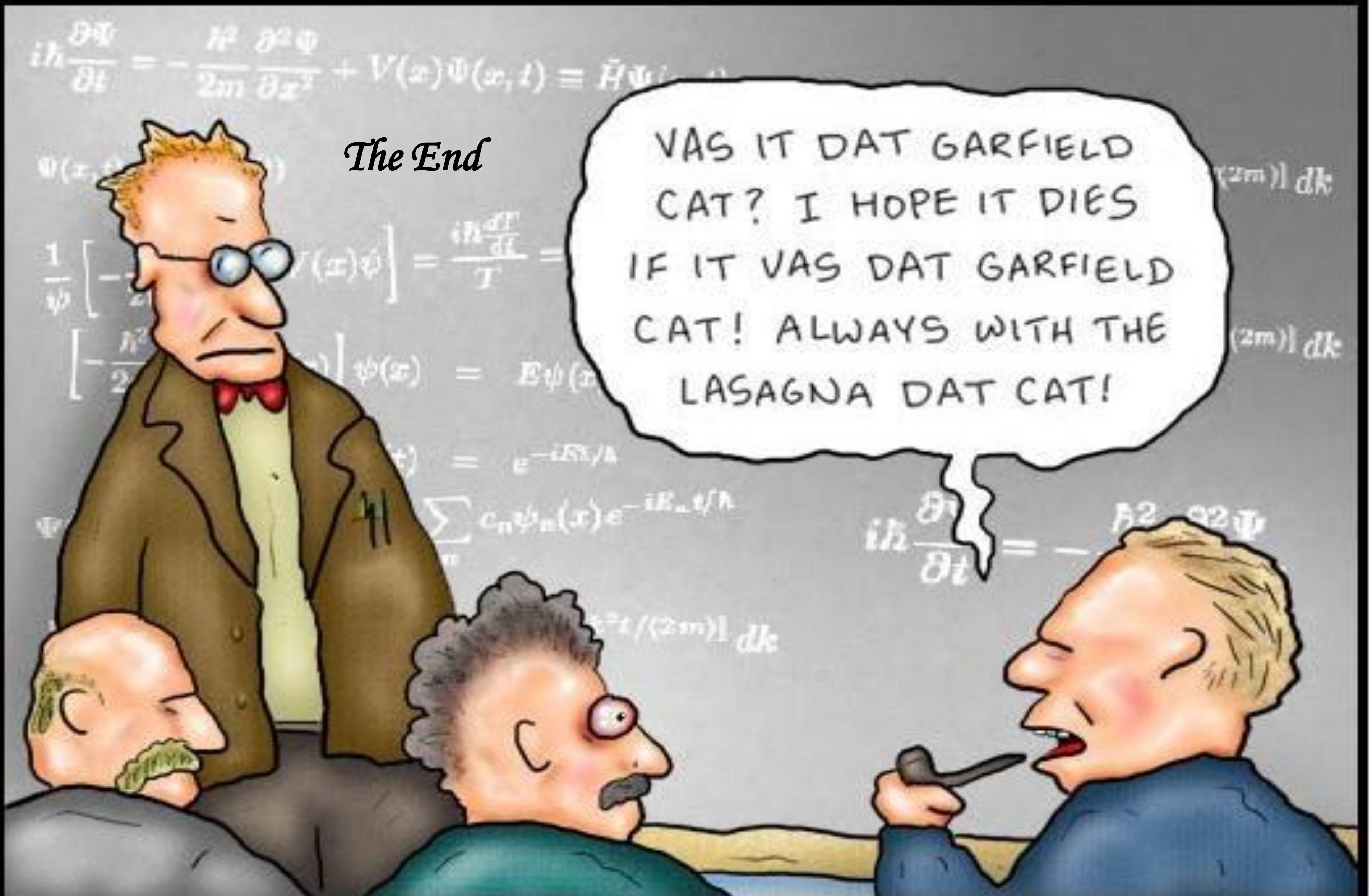
Deutsch-Jozsa Alogorithm

- We have n qubits $|x\rangle=|x_1,x_2,\ldots,x_n\rangle$
- Set all x_i's to 0 and add a $|1\rangle$ ← is crucial
- Input is $|0,0,\ldots,0\rangle|1\rangle$
- Apply a Hadamard gate
- Apply a f-CNOT gate
- Apply another Hadamard gate
- Result is $\sum|y_i\text{'s}\rangle|1\rangle$. Where $\sum|y_i\text{'s}\rangle$ is a sum (superposition) of all posible 2^n qubit states.

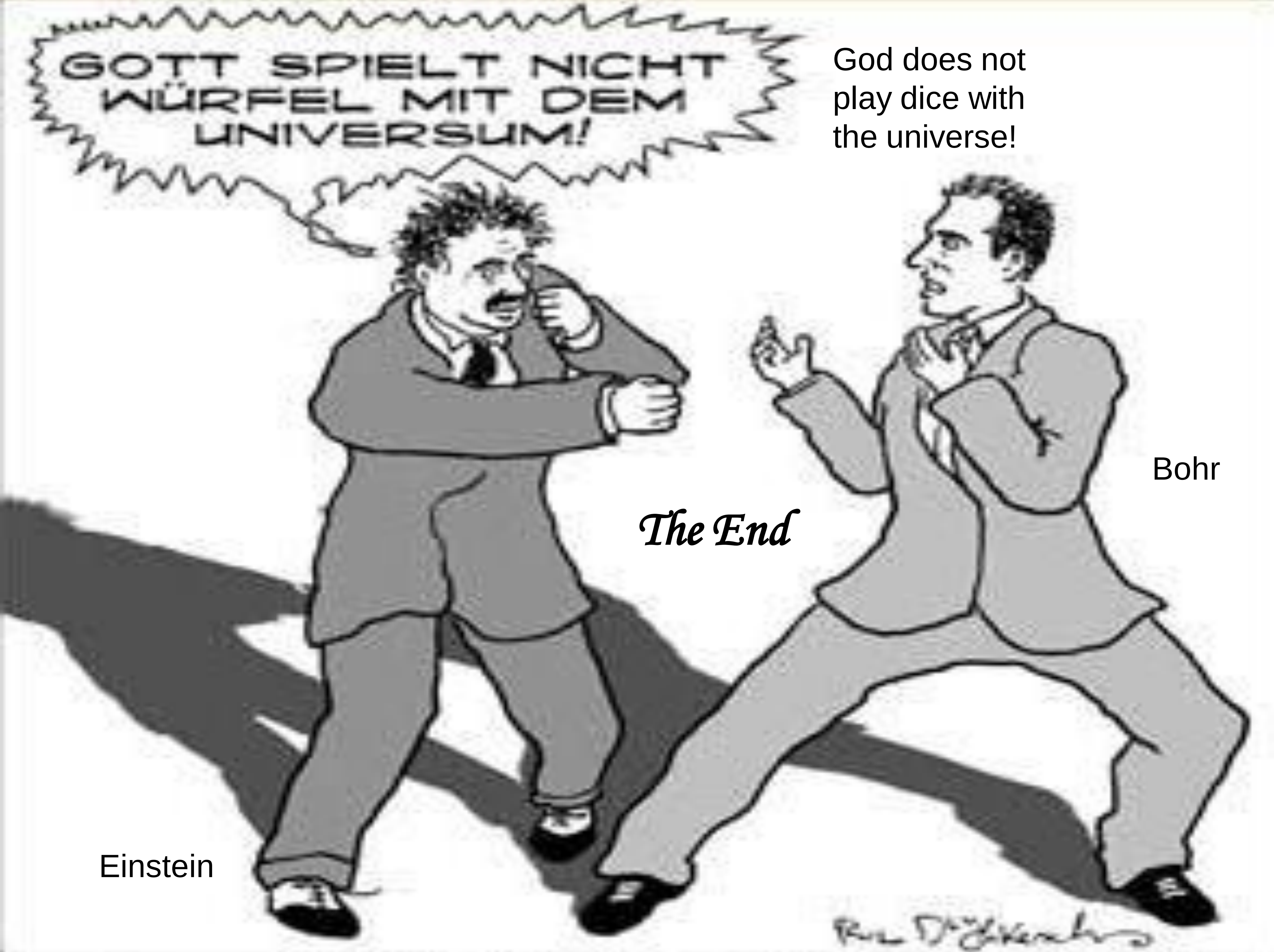
Conclusion

- $\sum|y_i$'s› $=A_0|0,0,\ldots,0\rangle + A_1|0,0,\ldots,1\rangle + \ldots + A_{2^n}|1,1,\ldots,1\rangle$
- MAKE A MEASUREMENT!
- See if $\sum|y_i$'s› is in state $|0,0,\ldots,0\rangle$
- Result is $P_0 = |A_0|^2$, the probability that $\sum|y_i$'s› is in state $|0,0,\ldots,0\rangle$
- If $P_0 = 1$ f is constant. If $P_0 = 0$ f is balanced. Any other probability f is neither.

The End



Once again Niels Bohr interrupted Schrödinger's lecture with his inane comments.



God does not play dice with the universe!

Bohr

The End

Einstein