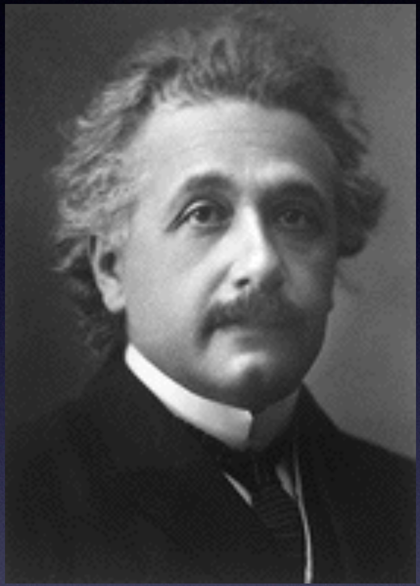
Special Relativity

Kristin Schleich and Don Witt
Department of Physics and Astronomy
University of British Columbia

1905 - Three papers were published by Einstein



Brownian Motion

Einstein discovered a kinetic theory to account for the properties of suspensions, i.e. liquids with solid particles suspended in them.

Photo Electric Effect

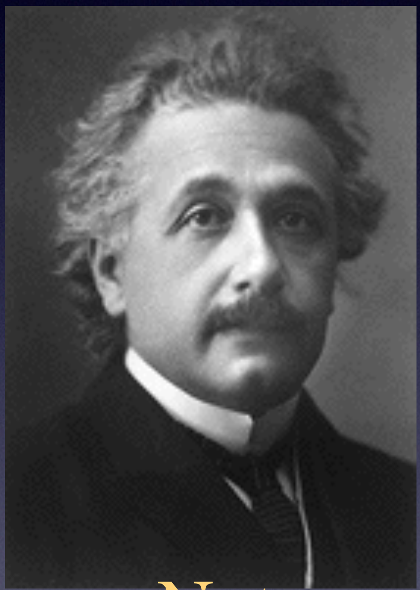
Einstein proposed the theory behind the photo electric effect.

Special Relativity

Einstein proposed the theory of special relativity.

Einstein also began his work on a theory of gravity consistent with special relativity. This theory, general relativity, was published in 1916.

1921 - Einstein awarded the Nobel prize in Physics



"... for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect ... "

Not specifically for relativity!

Russell A. Hulse

Joseph H. Taylor Jr.

The Nobel prize for relativity was awarded in 1993

"for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"

“On the Electrodynamics of Moving Bodies”

Not titled special relativity

3. *Zur Elektrodynamik bewegter Körper;* *von A. Einstein.*

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Magneten und einem Leiter. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung von Leiter und Magnet, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der bewegte sei, streng voneinander zu trennen sind. Bewegt sich nämlich der Magnet und ruht der Leiter, so entsteht in der Umgebung des Magneten ein elektrisches Feld von gewissem Energiewerte, welches an den Orten, wo sich Teile des Leiters befinden, einen Strom erzeugt. Ruht aber der Magnet und bewegt sich der Leiter, so entsteht in der Umgebung des Magneten kein elektrisches Feld, dagegen im Leiter eine elektromotorische Kraft, welcher an sich keine Energie entspricht, die aber — Gleichheit der Relativbewegung bei den beiden ins Auge gefaßten Fällen vorausgesetzt — zu elektrischen Strömen von derselben Größe und demselben Verlaufe Veranlassung gibt, wie im ersten Falle die elektrischen Kräfte.

Beispiele ähnlicher Art, sowie die mißlungenen Versuche, eine Bewegung der Erde relativ zum „Lichtmedium“ zu konstatieren, führen zu der Vermutung, daß dem Begriffe der absoluten Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik keine Eigenschaften der Erscheinungen entsprechen, sondern daß vielmehr für alle Koordinatensysteme, für welche die mechanischen Gleichungen gelten, auch die gleichen elektrodynamischen und optischen Gesetze gelten, wie dies für die Größen erster Ordnung bereits erwiesen ist. Wir wollen diese Vermutung (deren Inhalt im folgenden „Prinzip der Relativität“ genannt werden wird) zur Voraussetzung erheben und außerdem die mit ihm nur scheinbar unverträgliche

Special Relativity

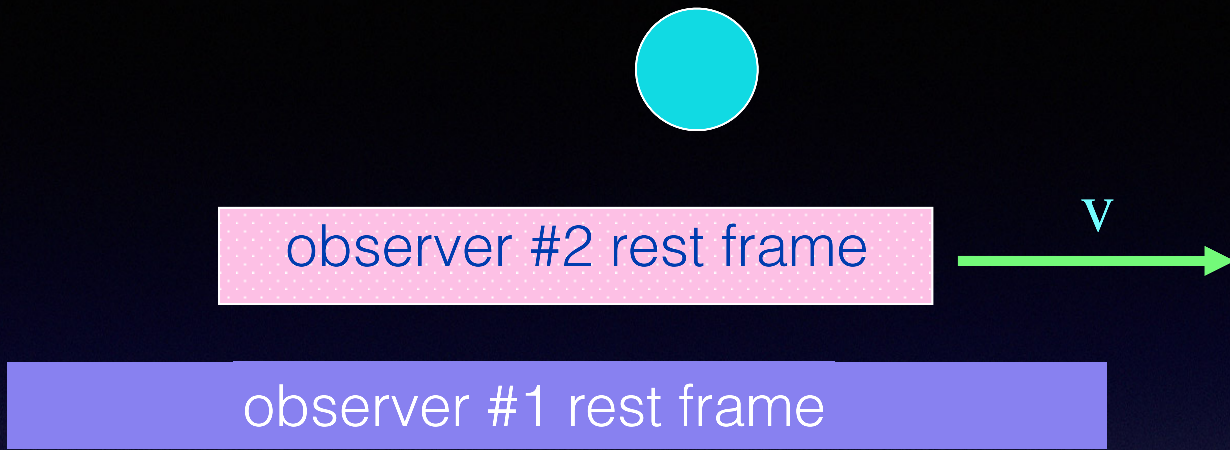
Not a theory about interactions rather a theory about invariance

Old theory of invariance - Galilean invariance

Physics is the same in all inertial frames

Time is absolute - the same in all inertial frames

inertial frame - frame travelling with constant velocity



Frame 2 travels with velocity v with respect to frame 1

velocity of blue ball in rest frame 1 is u

$a = \Delta u / \Delta t = F / m$ in frame 1

Frame 1 travels with velocity -v with respect to frame 2

velocity of blue ball in rest frame 2 is u-v

$a = \Delta\,(u\text{-}v) / \Delta\,t = (\Delta\,u\text{-}\Delta\,v) / \Delta\,t = \Delta\,u / \Delta\,t$

as v is constant

$a = \Delta\,u / \Delta\,t = F / m$ also in frame 2

Physics is the same

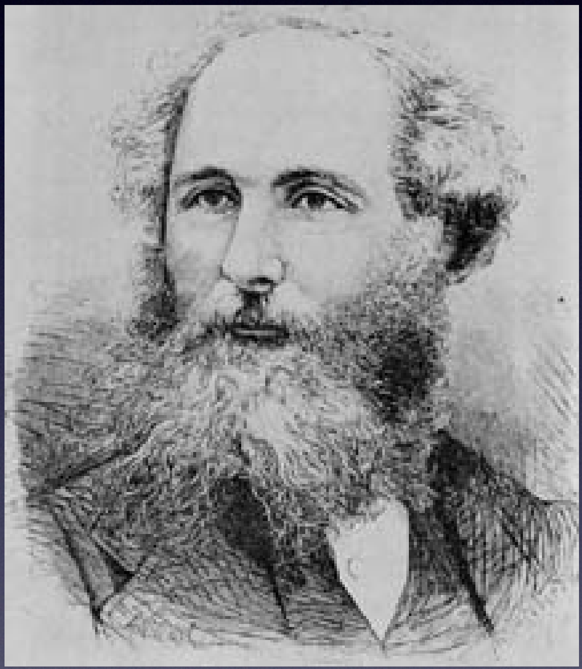
Worked great for Newton

Gravitational Force is
Galilean invariant

Sir Isaac Newton

Electromagnetism

But then came the first unified field theory, electromagnetism



James Clerk Maxwell

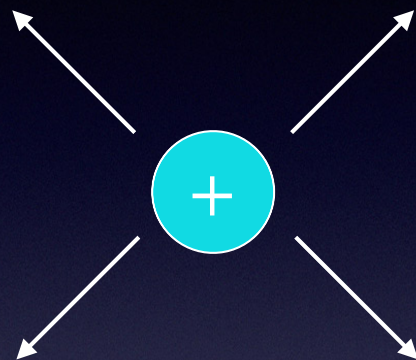
One defines a magnetic field B by its fundamental effect on a moving charged particle, namely $F = q\, v \times B$

a charge q in an electromagnetic field feels a force

$$F = q\,E + q\,v \times B$$

Problem: not Galilean invariant

Physics in frame 1



B=0

E

observer #1 rest frame

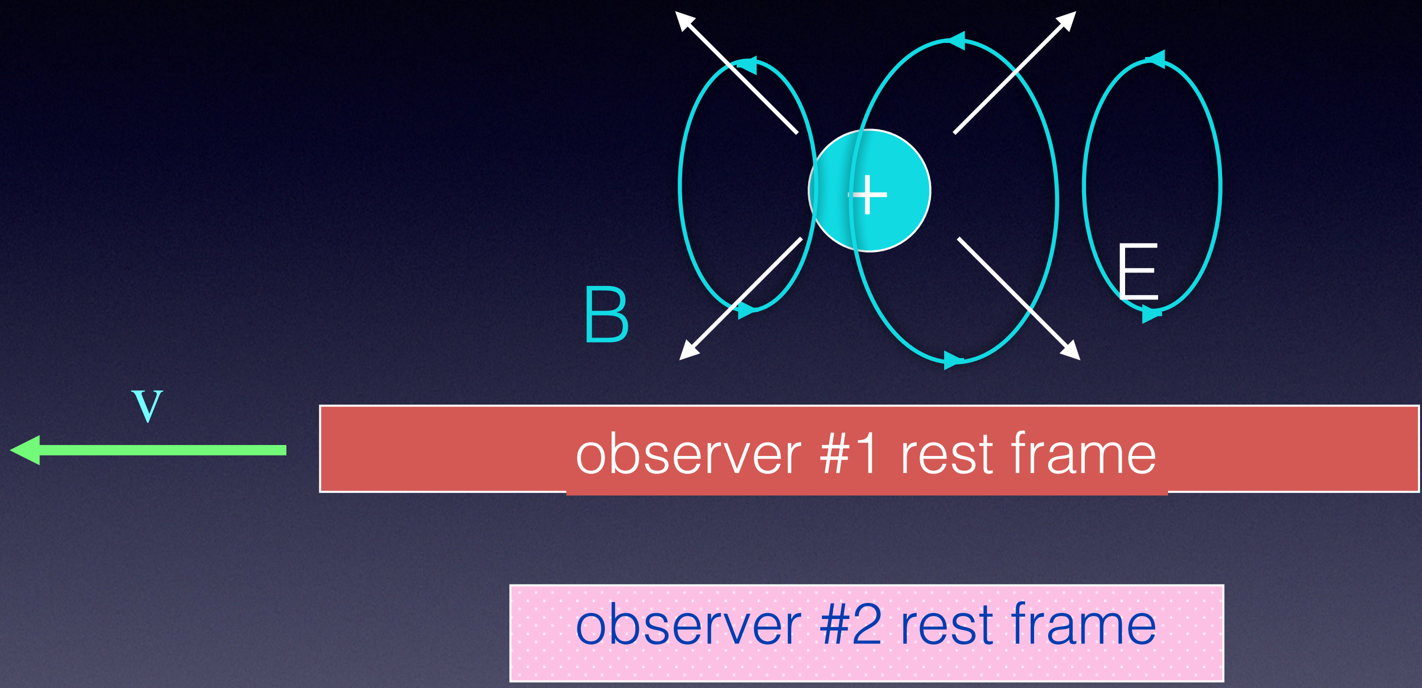
F=qE

observer #2 rest frame

v

Physics in frame 2

moving charge generates a magnetic field



$$F = q\,E + q\,v \times B$$

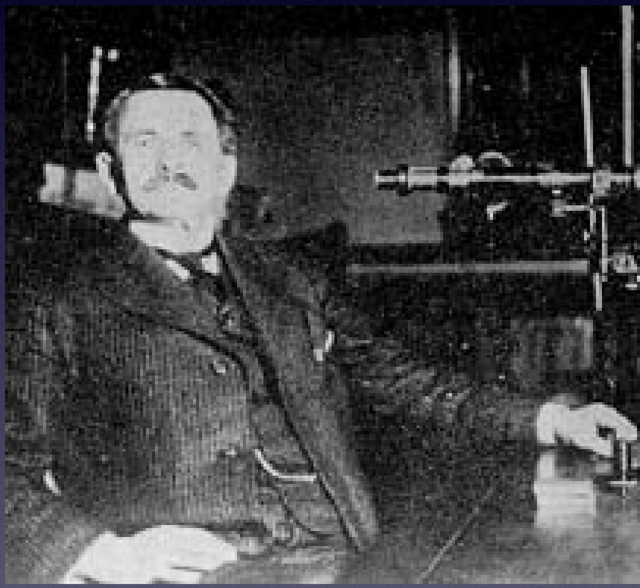
Force is different in frame 1 and frame 2

Not invariant!

Michelson-Morley experiment

The speed of light is the same in all directions

Implies that there cannot be an underlying "ether" with and "absolute rest frame"



Albert Michelson

Einstein - put it all together

Principles of Special Relativity

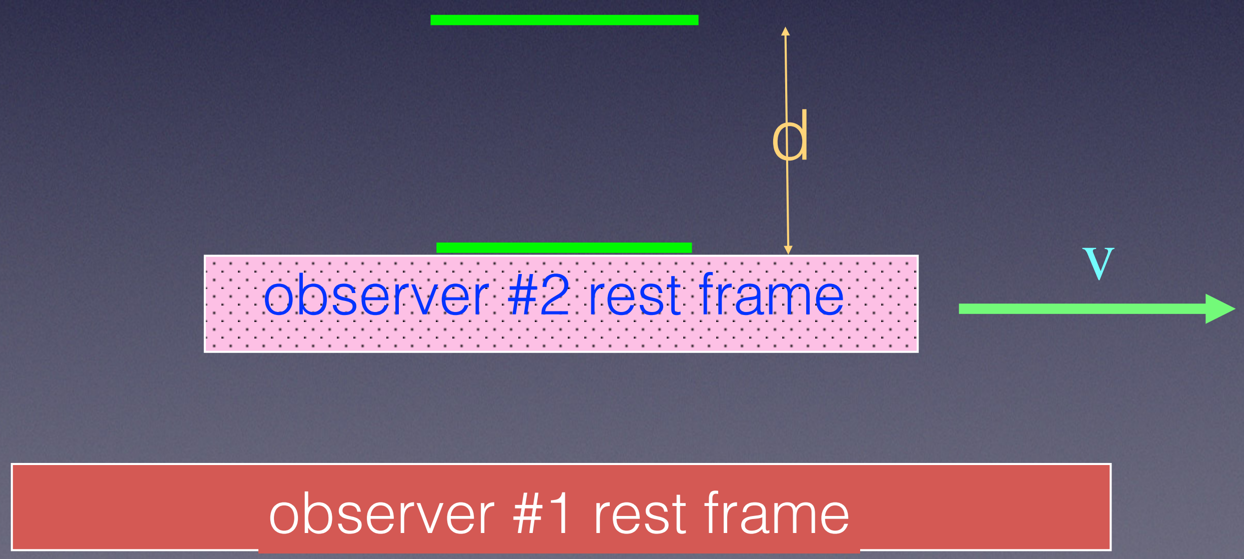
Physics is the same in all inertial frames.

The speed of light is the same constant and the limiting speed in all inertial frames.

- Note that the speed of light is the same to all inertial (eg constant velocity observers)
- Means that Galilean invariance does not apply to the laws of physics
- What does this imply about space and time?

Gedanken Experiment

Consider an “ideal clock” made of a light beam moving between two mirrors. Place it on a platform which itself moves at speed v with respect to observer #1. Observer #2 moves with the platform.



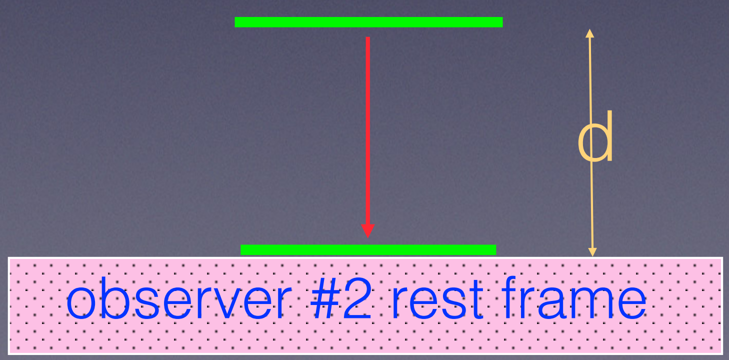
Gedanken Experiment

Time interval defined by successive events:

departure of light pulse from upper mirror

arrival of light pulse at lower mirror

First in observer #2's frame: clock is at rest



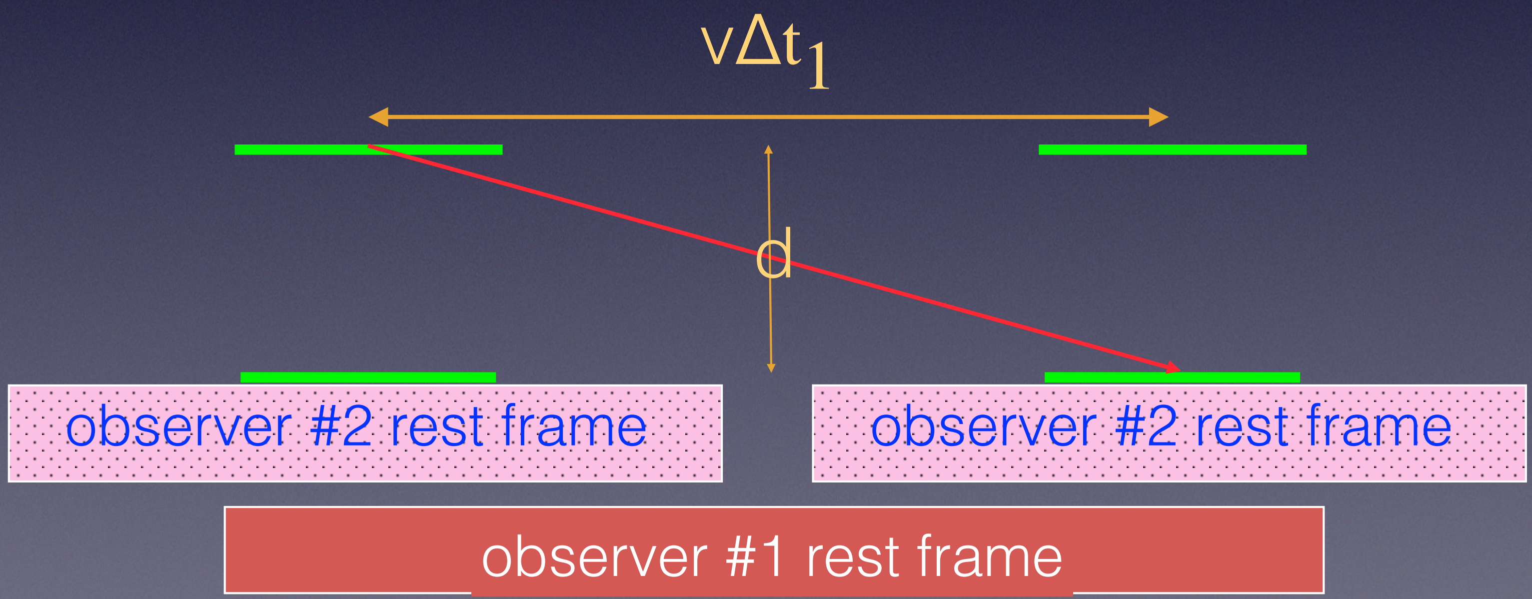
$\Delta t_2 = d/c$

Gedanken Experiment

Now consider from observer #1 rest frame.

Event 1: photon leaves top mirror

Event 2: photon arrives at bottom mirror



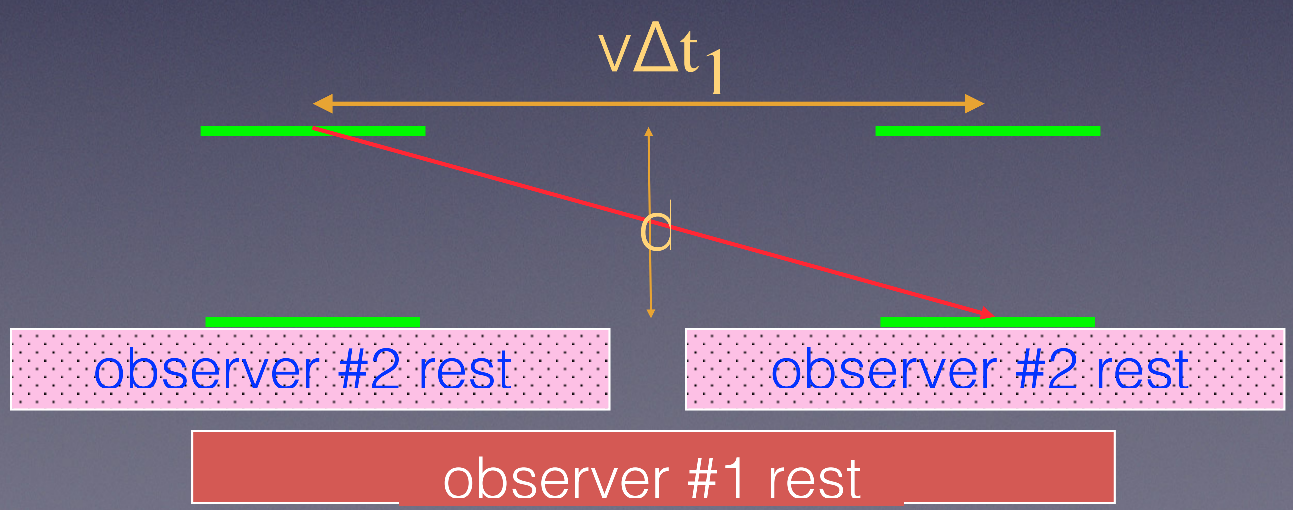
Gedanken Experiment

From observer #1 rest frame.

distance travelled by photon is hypotenuse of triangle with sides $v\Delta t_1$ and d.

light travels with speed c

$$(c\Delta t_1)^2 = (v\Delta t_1)^2 + d^2$$



Gedanken Experiment

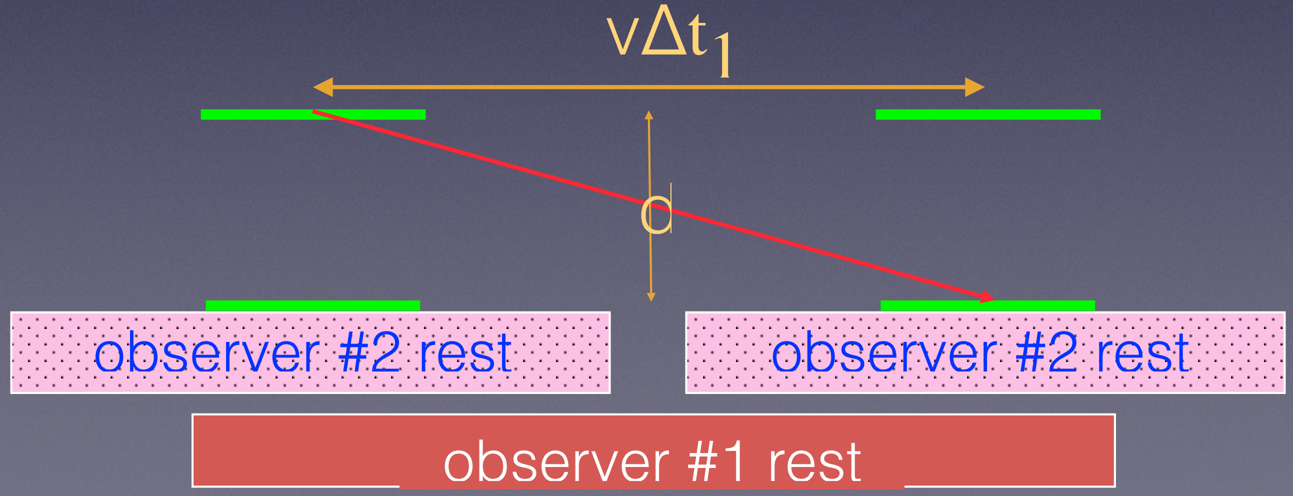
Solving,

$$\Delta t_1 = \frac{d/c}{\sqrt{1 - v^2/c^2}}$$

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_2 \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Observer 1 sees the moving clock of observer 2 run slow as gamma is always greater than 1

Time dilation



Lorentz transformation

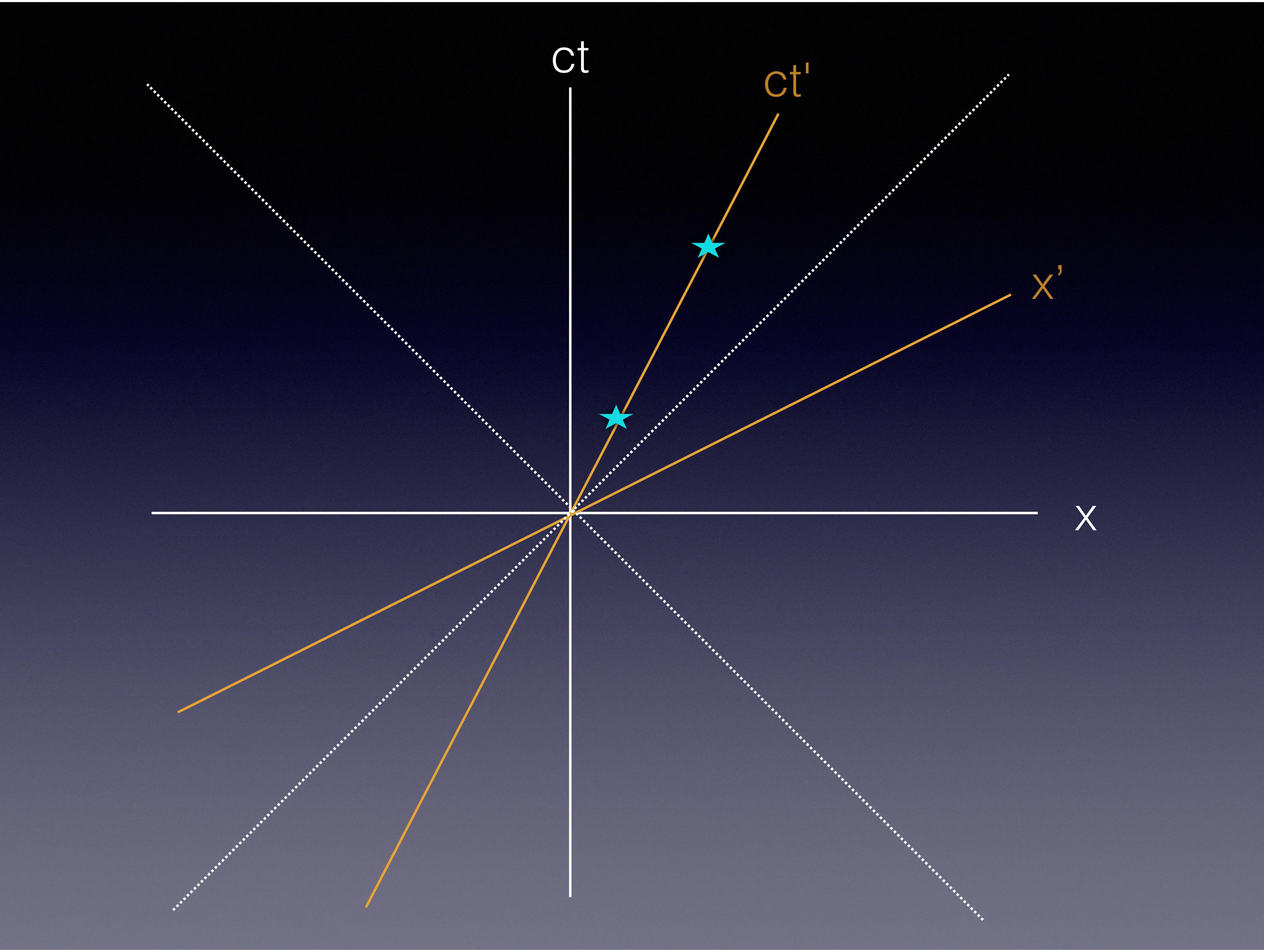
$$t' = \gamma(t - \frac{v}{c^2}x)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

Where t’ x’ y’ z’ are the coordinates of the reference frame moving with velocity v along the x axis of the first reference frame



ct
ct'
x'
x

4-vectors

No absolute time so need 4 coordinates, time and spatial position to describe any event

$$R=(ct, x, y, z)$$

velocities also must have 4 components

$$u=dR/d\tau=(cdt/d\tau, dx/d\tau, dy/d\tau, dz/d\tau)$$

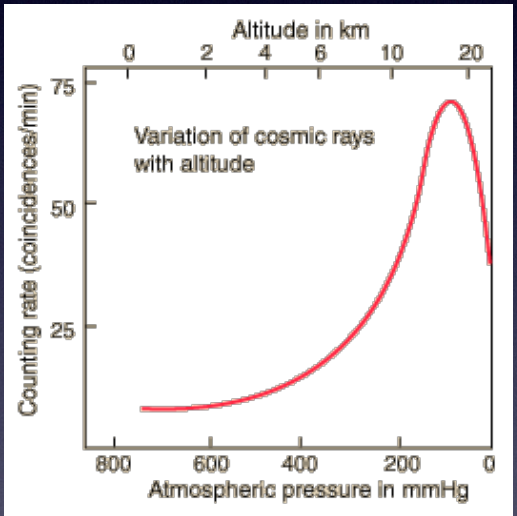
τ (tau) is called proper time - the time of the frame instantaneously at rest with respect to the moving particle

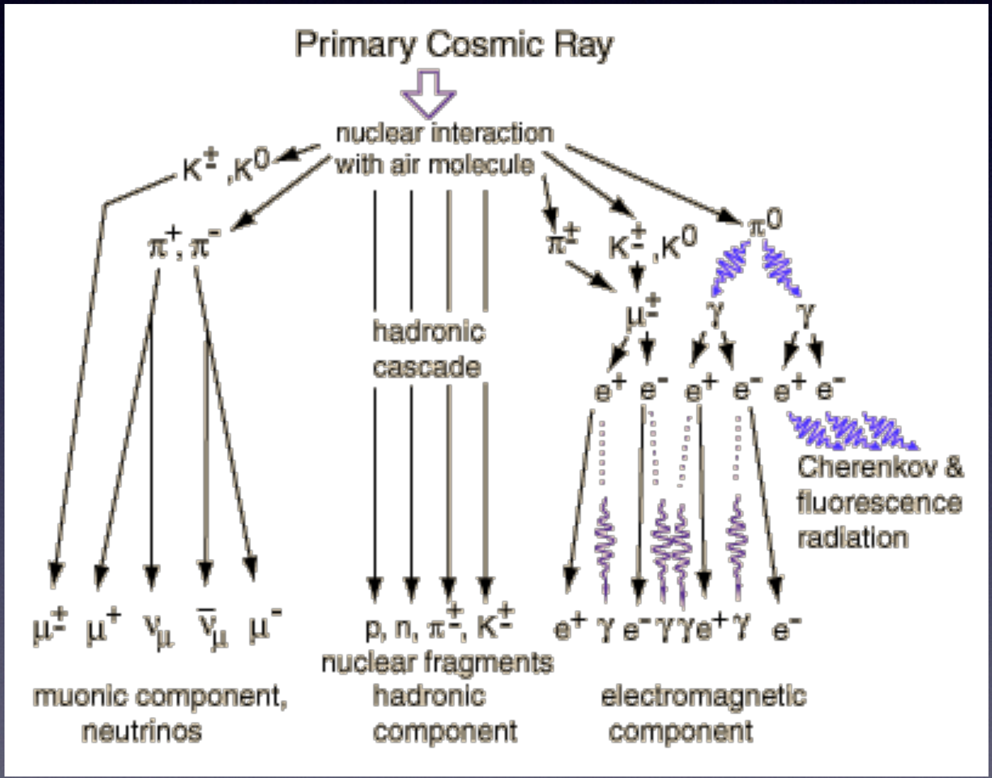
$$p = mu$$

muon - mass 207 electron masses

lifetime 2.20 microseconds

Flux of cosmic rays peaks about 15 km above surface of earth





How can we see them here at the surface of earth? Time dilation.

Energy of muon at surface is 4 GeV. Energy when produced at 15,000 m is 6 GeV. γ is about 40.

Consequences

- Time is not absolute.
  - Simultaneity is now observer dependent.
- Electromagnetism consistent with spacetime invariance (Lorentz invariance)
- Doppler shift

4-vectors

No absolute time so need 4 coordinates, time and spatial position to describe any event

$$R=(ct, x, y, z)$$

velocities also must have 4 components

$$u=dR/d\tau=(\cancel{c}dt/d\tau, dx/d\tau, dy/d\tau, dz/d\tau)$$

τ (tau) is called proper time - the time of the frame instantaneously at rest with respect to the moving particle

$$p = mu$$

Relativistic kinematics

$p = mv$ Newtonian momentum

$p = m\gamma v$

$E = m\gamma c^2$

$$E^2 - p^2c^2 = m^2c^4$$

E, p components of a 4-vector

Energy -momentum conserved in particle collisions
pion collision with stationary proton

$$\pi^- + p \rightarrow K^+ + \Sigma^-$$

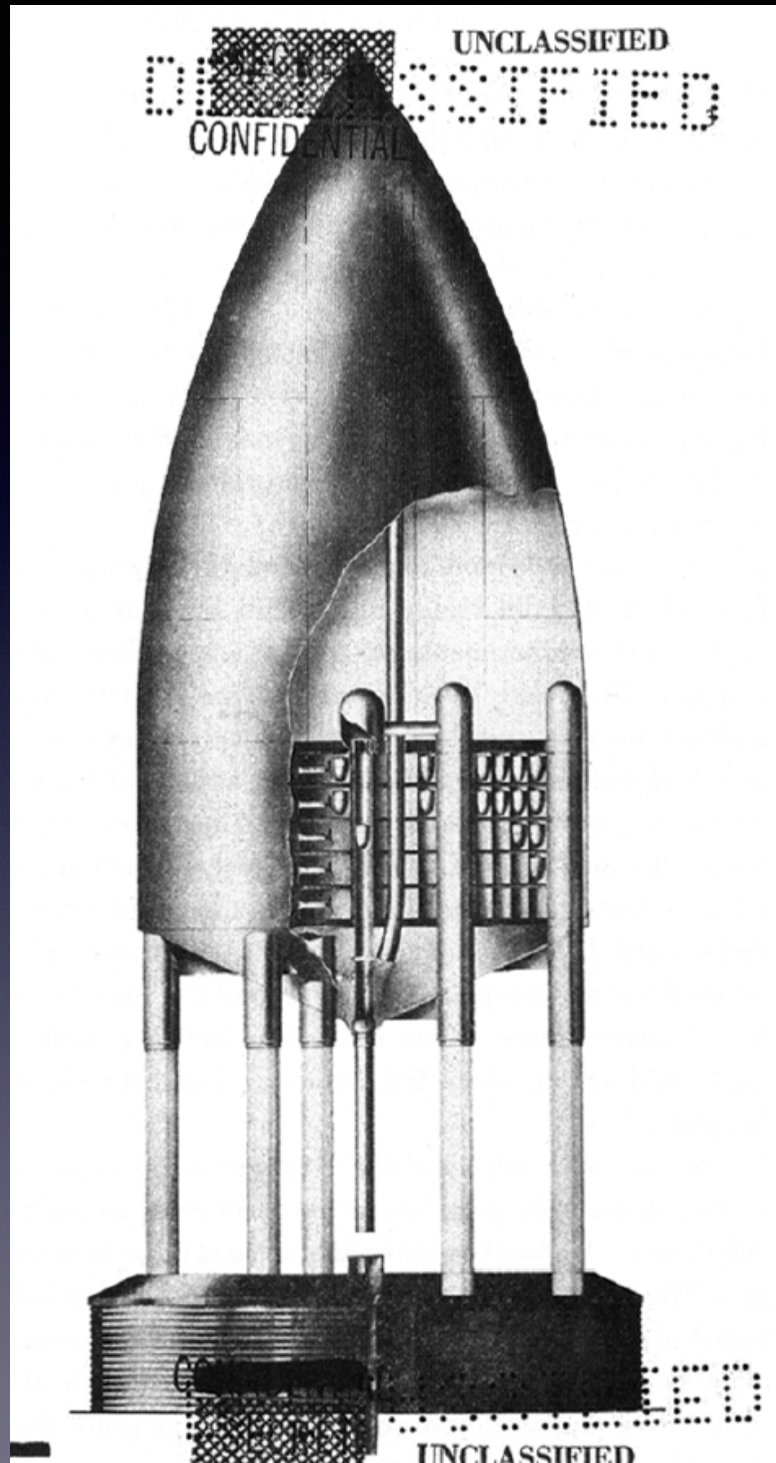
Energy -momentum conserved in particle collisions
pion collision with stationary proton

initial total mass 139.6 MeV/c^2 + 938.3 MeV/c^2 = 1077.9 MeV/c^2

final total mass 493.7 MeV/c^2 + 1189.4 MeV/c^2 = 1683.1 MeV/c^2

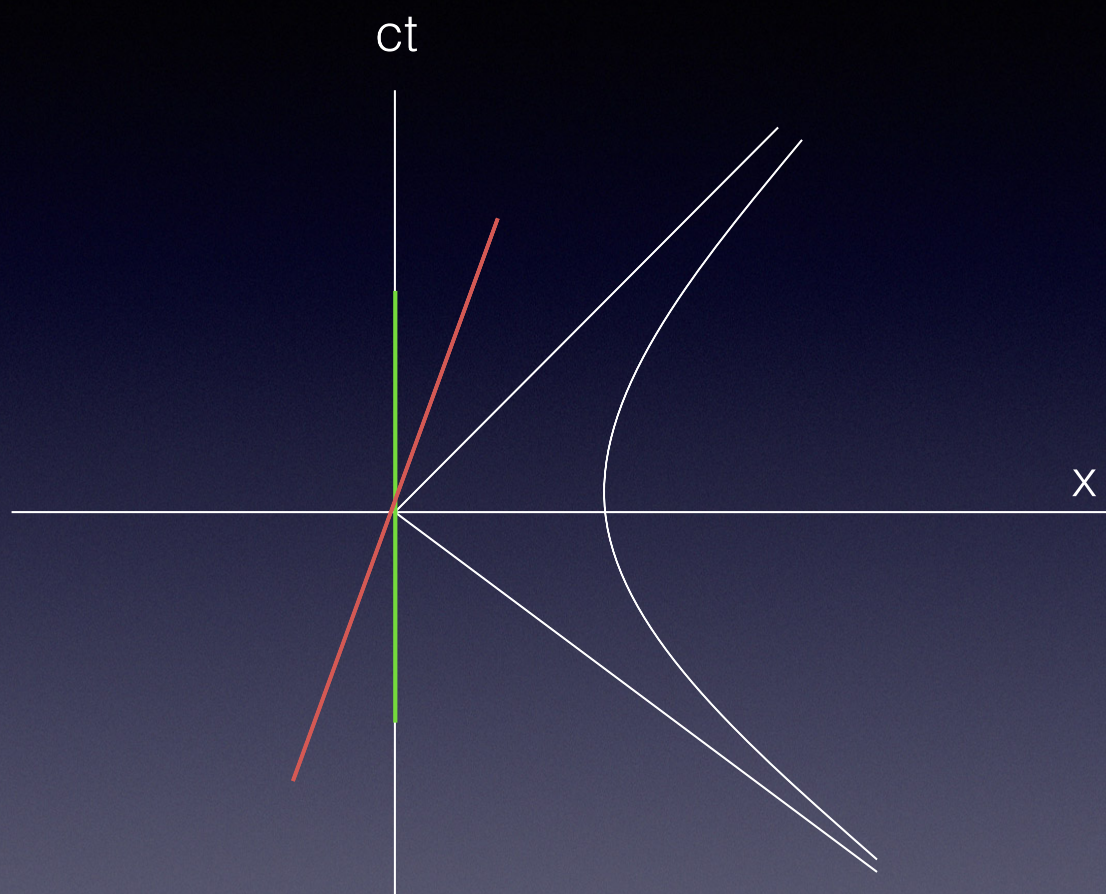
Energy converted to mass

Project Orion



Acceleration in Special Relativity

- Assume you have a rocket which can travel with constant acceleration g
- How far can you travel in a ship time of 20 years?


ct
x

Acceleration in Special Relativity

$$ct = \frac{c^2}{g}\sinh\left(\frac{g}{c}\tau\right)$$

$$x = \frac{c^2}{g}\cosh\left(\frac{g}{c}\tau\right)$$

$$\cosh b = \frac{1}{2}(e^b + e^{-b})$$

- If g = 9.8 m/s^2 and τ=20 years, x = 400 million light years

Consequences

- Newton's law of universal gravitation not invariant under special relativity.

$$F = -GMm/r^2$$

- Force acts instantaneously at a distance
- Inconsistent with the speed of light being the limiting speed

Need new theory of gravitation consistent with special relativity

Principle of Equivalence

acceleration = gravitation

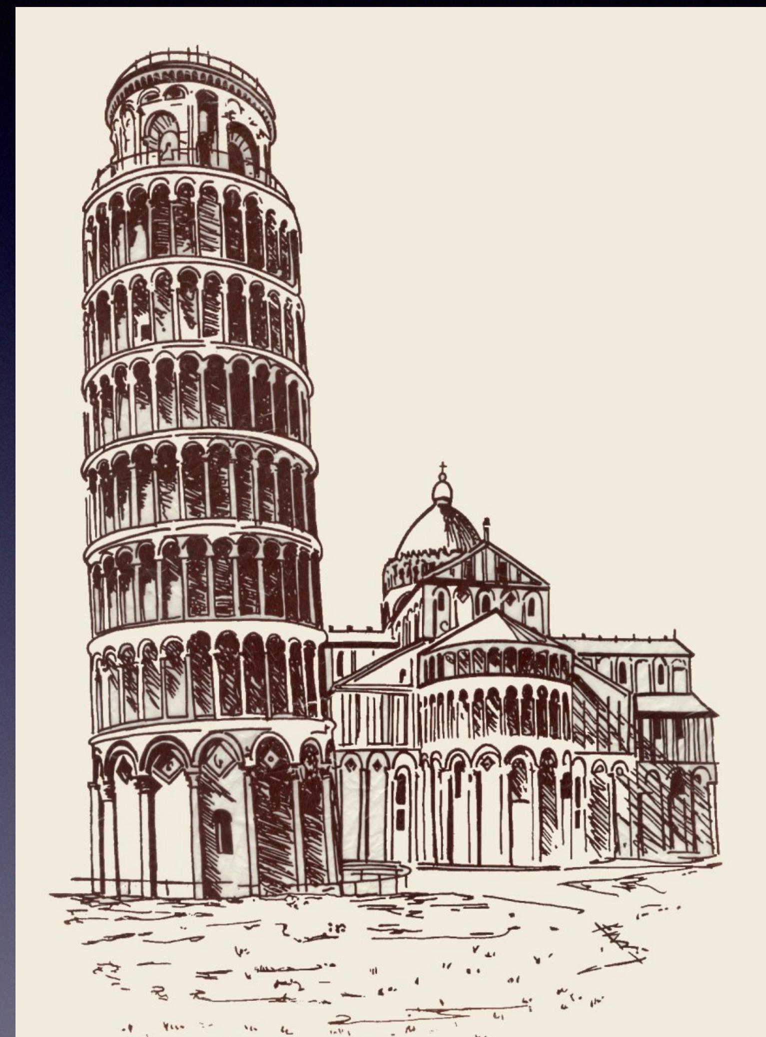
Physics is the same in a uniform gravitational field
as in a uniformly accelerated frame

Gravitational mass = inertial mass

g

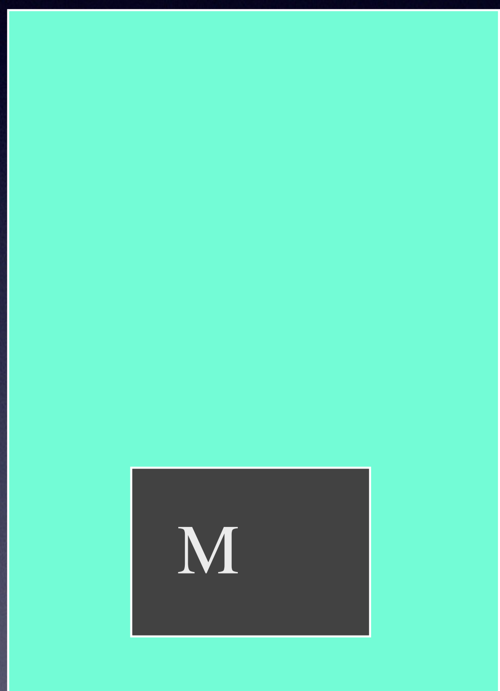
feathers

lead



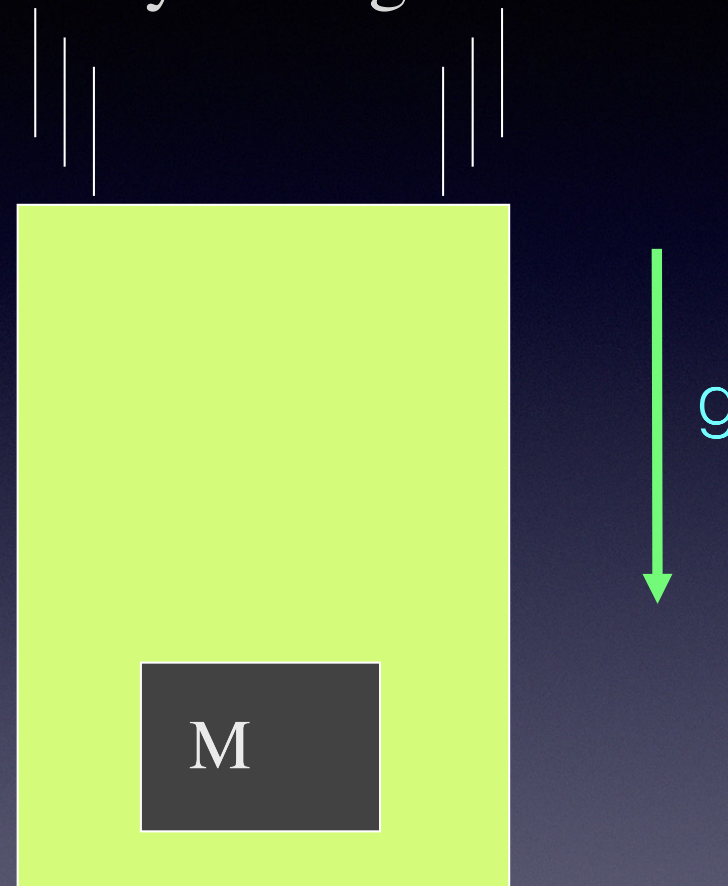
Lead and feathers fall with the same acceleration

verified to 1.5×10^{-13} through lunar ranging observer in outer space

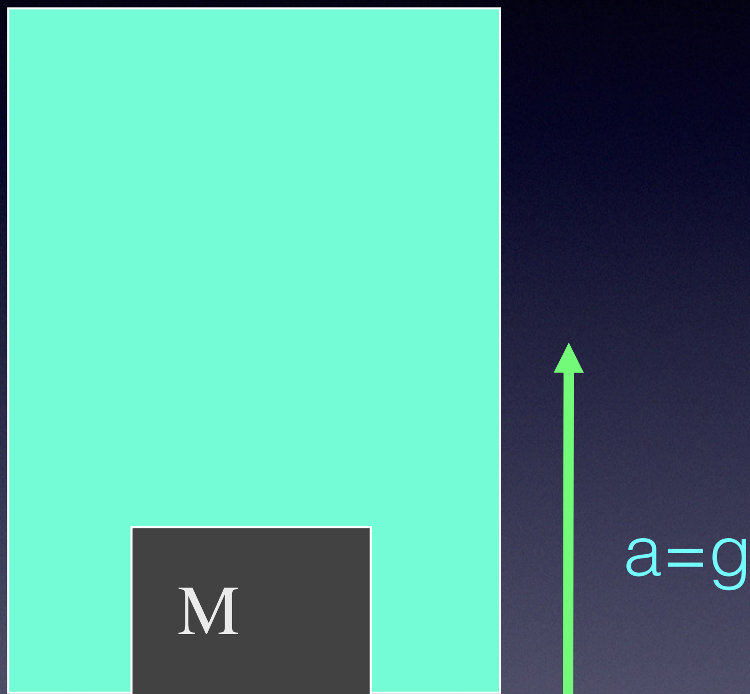


floor exerts no force on mass M

Freely falling in uniform g

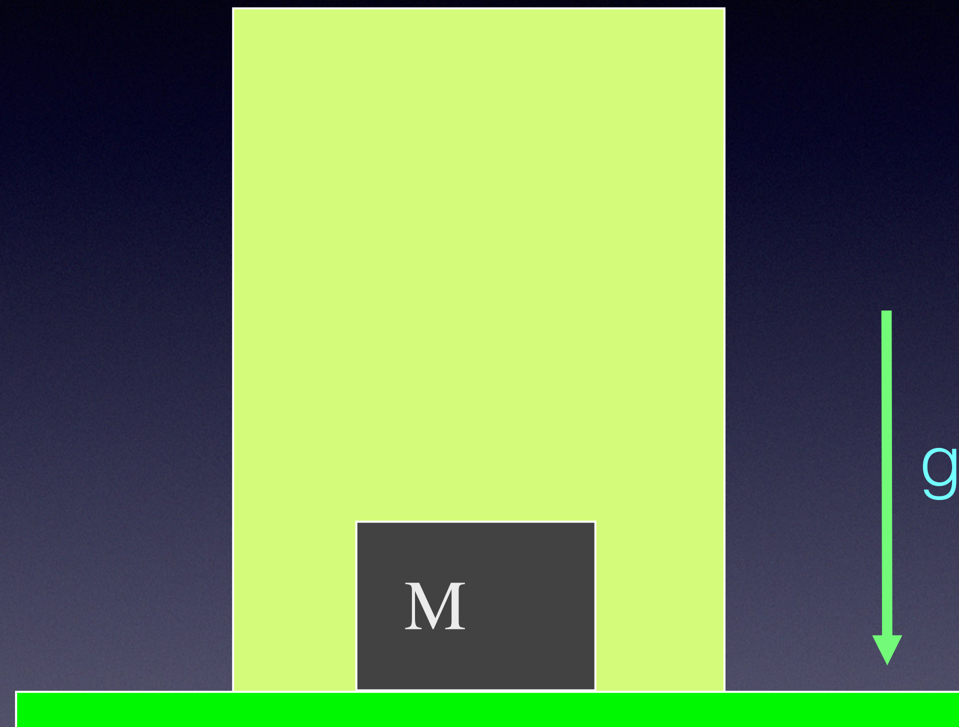


mass M falls at exactly the same rate as the elevator, floor exerts no force on M accelerated observer



mass M feels force Mg from floor

Freely falling in uniform g

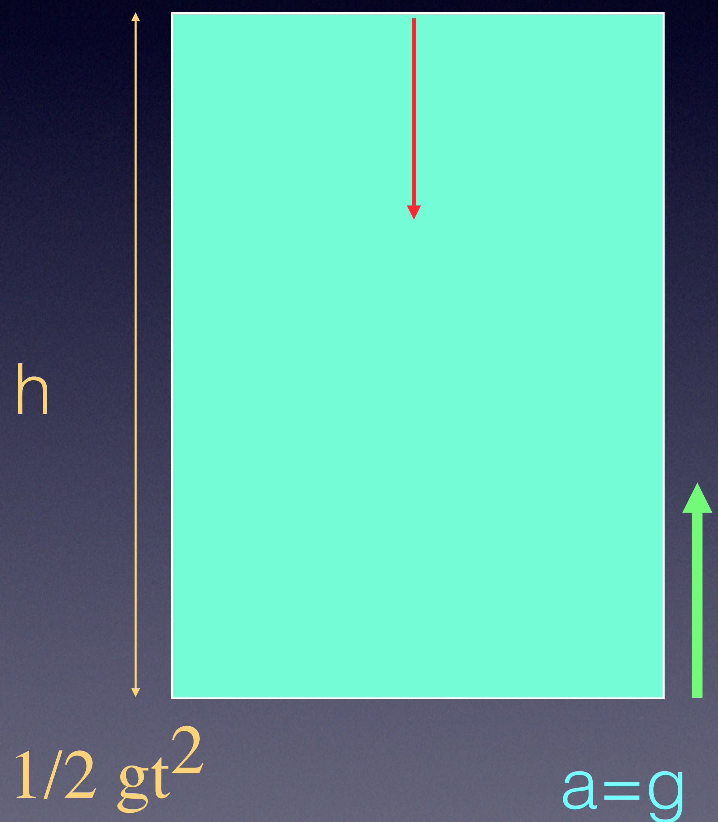


floor exerts force Mg on mass M

consequences - gravitational time dilation.

$$z_N(t) = d + 1/2\ gt^2$$

Place a clock in the nose (N) of the rocket and another in the tail (T) of the rocket. What is the time $\Delta\tau_T$ between receiving photons at the tail if they are released from the nose $\Delta\tau_N$ apart?



$$z_N(0) - z_T(t_1) = ct_1$$

$$z_N(\Delta\tau_N) - z_T(t_1+\Delta\tau_T) = c(t_1+\Delta\tau_T - \Delta\tau_N)$$

using t_1 = h/c a little algebra one arrives at the result

$$\Delta\tau_T = \Delta\tau_N(1 - gh/c^2)$$

gh - gravitational potential energy

clock at higher gravitational potential runs fast according to one at lower gravitational potential

equivalently clock deep in a gravitational potential runs slow according to one at higher gravitational potential

GPS - Accuracy (latitude, longitude, and altitude) 5-10 meters.
Network of 24 satellites in high orbit altitude
- 20,000 km above ground
- speed 14,000 km/hour
- clock - accurate to a nanosecond

Special Relativity - lose 7 microseconds a day relative to ground clock

General Relativity - gain
45 microseconds a day

Error in position = 10 km per day!!!

Principle of Equivalence is underpinning of General Relativity - Gravity as the Geometry of Curved Spacetime

Why was the Nobel Prize for relativity awarded 77 years after Einstein proposed the theory?

Partially, it is only in the last 30 years that we have been able to see some of its more spectacular consequences

Russell A. Hulse Joseph H. Taylor Jr.

PSR B1913+16 is two neutron stars in close orbit, one a pulsar emitting detectible radio waves

A pulsar is a neutron star

Mass 1.44 M_{sun}

Radius 10 km - i.e. incredibly dense

with an extremely rapid period of rotation

Period of rotation of .059 seconds

extremely accurate clocks (part in 10^{14})

system allows determination of relativistic effects to high precision

precession of periastron 4.22659 degrees/year

time delay

gravitational redshift

loss of energy due to gravitational radiation

Extremely strong astrophysical evidence of black holes

Newtonian escape velocity $v = \sqrt{2GM/R}$

$c = \sqrt{2GM/R}$ if the speed of light is the speed limit

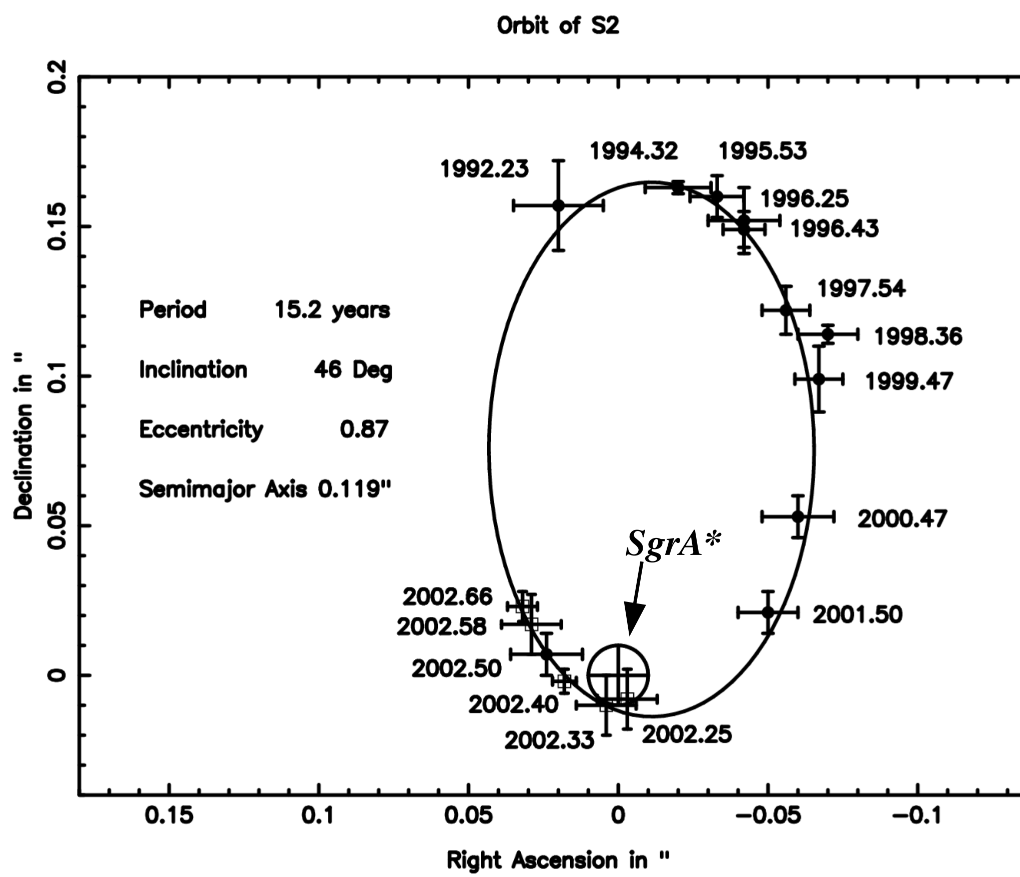
$R = 2GM/c^2$ is radius of mass for which light cannot escape - turns out to be exactly the Schwarzschild radius of a black hole

Recent spectacular evidence of a black hole

Mass- 3 x 10 ^{6} M_{sun}

in center of our galaxy

Strong evidence of other black holes both stellar sized and in centers of galaxies



Schodel et al (2002)

Detection of Gravitational Waves