1905 - Three papers were published by Einstein

Brownian Motion
Einstein discovered a kinetic theory to account for the properties of suspensions, i.e. liquids with solid particles suspended in them.

Photo Electric Effect
Einstein proposed the theory behind the photo electric effect.

Special Relativity
Einstein proposed the theory of special relativity.

Einstein also began his work on a theory of gravity consistent with special relativity. This theory, general relativity, was published in 1916.
1921 - Einstein awarded the Nobel prize in Physics

"... for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect ..."

Not specifically for relativity!
The Nobel prize for relativity was awarded in 1993

"for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"
"On the Electrodynamics of Moving Bodies"

Not titled special relativity


Beispiele ähnlicher Art, sowie die mißlungenen Versuche, eine Bewegung der Erde relativ zum "Lichtmedium" zu konstatieren, führen zu der Vermutung, daß dem Begriff der absoluten Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik keine Eigenschaften der Erscheinungen entsprechen, sondern daß vielmehr für alle Koordinatensysteme, für welche die mechanischen Gleichungen gelten, auch die gleichen elektrodynamischen und optischen Gesetze gelten, wie dies für die Größen erster Ordnung bereits erwiesen ist. Wir wollen diese Vermutung (denen Inhalt im folgenden "Prinzip der Relativität" genannt werden wird) zur Voraussetzung erheben und außerdem die mit ihm nur scheinbar unverträgliche
Special Relativity

Not a theory about interactions rather a theory about invariance
Old theory of invariance -
Galilean invariance

Physics is the same in all inertial frames

Time is absolute - the same in all inertial frames

inertial frame - frame travelling with constant velocity
Frame 2 travels with velocity $v$ with respect to frame 1

velocity of blue ball in rest frame 1 is $u$

$$a = \frac{\Delta u}{\Delta t} = \frac{F}{m} \quad \text{in frame 1}$$
Frame 1 travels with velocity \(-v\) with respect to frame 2.

Velocity of blue ball in rest frame 2 is \(u-v\).

\[
a = \frac{\Delta (u-v)}{\Delta t} = \frac{(\Delta u - \Delta v)}{\Delta t} = \frac{\Delta u}{\Delta t}
\]

as \(v\) is constant.

\[
a = \frac{\Delta u}{\Delta t} = \frac{F}{m}
\]

also in frame 2.

Physics is the same.
Worked great for Newton

Gravitational Force is Galilean invariant

Sir Isaac Newton
Electromagnetism

But then came the first unified field theory, electromagnetism

One defines a magnetic field $B$ by its fundamental effect on a moving charged particle, namely $F = q \, v \times B$

A charge $q$ in an electromagnetic field feels a force

$$F = q \, E + q \, v \times B$$

Problem: not Galilean invariant
Physics in frame 1

\[ B = 0 \]

\[ E \]

\[ F = qE \]

observer #1 rest frame

observer #2 rest frame
Physics in frame 2

moving charge generates a magnetic field

\[ F = qE + qv \times B \]

Force is different in frame 1 and frame 2
Not invariant!
Michelson-Morley experiment

The speed of light is the same in all directions

Implies that there cannot be an underlying “ether” with and “absolute rest frame”

Albert Michelson
Einstein - put it all together

Principles of Special Relativity

Physics is the same in all inertial frames.

The speed of light is the same constant and the limiting speed in all inertial frames.
• Note that the speed of light is the same to all inertial (e.g., constant velocity) observers.

• Means that Galilean invariance does not apply to the laws of physics.

• What does this imply about space and time?
Gedanken Experiment

Consider an “ideal clock” made of a light beam moving between two mirrors. Place it on a platform which itself moves at speed $v$ with respect to observer #1. Observer #2 moves with the platform.
Gedanken Experiment

Time interval defined by successive events:
departure of light pulse from upper mirror
arrival of light pulse at lower mirror

First in observer #2’s frame: clock is at rest

\[ \Delta t_2 = \frac{d}{c} \]
Gedanken Experiment

Now consider from observer #1 rest frame.

Event 1: photon leaves top mirror

Event 2: photon arrives at bottom mirror

(observer #2 rest frame)

(observer #2 rest frame)

(observer #1 rest frame)
Gedanken Experiment

From observer #1 rest frame.

distance travelled by photon is hypotenuse of triangle with sides $v \Delta t_1$ and $d$.

light travels with speed $c$

$$(c\Delta t_1)^2 = (v\Delta t_1)^2 + d^2$$
Gedanken Experiment

Solving,

\[ \Delta t_1 = \frac{d/c}{\sqrt{1 - v^2/c^2}} \]

\[ \Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_2 \]

\[ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \]

Observer 1 sees the moving clock of observer 2 run slow as gamma is always greater than 1

Time dilation

\[ \gamma \Delta t_1 \]

observer #1 rest

observer #2 rest

observer #2 rest
Lorentz transformation

\[ t' = \gamma (t - \frac{v}{c^2} x) \]
\[ x' = \gamma (x - vt) \]
\[ y' = y \]
\[ z' = z \]

Where \( t' \) \( x' \) \( y' \) \( z' \) are the coordinates of the reference frame moving with velocity \( v \) along the \( x \) axis of the first reference frame.
4-vectors

No absolute time so need 4 coordinates, time and spatial position to describe any event

\[ R = (ct, x, y, z) \]

velocities also must have 4 components

\[ u = \frac{dR}{d\tau} = (\frac{cdt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}) \]

\( \tau \) (tau) is called proper time - the time of the frame instantaneously at rest with respect to the moving particle

\[ p = mu \]
muon - mass 207 electron masses
lifetime 2.20 microseconds
Flux of cosmic rays peaks about 15 km above surface of earth

How can we see them here at the surface of earth? Time dilation.
Energy of muon at surface is 4 GeV. Energy when produced at 15,000 m is 6 GeV. \( \gamma \) is about 40.
Consequences

- Time is not absolute.
- Simultaneity is now observer dependent.
- Electromagnetism consistent with spacetime invariance (Lorentz invariance)
- Doppler shift
4-vectors

No absolute time so need 4 coordinates, time and spatial position to describe any event

\[ R = (ct, x, y, z) \]

velocities also must have 4 components

\[ u = \frac{dR}{d\tau} = (\frac{cdt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}) \]

\( \tau \) (tau) is called proper time - the time of the frame instantaneously at rest with respect to the moving particle

\[ p = m u \]
Relativistic kinematics

\[ p = m v \]  \hspace{1cm} \text{Newtonian momentum}

\[ p = m \gamma v \]

\[ E = m \gamma c^2 \]

\[ E^2 - p^2 c^2 = m^2 c^4 \]

E, p components of a 4-vector
Energy -momentum conserved in particle collisions
pion collision with stationary proton

\[ \pi^- + p \rightarrow K^+ + \Sigma^- \]

Energy -momentum conserved in particle collisions
pion collision with stationary proton

Initial total mass \( 139.6 \text{ MeV/c}^2 + 938.3 \text{ MeV/c}^2 = 1077.9 \text{ MeV/c}^2 \)

Final total mass \( 493.7 \text{ MeV/c}^2 + 1189.4 \text{ MeV/c}^2 = 1683.1 \text{ MeV/c}^2 \)

Energy converted to mass
Project Orion
• Assume you have a rocket which can travel with constant acceleration $g$

• How far can you travel in a ship time of 20 years?
Acceleration in Special Relativity

\[ ct = \frac{c^2}{g} \sinh \left( \frac{g}{c} \tau \right) \]

\[ x = \frac{c^2}{g} \cosh \left( \frac{g}{c} \tau \right) \]

\[ \cosh b = \frac{1}{2} (e^b + e^{-b}) \]

- If \( g = 9.8 \text{ m/s}^2 \) and \( \tau = 20 \text{ years} \), \( x = 400 \text{ million light years} \)
Consequences

- Newton’s law of universal gravitation not invariant under special relativity.

\[ F = -\frac{GMm}{r^2} \]

- Force acts instantaneously at a distance
- Inconsistent with the speed of light being the limiting speed

Need new theory of gravitation consistent with special relativity
Principle of Equivalence

\[ \text{acceleration} = \text{gravitation} \]

Physics is the same in a uniform gravitational field as in a uniformly accelerated frame.
Gravitational mass = inertial mass

Lead and feathers fall with the same acceleration

verified to $1.5 \times 10^{-13}$ through lunar ranging
observer in outer space

floor exerts no force on mass M

Freely falling in uniform g

mass M falls at exactly the same rate as the elevator, floor exerts no force on M
Freely falling in uniform g

mass M feels force Mg from floor

accelerated observer

floor exerts force Mg on mass M

g = a
Place a clock in the nose (N) of the rocket and another in the tail (T) of the rocket. What is the time $\Delta \tau_T$ between receiving photons at the tail if they are released from the nose $\Delta \tau_N$ apart?

$$z_N(t) = d + \frac{1}{2} gt^2$$

$$z_N(0) - z_T(t_1) = ct_1$$

$$z_N(\Delta \tau_N) - z_T(t_1 + \Delta \tau_T) = c(t_1 + \Delta \tau_T - \Delta \tau_N)$$

consequences - gravitational time dilation.
using \( t_1 = \frac{h}{c} \) a little algebra one arrives at the result

\[
\Delta \tau_T = \Delta \tau_N (1 - \frac{gh}{c^2})
\]

\( gh \) - gravitational potential energy

clock at higher gravitational potential runs fast according to one at lower gravitational potential

equivalently clock deep in a gravitational potential runs slow according to one at higher gravitational potential
GPS - Accuracy (latitude, longitude, and altitude) 5-10 meters. Network of 24 satellites in high orbit altitude
- 20,000 km above ground
- speed 14,000 km/hour
- clock - accurate to a nanosecond

**Special Relativity** - lose 7 microseconds a day relative to ground clock

**General Relativity** - gain 45 microseconds a day

**Error in position** = 10 km per day!!!
Principle of Equivalence is underpinning of General Relativity - Gravity as the Geometry of Curved Spacetime
Why was the Nobel Prize for relativity awarded 77 years after Einstein proposed the theory?

Partially, it is only in the last 30 years that we have been able to see some of its more spectacular consequences.
PSR B1913+16 is two neutron stars in close orbit, one a pulsar emitting detectible radio waves
A pulsar is a neutron star

**Mass** $1.44 \, M_{\text{sun}}$

**Radius** $10 \, \text{km}$ - i.e. incredibly dense

with an extremely rapid period of rotation

**Period of rotation** of $0.059 \, \text{seconds}$

extremely accurate clocks (part in $10^{14}$)

system allows determination of relativistic effects to high precision

precession of periastron $4.22659 \, \text{degrees/year}$

time delay

gravitational redshift

loss of energy due to gravitational radiation
Extremely strong astrophysical evidence of black holes

Newtonian escape velocity \( v = \sqrt{\frac{2GM}{R}} \)

\( c = \sqrt{\frac{2GM}{R}} \) if the speed of light is the speed limit

\( R = \frac{2GM}{c^2} \) is radius of mass for which light cannot escape - turns out to be exactly the Schwarzschild radius of a black hole
Recent spectacular evidence of a black hole 
\textbf{Mass-} \(3 \times 10^6 \, M_{\odot}\) in center of our galaxy 

Strong evidence of other black holes both stellar sized and in centers of galaxies

Schodel et al (2002)
Detection of Gravitational Waves