Symmetries, Parity-Violation & the Structure of the Proton

The G⁰ & Q_{weak} Experiments

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Symmetries, Parity-Violation & the Structure of the Proton

Parity Violation as a Tool for Probing the Structure of the Proton

1) Parity Conservation and Parity Violation

Symmetries in Physics, Noether's Theorem Parity Transformations & Observables

2) Probing Microscopic objects (e.g. proton, nucleus)

What/How?

Diffraction Patterns & Scattering objects

Form Factors

3) Electroweak Structure of the Proton: G⁰ & Q_{weak}

G⁰ and Q_{weak} experiments @ Jefferson Lab (JLab)

Our present picture/model of the proton (e.g. quark models,...)

(a) How do building blocks manifest themselves?

Our present picture of the Standard Model

(b) Are there extensions beyond the Standard Model?

(1) <u>Symmetries in Physics</u>

Everyday use (implicit)
Mathematical/Quantitative counterpart (e.g. Rotational Symmetry)

Link between Symmetries & Conservation Laws: Noether's Theorem

Wikipedia:

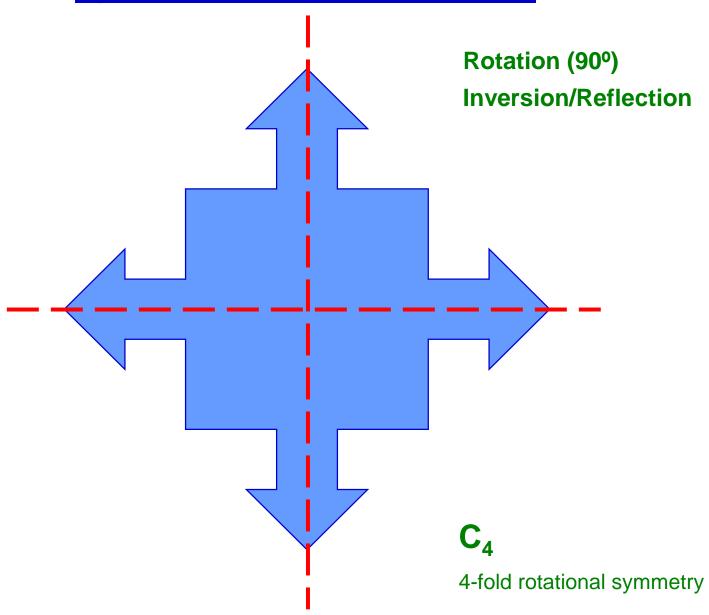
Noether's theorem (1915, Emmy Noether) is a central result in theoretical physics that expresses the one-to-one correspondence between the symmetries and the conservation laws.

Einstein (in reference to Noether): "...penetrating mathematical thinking..."

Some important examples of Noether's theorem are the following:

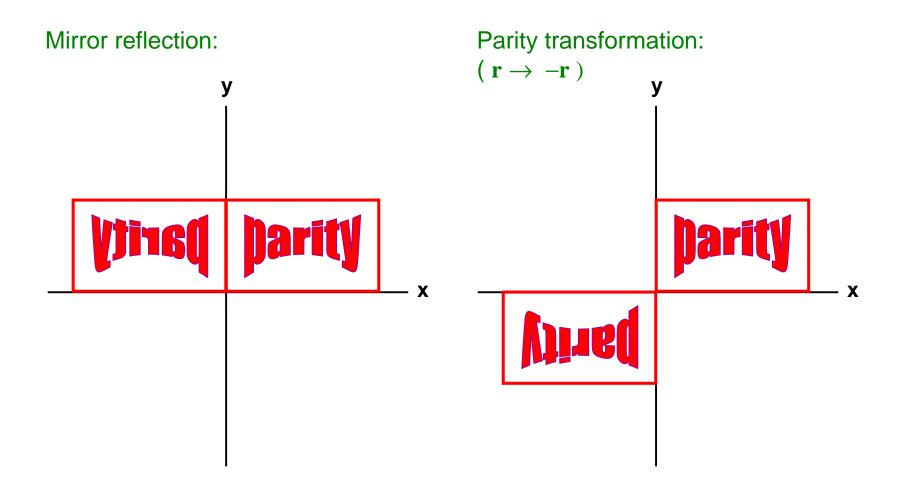
- The energy is conserved if and only if the physical laws are invariant under time translation (if their form does not depend on time Time-translation symmetry)
- The momentum is conserved iff the physical laws are invariant under spatial translations (if the laws do not depend on the position Space-translation symmetry)
- The angular momentum is conserved iff the physical laws are invariant under rotations (if the laws do not care about the orientation Rotational Symmetry)

Symmetries (Spatial, Discrete)

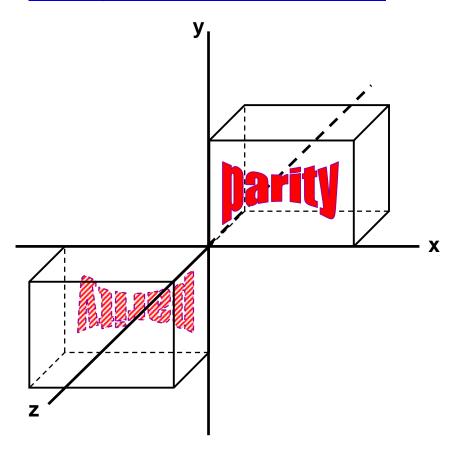


Parity Operation (in 2D)

Space Reflection (Inversion) Symmetry



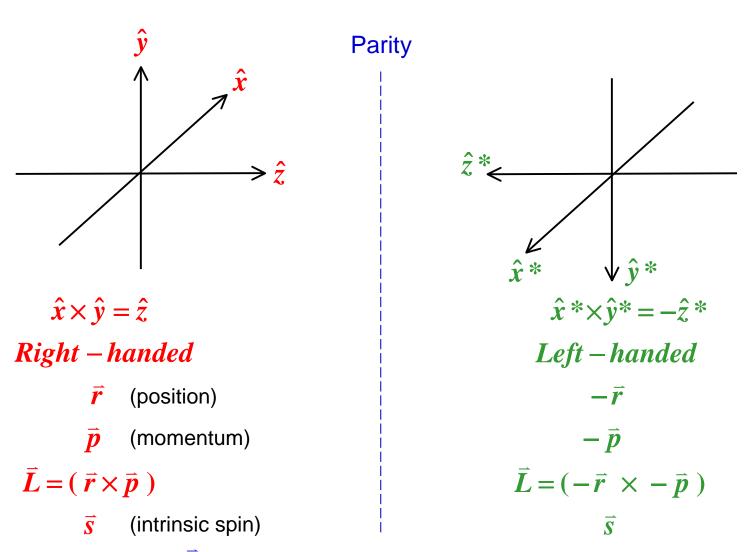
Parity Operation (in 3D)



The Parity transformation: ($r \rightarrow \ -r$)

- simultaneous reflection of all space coordinates through the origin
- equivalent to reflection plus 180° rotation
- if we assume rotational invariance, it is a mirror reflection

The Parity Transformation



 \bar{L} , \bar{s} are pseudo or axial vectors

Distinct: Cannot "rotate" RHS into LHS

The Parity Transformation as Depicted in Popular Culture



1968 **Doppelganger Journey To The Far Side Of The Sun** (US Title)

Parity analogue systems: "Right-handed" vs "Left-handed" systems

Parity Conservation (PC)

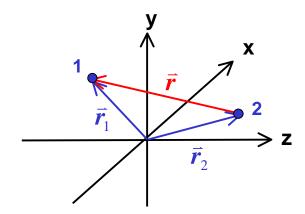
The absence of physics that would select a particular *handedness*



independent of *handedness* (PC)

handedness-dependent (PNC)

Example: Consider the Coulomb interaction of 2 protons



$$\vec{r} = \vec{r}_1 - \vec{r}_2$$
 ; $|\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}}$

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \quad ; \quad |\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}}$$

$$V(\vec{r}) = \frac{e^2}{4\pi\epsilon_0} \frac{1}{|\vec{r}|} = V(-\vec{r})$$

Conserves parity!

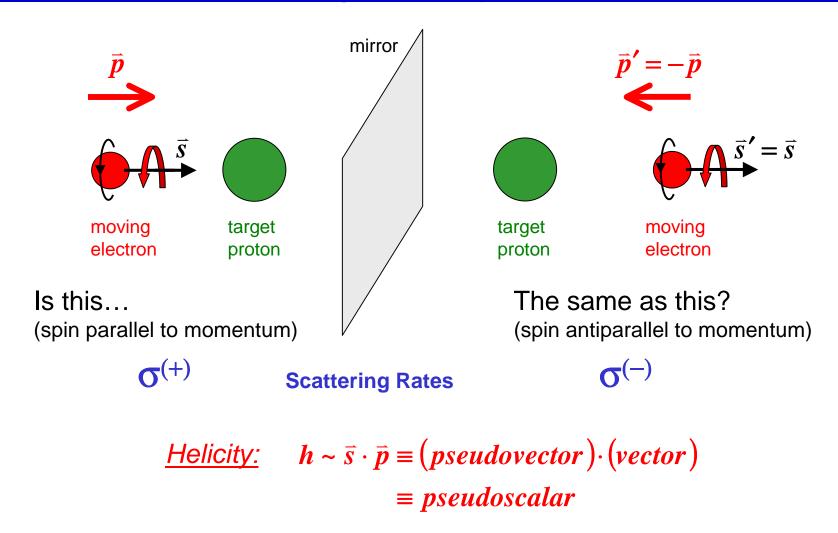
Counter example:

Weak Interaction violates parity **(PNC)**

→ has the form (Vector – Axial vector) → handedness built-in

$$\sim (\vec{v} - \vec{a}) \cdot (\vec{v} - \vec{a}) \approx \vec{v} \cdot \vec{v} + \vec{a} \cdot \vec{a} - 2(\vec{v} \cdot \vec{a})$$

Principle of a "beam-target" Parity-violation Measurement



>To replace the experiment with its mirror image, you flip the beam helicity

Parity-sensitive Observables

In accelerator-based scattering experiments:

- → Prepare the beam (e,p,n,...) in (+) or (-) helicity states (Parity analogue states)
- → Observable: Parity-Violating Longitudinal Asymmetry, A Difference/sum of Parity analogue reactions rates

$$A = \frac{\sigma^{(+)} - \sigma^{(-)}}{\sigma^{(+)} + \sigma^{(-)}}$$

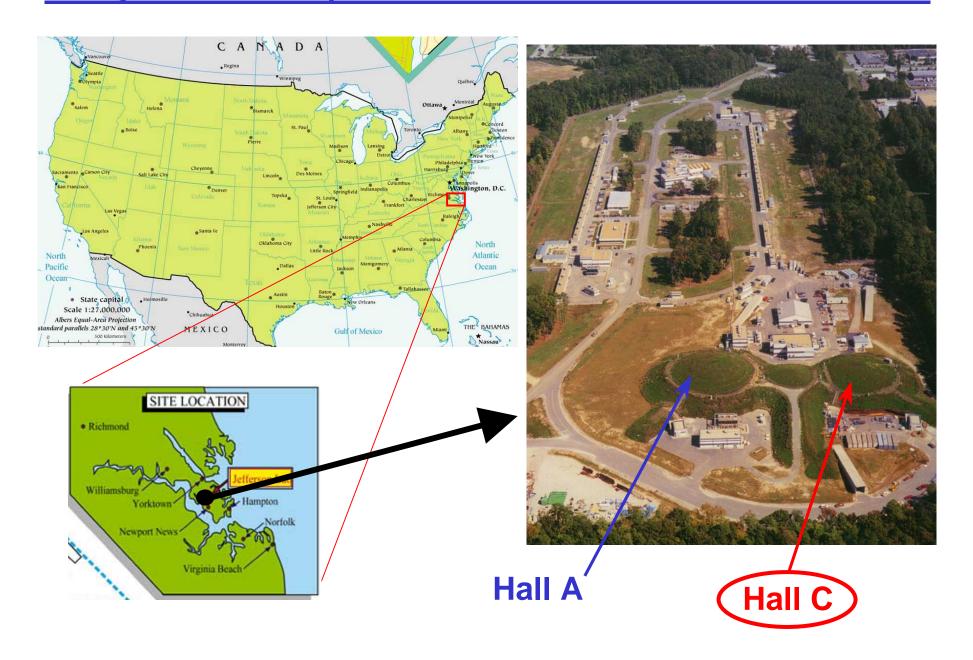
→ A is usually very very small ~ $(1-10) \times 10^{-7}$ [0.1–1 ppm] (Signature of the Weak interaction)

Conceptually straightforward → but False Asymmetries!!

$$A_{measured} = A_{physics} + A_{false1} + A_{false2} + \dots$$

Many many sources of false asymmetries: keep \leq (1–10) x 10⁻⁸ \rightarrow High Precision Experiments [0.01–0.1 ppm]

Parity-Violation Experiments w/ e⁻ beams @ Jefferson Lab



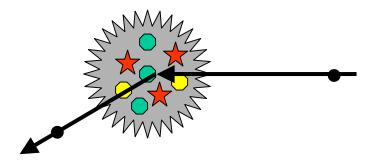
(2) Probing the Nucleus/Proton using subatomic beams

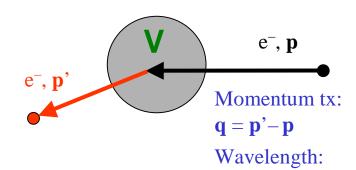
Beam on Target → Scattering Amplitudes → Target structure (form factor)

"The Accelerator as a Microscope".

Just bashing small things into other small things?

Consider Coulomb (electric) Scattering (eg, e^- from a Nucleus \leftarrow charge distrⁿ)





 $\lambda \sim 1/q$

Q.M. Scattering Amplitude $\mathbf{M} \sim \langle \Psi^{\mathbf{e}}_{\text{out}} | V_{\text{op}} | \Psi^{\mathbf{e}}_{\text{in}} \rangle$ bra vector tx operator/potential ket vector

Lowest Order (plane wave)

Approx: $M \approx \int e^{i\vec{p}\cdot\vec{r}} V(\vec{r}) e^{-i\vec{p}\cdot\vec{r}} d^3r$

$$M \approx \int V(\vec{r}) e^{i(\vec{p}' - \vec{p}) \cdot \vec{r}} d^3r = \int V(\vec{r}) e^{i\vec{q} \cdot \vec{r}} d^3r$$

$$M \approx \widetilde{V}(\vec{q}) \equiv [$$
 Fourier Transform of $V(\mathbf{r})]$

Tx. Potential

Aside/Reminder: Fourier Transform (FT)

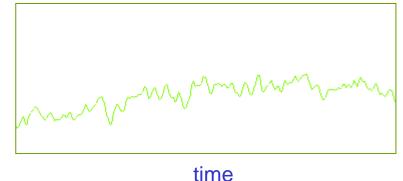
Function $f(x) \rightarrow \text{independent variable } x$ also described by

Fourier Transform $\mathbf{F}(k) \rightarrow \text{ conjugate variable } k$

$$F(k) = \int f(x) e^{i k \cdot x} dx$$

eg) Electrical Signal / Sound wave (mp3, WMP)

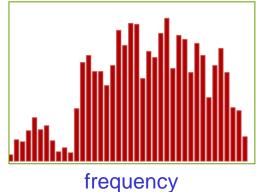
Amplitude



F.T.

Freq. Spectrum (Spectral Components)

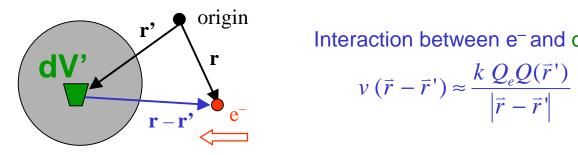
Amplitude or Power



Transition Potential & Amplitude

$$M_{scat} \approx \widetilde{V}(\vec{q}) = \int V(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3r \equiv F.T. of V(\vec{r})$$

Consider a nucleus: Nuclear Charge Distribution $\rho(\mathbf{r}')$



Interaction between e⁻ and dV' (at r')

$$v\left(\vec{r} - \vec{r}'\right) \approx \frac{k Q_e Q(\vec{r}')}{\left|\vec{r} - \vec{r}'\right|}$$

Total Interaction between e⁻ and Nucleus:

$$V(\vec{r}) = \int v(\vec{r} - \vec{r}') \rho(\vec{r}') d^3r'$$
Convolution of V*p

Convolution Theorem:

The F.T. of a convolution of functions ≡ Product of the Individual transforms

$$\widetilde{V}(\vec{q})$$
 $V(\vec{r}) = v * \rho$ $\widetilde{v}(\vec{q}) \widetilde{\rho}(\vec{q})$

$$\therefore M_{scat} \approx \widetilde{V}(\vec{q}) \approx \widetilde{v}(\vec{q}) \cdot \widetilde{\rho}(\vec{q})$$
Force/Int
$$\approx \int_{|\vec{r}|}^{1} e^{i\vec{q} \cdot \vec{r}} d^{3}r \approx \frac{1}{q^{2}}$$

$$\approx \int \rho(\vec{r}) e^{i\vec{q} \cdot \vec{r}} d^{3}r \equiv F(\vec{q}) \text{ (or } G(\vec{q}))$$

Cross Sections

Experiments measure "Cross Sections" (Rate/Prob./Intensity)

$$\sigma \propto \left| M_{scat} \right|^2 \approx \left| \widetilde{v}(q) \, \widetilde{\rho}(q) \right|^2 \approx \left| \frac{1}{q^2} G(q) \right|^2$$

$$\sigma \equiv \sigma(\theta) \equiv \sigma(q) \sim I$$

- **"Probe"** \rightarrow By looking at $\sigma(q)$
- Really looking @ diffraction pattern

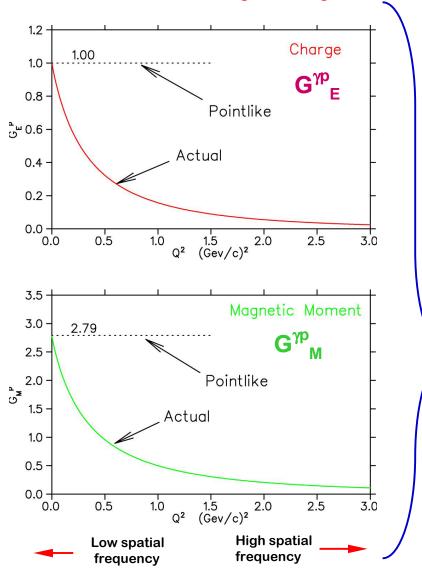
 θ or q

Infer the shape/distribution of scattering object

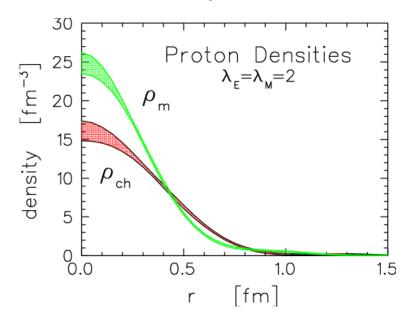
"Listening to the shape of the drum"

Example: Proton EM Form Factors from Electron Scattering

"Measure" Electric Charge & Magnetic Moment as functions of Q2 (momentum transfer)



- Sachs Form Factors (well established) (many, extensive measurements)
- ➤ Small proton → slow fall-off
- ▶ Large proton → rapid fall-off
- Form Factor is essentially the Fourier transform of the spatial distribution.



Parity-Violation in e-p Scattering:

JLab-Hall C "Parity" Experiments: G0 and Qweak

Using spin-polarized electrons and Parity-violation:

Probe the "weak" structure of the proton → learn something about:

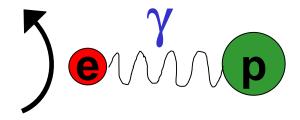


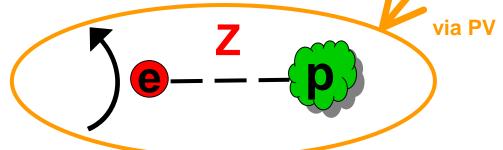
The strange quark currents in the proton (**G0**) [Role of the sea quarks at low energy]



The weak charge of the proton (**Qweak**) [Search for new physics beyond the Standard Model]

electron - proton scattering: Electroweak Interaction





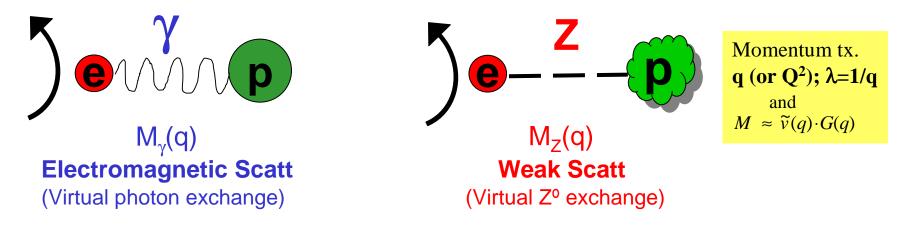
Electromagnetic

(Virtual photon exchange)
Probes proton's EM structure, **G**^γ

Weak

(Virtual Z^o exchange)
Probes proton's "Weak" structure, G^Z

Electromagnetic vs Weak Processes and Parity-Violation



Scattering Amplitude $M = M_{\gamma} + M_{Z}$; with $M_{\gamma} >> M_{Z}$ (factor of ~ 10⁵)

But: $M_{\gamma} \Rightarrow$ parity conserving , while $M_{Z} \Rightarrow$ parity non-conserving

Compare 2 parity-sensitive cross-sections (whose parity-conserving parts are identical)



Scattering Asymmetry:

$$A = (\sigma_{+} - \sigma_{-})/(\sigma_{+} + \sigma_{-}) \sim M_{\gamma}M_{Z}/|M_{\gamma}|^{2}$$

Recap (so far)

Scattering Processes (Cross Sections/Diffraction Pattern) → Form Factors

$$\sigma \propto \left| M_{scat} \right|^2 \approx \left| \widetilde{v}(q) \, \widetilde{\rho}(q) \right|^2 \approx \left| \frac{1}{q^2} G(q) \right|^2$$

Parity-Violation → "Weak" Interaction components (of F.F.)

$$A = (\sigma_{+} - \sigma_{-})/(\sigma_{+} + \sigma_{-}) \sim M_{\gamma}M_{Z}/|M_{\gamma}|^{2}$$

$$A = (\sigma_{+} - \sigma_{-})/(\sigma_{+} + \sigma_{-}) \sim f(q) \times G_{\gamma}G_{Z} / |G_{\gamma}|^{2}$$

(3a) Proton Structure: An open question

What does a proton look like?



$$|p\rangle \approx \alpha |qqq\rangle + \beta |qqqg\rangle + \gamma |qqqq\overline{q}\rangle + \dots$$

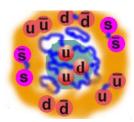
A Proton has 3 Quarks (Valence Quarks) (uud)



There may be contributions from gluons to the Proton's structure (?)



There may be contributions from a "sea" of Quark-AntiQuark pairs (?) {strange-antistrange quark pairs (??)}



Strange quarks present only in the sea

(Proton has no "net" strangeness)

G0: JLab E00-06

Measurement of the Strange Quark Currents in the Proton

G0 Collaboration:

Canada (TRIUMF, U.Manitoba, UNBC, U.Winnipeg), France, USA

Measure G^{Z,p}: Weak *vector-current* form factor

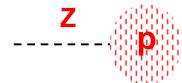
- a "new" & fundamental property of the proton
- Similar to normal electric & magnetic form factors, but for "weak" interaction (weak charge, weak magnetic)

$$\rho \rightarrow G_E$$

EM charge & $\rho \to G_E$ G^Z_E, G^Z_M : weak charge & magnetic FF $J \to G_M$ $J \to G_M$



$$j \rightarrow G_M$$



$$G^{Zp}$$
, $G^{\gamma p}$, $G^{\gamma n} \Rightarrow G^{s,p}$

Strangeness form factor of proton Role of sea quarks at low E

$$\mathbf{G^{s,p}_{E}} = (1 - 4\sin^{2}\theta_{W}) \mathbf{G^{\gamma p}_{E}} - \mathbf{G^{\gamma n}_{E}} - \mathbf{G^{Z p}_{E}}$$

$$\mathbf{G^{s,p}_{M}} = (1 - 4\sin^{2}\theta_{W}) \mathbf{G^{\gamma p}_{M}} - \mathbf{G^{\gamma n}_{M}} - \mathbf{G^{Z p}_{M}}$$

Form Factors from Parity-Violating Asymmetry

Parity-Violating Asymmetry

Parity-violating Asymmetry
$$A = \frac{\sigma^{(+)} - \sigma^{(-)}}{\sigma^{(+)} + \sigma^{(-)}} \sim \frac{f(q) \cdot G_{\gamma}G_{Z}}{\left|G_{\gamma}\right|^{2}}$$

$$A(Q^{2}) = -\frac{(G_{F}Q^{2})}{\pi\alpha\sqrt{2}} \underbrace{\left\{ \epsilon G^{\gamma}_{E}G^{Z}_{E} + \tau G^{\gamma}_{M}G^{Z}_{M} + \eta G^{\gamma}_{M}G^{Z}_{A} \right\} \left[1/P_{Z} \right]}_{\epsilon(G^{\gamma}_{E})^{2} + \tau(G^{\gamma}_{M})^{2}}$$

(where ε , τ , η are kinematical parameters)

Want to determine G^{Z}_{F} and G^{Z}_{M}

Do 2 measurements of A at each momentum tx value (q or Q²)

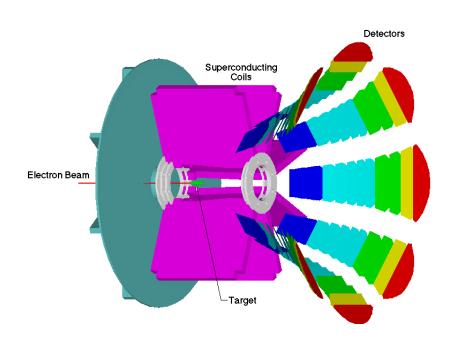
For a given Q^2 , ε ranges from 1 (small angles) \rightarrow 0 (large angles)

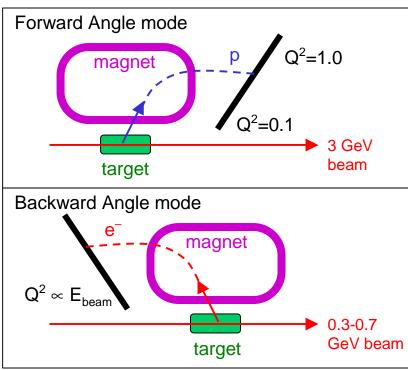
- At small (forward) angles, measure a combination of G^{Z}_{E} and G^{Z}_{M} i)
- At large (backward) angles, measure G^{Z}_{M} ii) → Combine both measurements to **extract** G^Z_F

GO MEASUREMENTS:

- A) Forward Angle Mode
- B) Backward Angle Mode

G0 Schematic Layout and Experiment Parameters

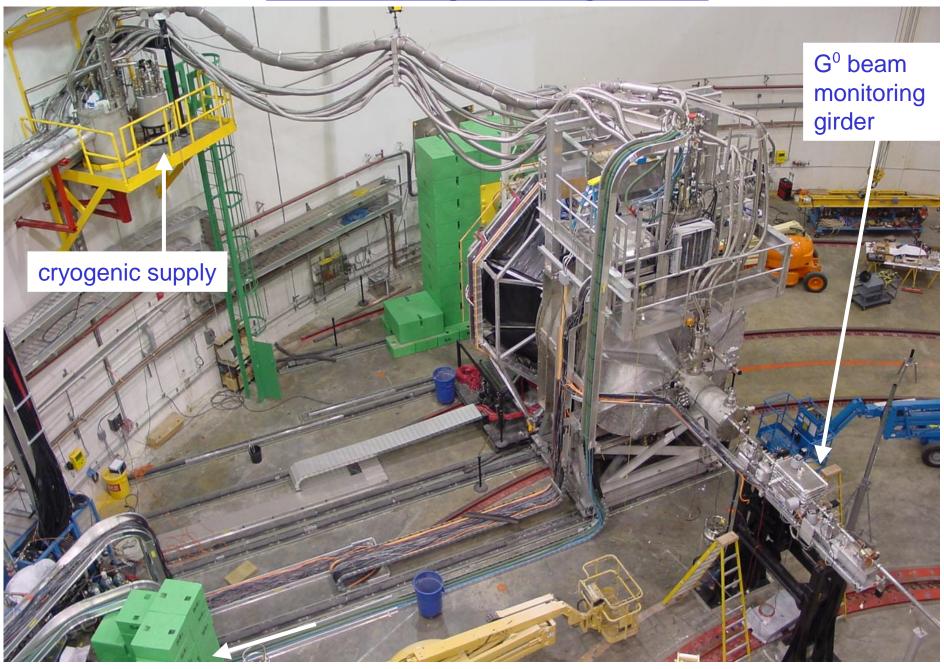


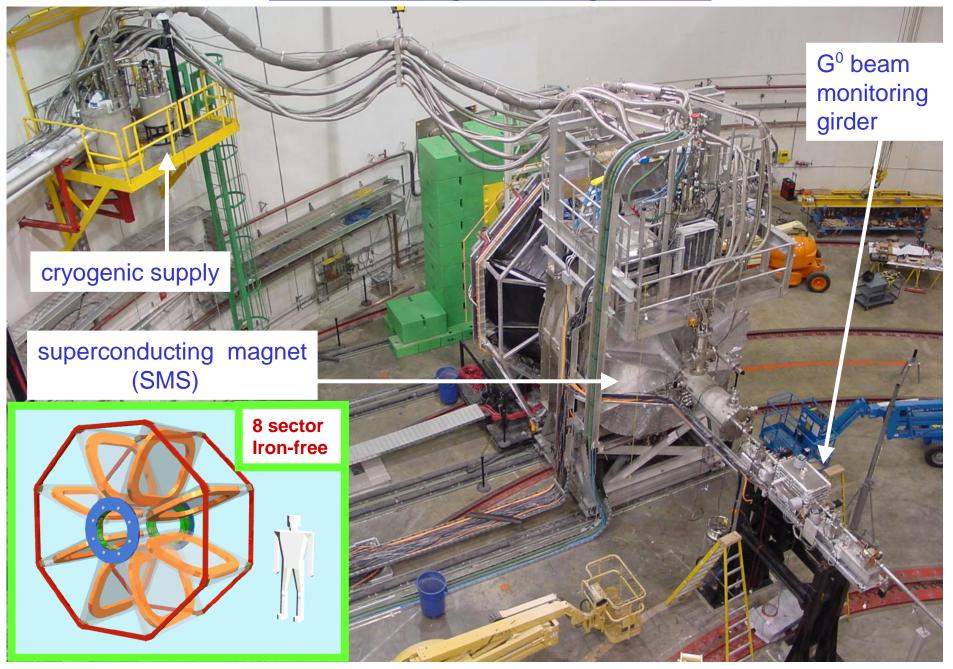


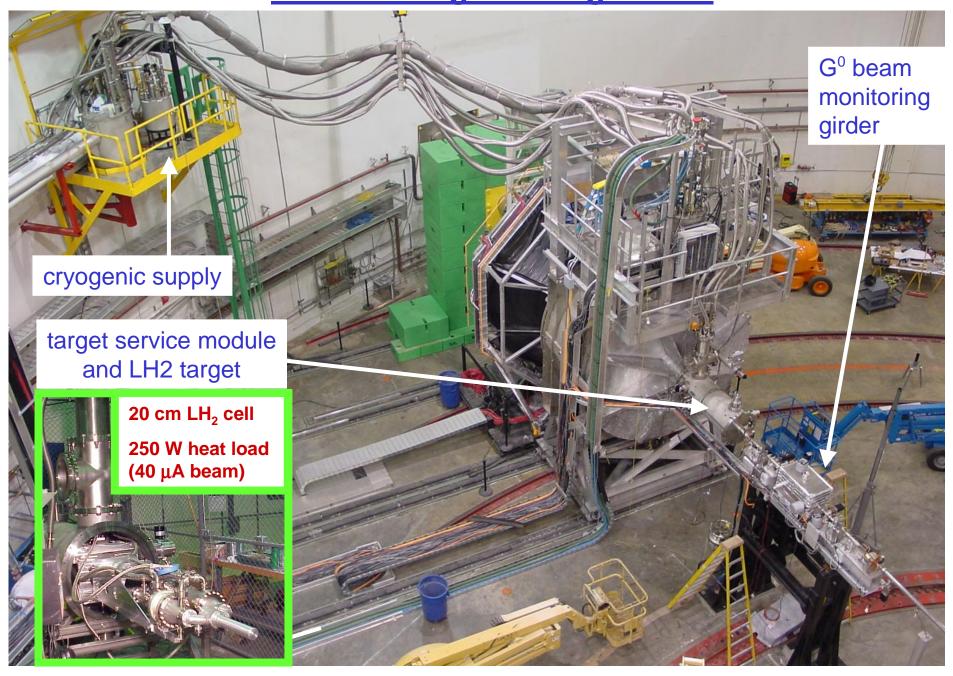
Goal: Measure $A_z \sim 5 \times 10^{-6}$ to $(\Delta A/A) \sim \pm 5 \% (10^{-7})$

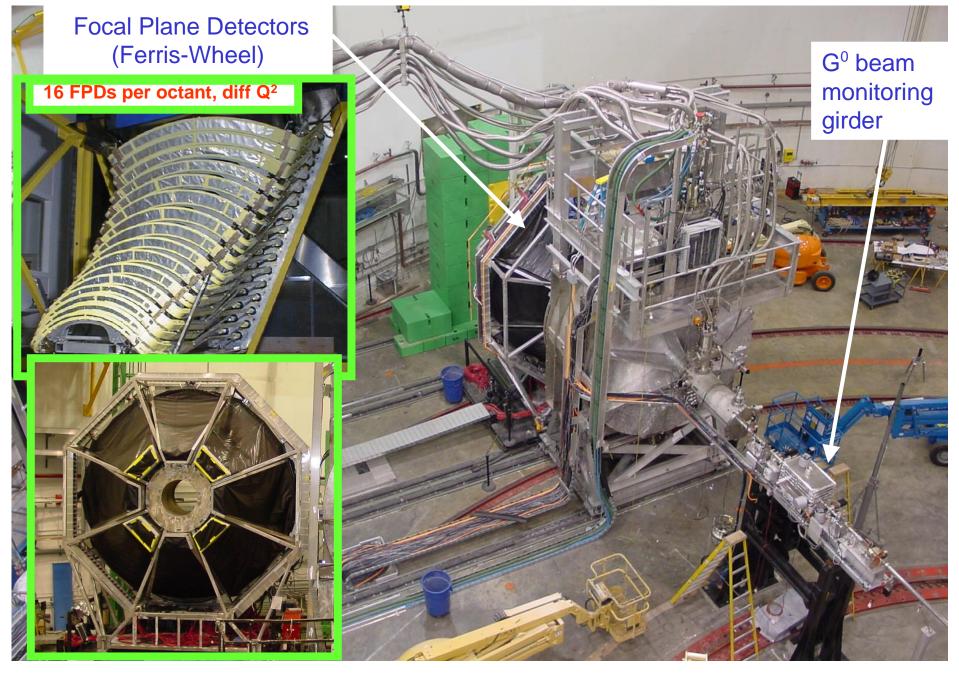
Systematics: (h.c. variables)

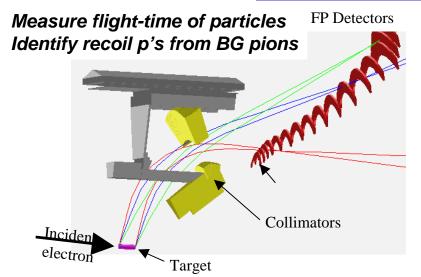
$$\begin{array}{lll} \Delta E & < 2.5 \times 10^{\text{-8}} \\ \Delta I_b / I_b & < 1 \text{ ppm} \\ \Delta x & < 20 \text{ nm} \\ \Delta \theta & < 2 \text{ nrad} \\ \end{array}$$

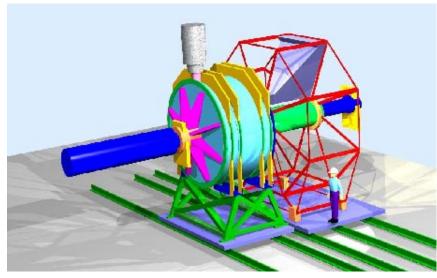


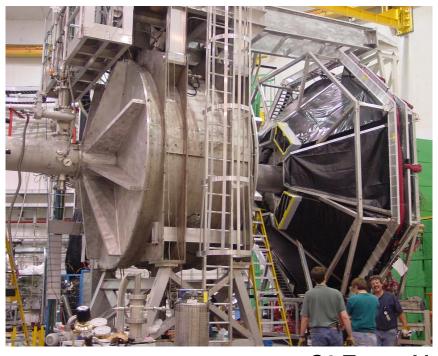










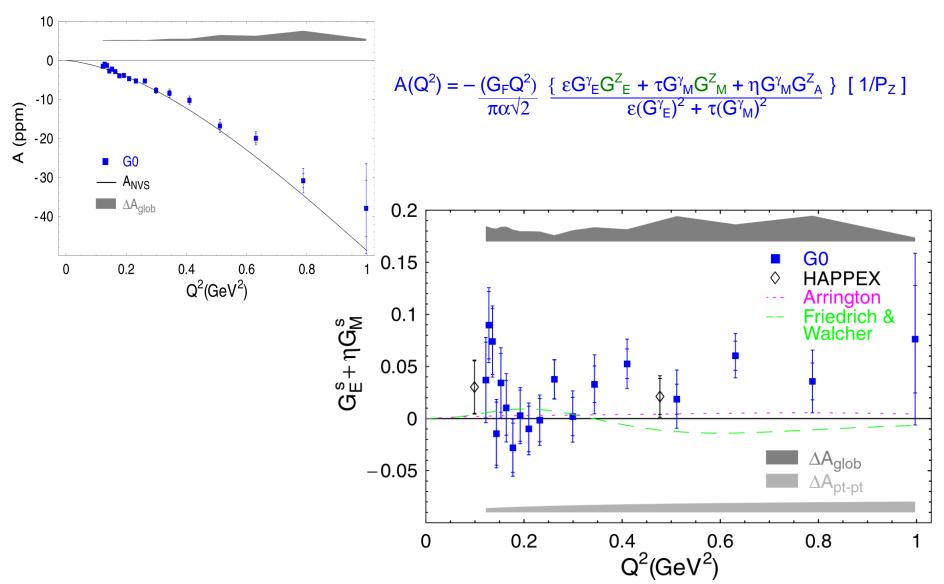




G0 Target-Magnet-FPD in Hall C

First Results from G0 Experiment

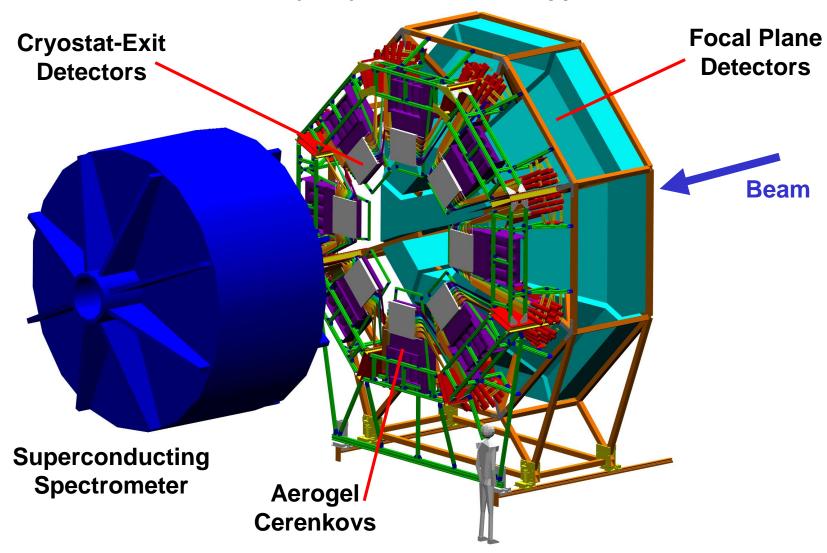
Phase I (Forward Angle Measurement, Fall/2002 – Summer/2004)



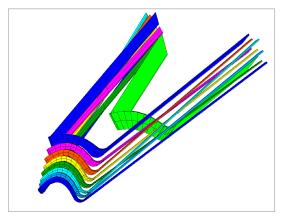
G0 Back-Angle Configuration

Magnet & Detector Package turned around; Detect back-scattered electrons

Additional Detectors; 2nd (Mini) Ferris-Wheel Support Structure



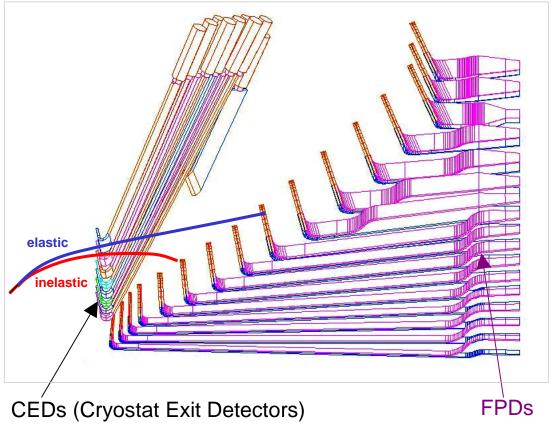
Back-Angle Configuration: Cryostat-Exit Detectors

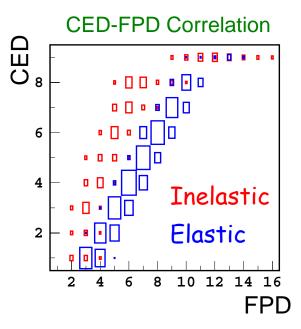


Magnet & Detector Package turned around

Measure back-scattered electrons (instead of recoil p's)

- Require additional det.^s: Cryostat-Exit Detectors (CED)
 (9 detectors per octant, fabricated at TRIUMF)
- CED-FPD coincidence to separate elastics/inelastics e⁻ (cannot use Time-of-Flight → electrons are relativistic)





Back-Angle Configuration: Aerogel Cerenkov Detectors

Back-scattered electron:

CED-FPD coinc. separates elastic & inelastic e⁻ 's

Background pions:

Need to separate π^- (backgnd) from e⁻ (signal)

Cerenkov Detectors:

"Discriminate" electrons from pions by their velocity (e's create light in Cerenkov det., but pions do not).

8 detectors: 4 Canadian

4 French

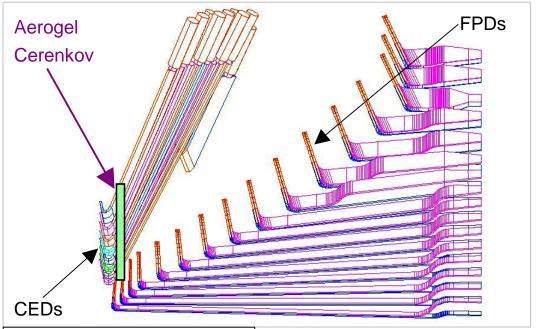
Aerogel Cerenkov

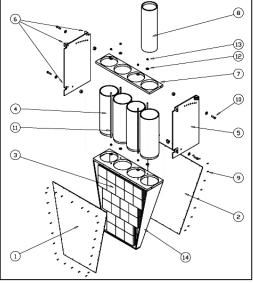
11 x 11 x 1 cm³ tiles (5 deep)

Aerogel, n ~ 1.03

If v > 0.97c (speed of light in aerogel medium), Cerenkov radiation created

(FPD, CED) plus Cerenkov Arrays

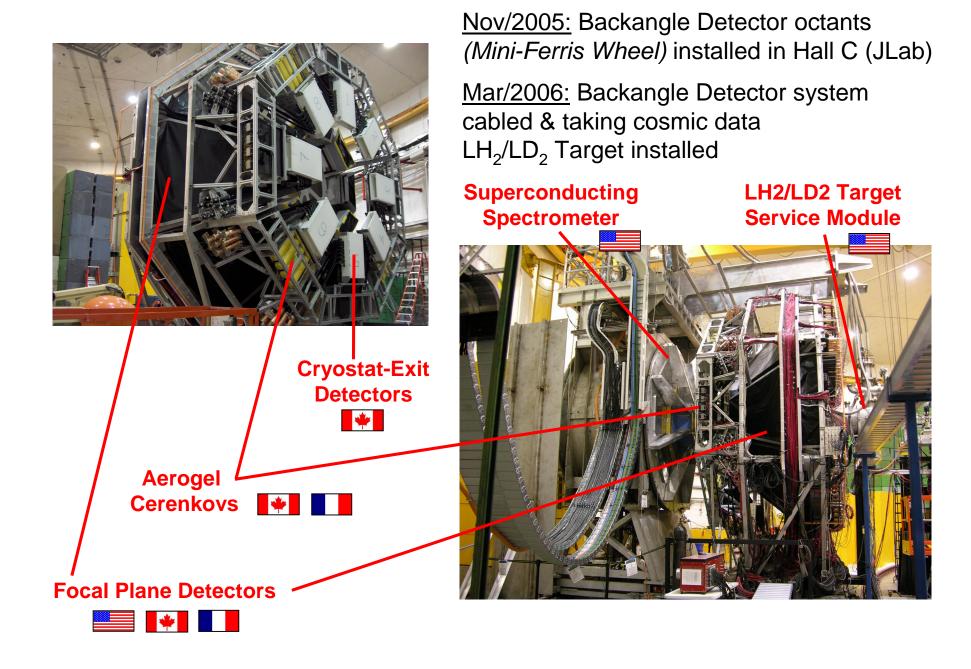








Backward Angle Measurement (Recently started Mar/2006)



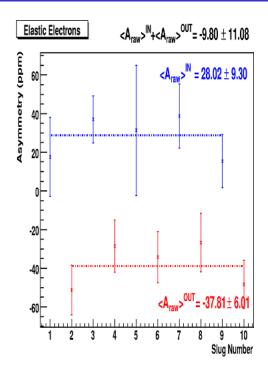
Backward Angle Measurement (Recently started Mar/2006)

Spring 2006: Commissioning and 1st Data Run in Backward Angle Mode

System (Detectors/Target/Beam) checked out 1st set of (preliminary) production data obtained 1st set of Parity-violating asymmetries measured

Present Status of the G0 Experiment

Forward Angle Measurement Completed Presently, in Backward Angle Configuration



G0 Backangle Measurements @ Q² = 0.6, 0.23 GeV²

- Run in Mar-Apr '06 at $Q^2 = 0.6 \text{ GeV}^2$ (Commissioning & H target)
- Run in July-Aug '06 at $Q^2 = 0.23 \text{ GeV}^2 \text{ next}$ (H and D targets)
- Run in Sep-Dec '06 at Q² = 0.6 GeV² (H and D targets)
- Run in Jan-Feb '07 at $Q^2 = 0.23 \text{ GeV}^2$ (H and D targets)

(3b) Physics Beyond the Standard Model?

Standard Model: "Pillar of fundamental physics for ~ 30 years" Electromagnetic, Weak, Strong interactions

Fermions ("matter particles"): leptons $(e, v_e, \mu, v_\mu, \tau, v_\tau)$ quarks (u, d, s, c, b, t) Gauge Bosons ("force carriers"): $(\gamma, W^+, W^-, Z, g), H$

Electroweak sector:

Unification of Electromagnetic (EM) and Weak Interactions

→ Electroweak Interaction (EW)

Proposed by Weinberg-Salam (γ , Z, W[±]) - predicted existence Z, W[±] \rightarrow discovered later at CERN Impressive agreement between Standard Model & EW observables

Issues: (Did not explain)

Large number of parameters (masses, couplings, mixing angles)
Origin of: quantization of EM charge and weak hypercharge,
P and CP violation, number of generations,...

Neutrino oscillations

Mass (scalar) hierarchy problem

Q: Extensions ("new" physics) beyond the Standard Model?

Qweak : JLab E02-020

A Search for Physics at the TeV Scale Via a Measurement of of the Proton's Weak Charge

Q_{weak} Collaboration:

Canada (TRIUMF, U.Manitoba, UNBC, U.Winnipeg), USA

Measure $\mathbf{Q}_{\text{weak}}^{\text{p}}$: Weak charge of the proton; $\mathbf{Q}_{\text{weak}}^{\text{p}} = 1 - 4 \cdot \sin^2 \theta_{\text{W}}$

- first precision measurement of this property of the proton
- fundamental measurement of the running of $\sin^2\theta_w$ at low energies

At low energies and small scattering angles, the "weak charge" of the proton is proportional to the Parity-Violating asymmetry (in e-p scattering):

$$(As \ Q^2 \to 0, \theta \to 0): \quad A \sim \left[Q^2 Q_{\text{weak}}^p + Q^4 B(Q^2)\right] \sim \left[Q^2 Q_{\text{weak}}^p\right] \quad ; \quad \left\{A = \frac{\sigma^{(+)} - \sigma^{(-)}}{\sigma^{(+)} - \sigma^{(-)}}\right\}$$

$$\text{contains } \mathbf{G}_{\text{E,M}}^{\gamma}, \mathbf{G}_{\text{E,M}}^{\text{Z}}, \text{ form factors}$$

 $Q^{p}_{weak} = 1 - 4 \sin^{2}\theta_{W} \sim 0.072$ \rightarrow a well-defined experimental observable \rightarrow has a definite prediction in the Standard Model

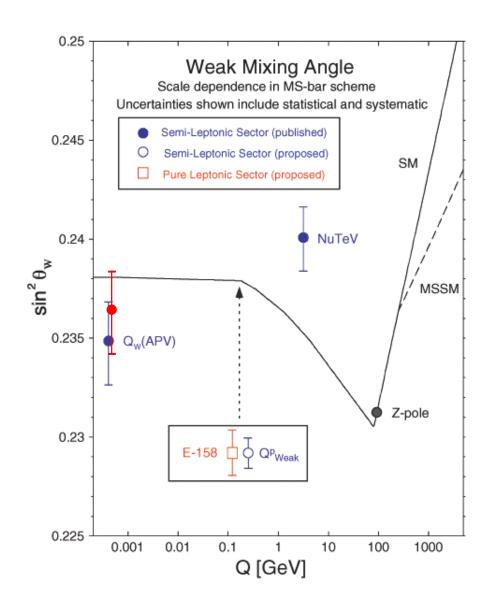
ightarrow Goal: Asymmetry Measurement: $\Delta A \sim 1 \times 10^{-8}
ightarrow \Delta Q^p_{weak} \sim \pm 4\%
ightarrow \Delta \sin^2\!\theta_W \sim \pm 0.3\%$ ightarrow Sensitive to *new* physics (particles) at ~4.6 TeV scale

Running of the Weak Mixing Angle

Electroweak radiative corrections $\Rightarrow \sin^2 \theta_W$ varies with Q

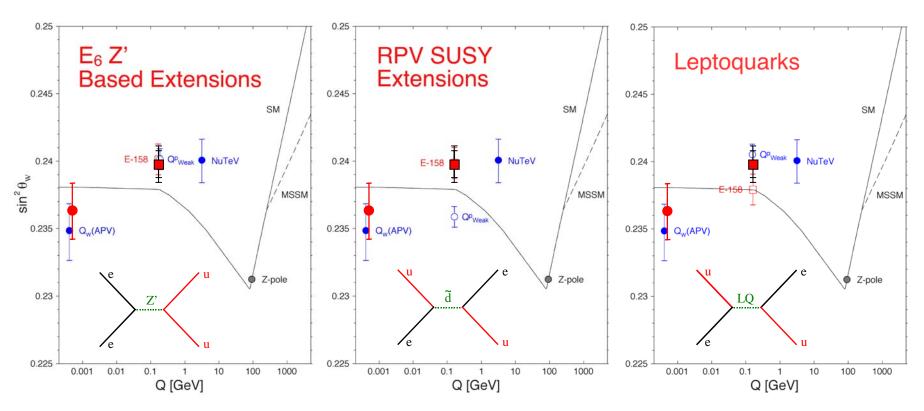
New Physics ??

Value of sin²θ_w at ~0.17 GeV/c: SLAC E158: (e-e) PV Moller Scattering \Rightarrow Q^e_{weak} Complementary JLab Q^p_{weak} (e-p)



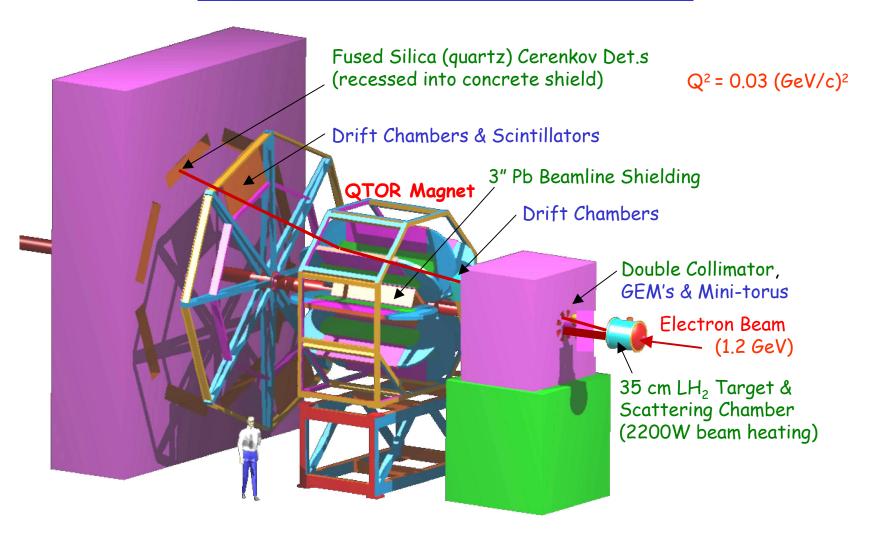
Useful Diagnostic for New Physics

Weak charge measurements: Q_{weak}^p , Q_{weak}^e



E158 recent results: $\sin^2 \theta_W = 0.2397 \pm 0.0010$ (stat) ± 0.0008 (sys)

Layout of the Qweak Experiment



Qweak Toroidal (QTOR) Magnet

- 8 sector toroidal magnet
- water cooled copper coils
- 8600 A, 1.2 MW
- 4.3 m long, 1.5 m wide coils, simple racetrack shape
- ~3300 kg per coil
- Magnet coils → Canadian
- Field mapping with G0 Mapper (TRIUMF-designed & built)



$$\int \vec{B} \cdot d\vec{l} = 0.67 \text{ T.m}$$

Qweak experiment presently in design & prototyping phase

SUMMARY

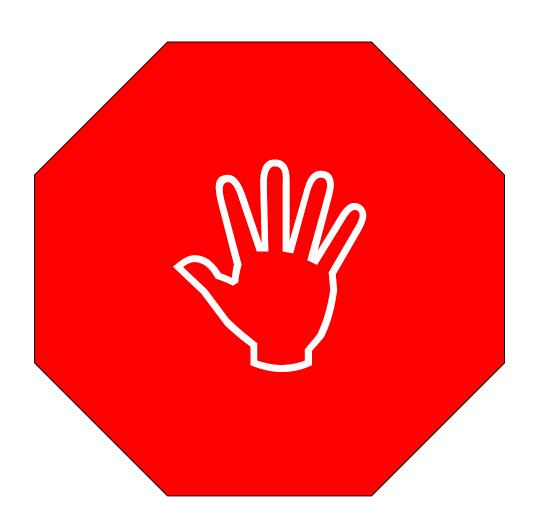
G₀

- G0 will measure a new, fundamental property of the proton.
 (G^{Zp}, weak current distribution)
- Measurement of G^{Zp} allows decomposition of the proton ground state matrix elements into quark flavour contributions (G^{u,p}, G^{d,p}, & G^{s,p} proton strange quark current distribution; direct measurement of the quark sea). The physics is of great current interest.

Q_{weak}

- First precision measurement of the weak charge of the proton. (Q_{weak})
- Fundamental measurement of the running of $\sin^2 \theta_{\mathbf{W}}$ at low energy.
- Sensitive search for new physics at the ~4.6 TeV scale.

Parity-violation is a useful tool indeed.



Proton "Weak" Form Factors & Strange Quarks

Flavour decomposition ("Proton is made up of quarks of different flavours")

$$G^{\gamma p}_{E,M} \rightarrow \Sigma \text{ (electric charge of quark } q_j) \times G^{q_j,p}_{E,M}$$

$$G^{Zp}_{E,M} \rightarrow \Sigma \text{ (weak charge of quark } q_j) \times G^{q_j,p}_{E,M}$$

$$G^{q_j,p}_{E,M} \rightarrow \Sigma \text{ (weak charge of quark } q_j) \times G^{q_j,p}_{E,M}$$

$$G^{q_j,p}_{E,M} \rightarrow \Sigma \text{ (weak charge of quark } q_j) \times G^{q_j,p}_{E,M}$$

Rewrite as a sum of contribution from each flavour (neglect heavy quarks c, t, b)

$$G^{\gamma p}_{E,M} = (2/3) G^{u,p}_{E,M} - (1/3) G^{d,p}_{E,M} - (1/3) G^{s,p}_{E,M}$$

$$G^{Z p}_{E,M} = (1/4 - 2/3 \sin^2 \theta_W) G^{u,p}_{E,M} - (1/4 - 1/3 \sin^2 \theta_W) G^{d,p}_{E,M} - (1/4 - 1/3 \sin^2 \theta_W) G^{s,p}_{E,M}$$

$$G^{\gamma n}_{E,M} = (2/3) G^{d,p}_{E,M} - (1/3) G^{u,p}_{E,M} - (1/3) G^{s,p}_{E,M}$$

Assuming **Charge Symmetry** between the proton and neutron $\begin{cases} G^{u,p} = G^{u,n} \\ G^{d,p} = G^{u,n} \\ G^{s,p} = G^{u,n} \end{cases}$

$$\left\{ \begin{array}{l} G^{u,p}_{E,M} = G^{d,n}_{E,M} \\ G^{d,p}_{E,M} = G^{u,n}_{E,M} \\ G^{s,p}_{E,M} = G^{s,n}_{E,M} \end{array} \right.$$

$$\mathbf{G}^{\mathsf{u},\mathsf{p}}_{\mathsf{E},\mathsf{M}} = (3 - 4 \sin^2 \theta_{\mathsf{W}}) \ \mathbf{G}^{\gamma \mathsf{p}}_{\mathsf{E},\mathsf{M}} - \mathbf{G}^{\mathsf{Z} \, \mathsf{p}}_{\mathsf{E},\mathsf{M}}$$
$$\mathbf{G}^{\mathsf{d},\mathsf{p}}_{\mathsf{E},\mathsf{M}} = (2 - 4 \sin^2 \theta_{\mathsf{W}}) \ \mathbf{G}^{\gamma \mathsf{p}}_{\mathsf{E},\mathsf{M}} + \mathbf{G}^{\gamma \mathsf{n}}_{\mathsf{E},\mathsf{M}} - \mathbf{G}^{\mathsf{Z} \, \mathsf{p}}_{\mathsf{E},\mathsf{M}}$$

$$\mathbf{G^{s,p}}_{\text{E,M}} = (1 - 4\text{sin}^2\theta_{\text{W}}) \; \mathbf{G^{\gamma p}}_{\text{E,M}} - \mathbf{G^{\gamma n}}_{\text{E,M}} - \mathbf{G^{Z p}}_{\text{E,M}}$$