Zeeman-Spectroscopy in Penning traps

Zeeman Spectroscopy:
Measurement of magnetic moments (g-factors)

Definition of the g factor:

The g factor relates the magnetic moment of a particle (in units of the Bohr Magneton μ_B) to its angular momentum J (in units of $h/2\pi$)

$$\vec{\mu} = g\mu_B\vec{J}$$

- **g factor of the electron in singly charged ions**

  → Test of atomic wave functions

- **g factor of the free electron**

  → Test of quantum electrodynamics (QED) for free particles

- **g factor of the electron in highly charged ions**

  → Test of bound-state QED

g factor of atomic energy lvels

For bound electrons the magnetic moment (or g factor) is linked to the total angular momentum *J*, formed by vector coupling of the electron spin *S* and the orbital angular momentum *L*

$$J = L + S$$

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

Measurements of g and deviations from the expected L-S value are due to QED and relativistic effects. Their calculation requires the knowledge of the atomic wave functions. Thus g factor measurements serve as test of atomic theory

The determination of the g factor requires the measurement of the energy separation of two angular momentum states (Zeeman effect) in a known magnetic field B

$$\Delta E = g\mu_B B$$

Advantage of ion traps for g factor determinations:

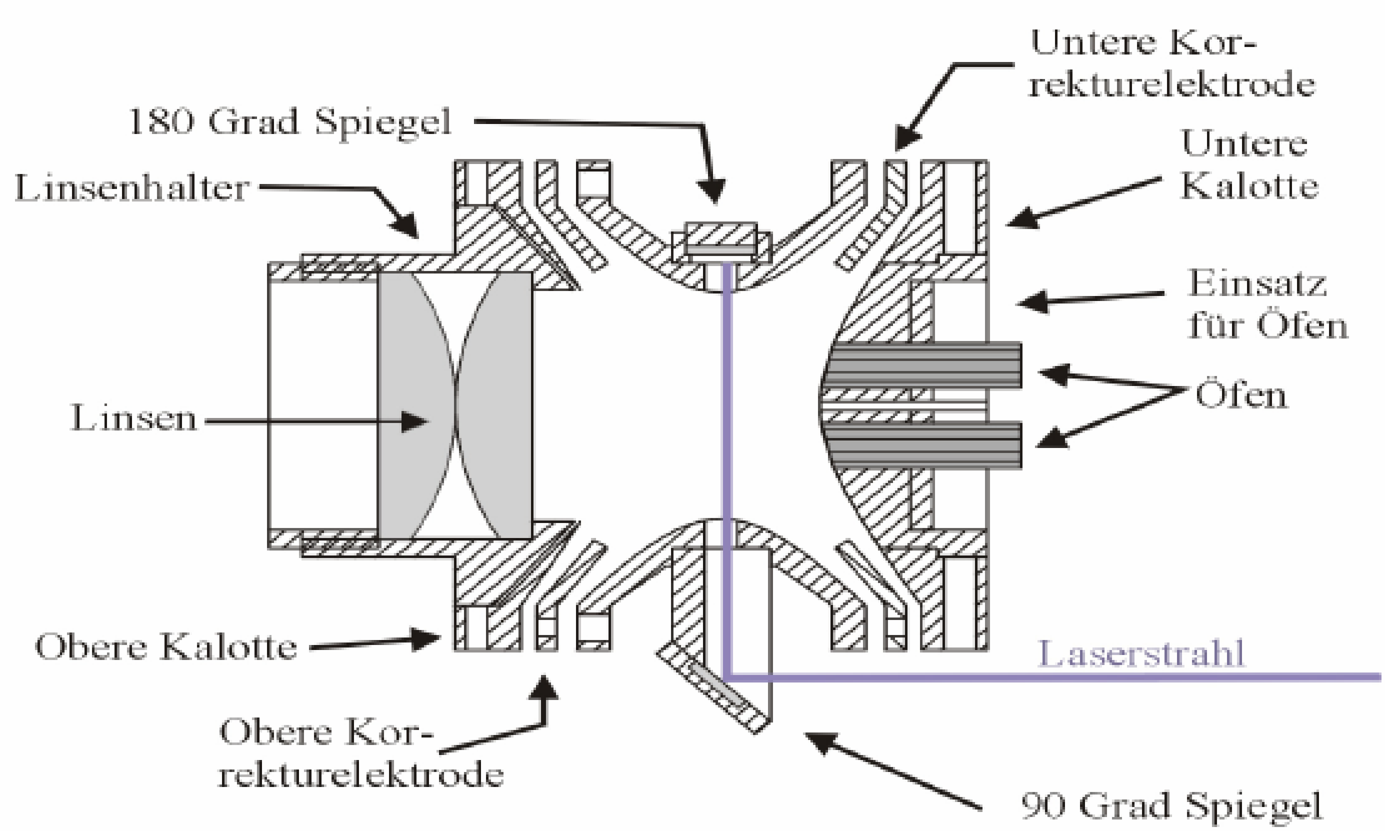
Long observation and coherence time
Magnetic field needs to be known only in a small volume
Stable fields available by superconducting magnets
No first order Doppler effect

Experimental method:

Microwave-optical double resonance technique

Selective depopulation of one Zeeman level in the electronic ground state by laser excitation (optical pumping). Microwave or radio-frequency induced transition between the depleted and a different Zeeman level. Succesful transition is indicated by a change in the observed fluorescence intensity

Sketch of a Penning trap for Zeeman spectroscopy

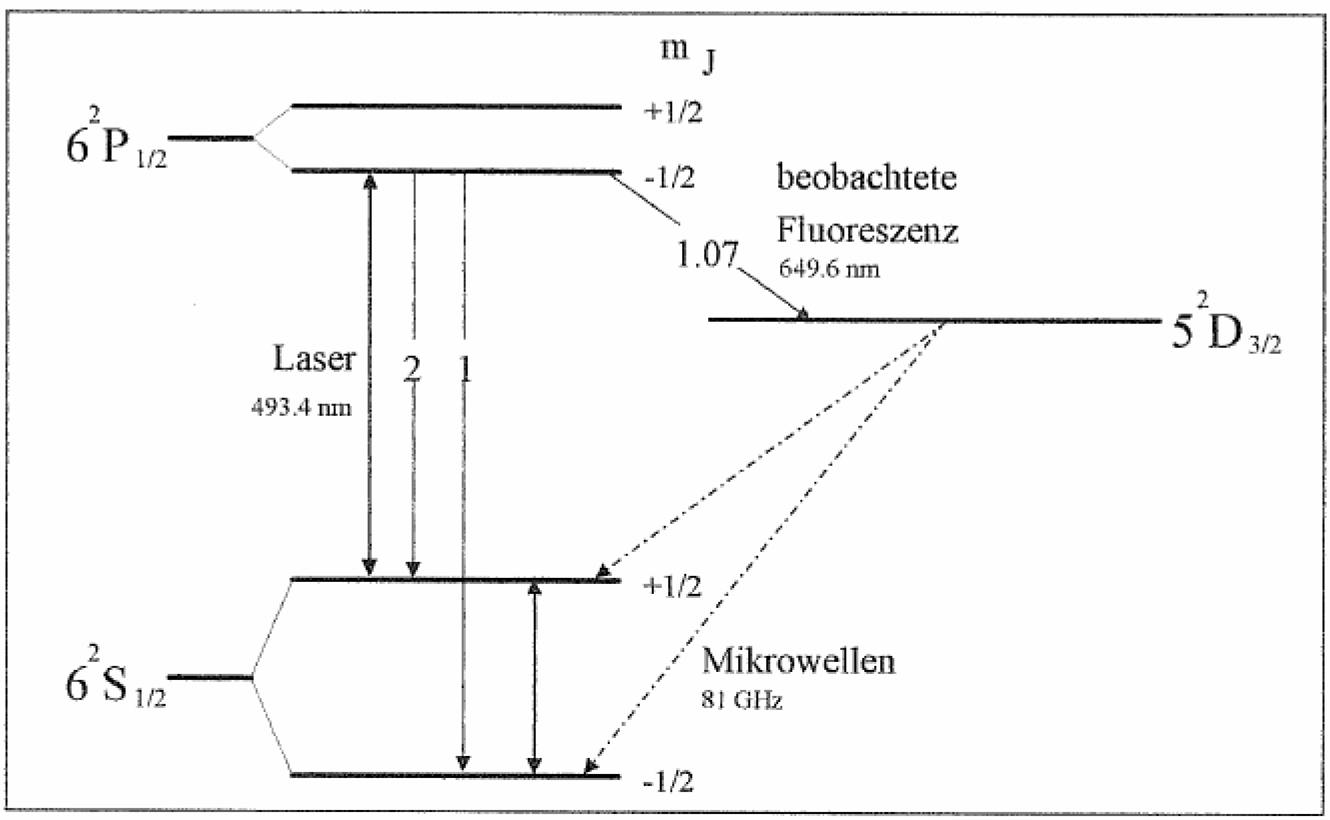


B field direction
(superconducting solenoid)

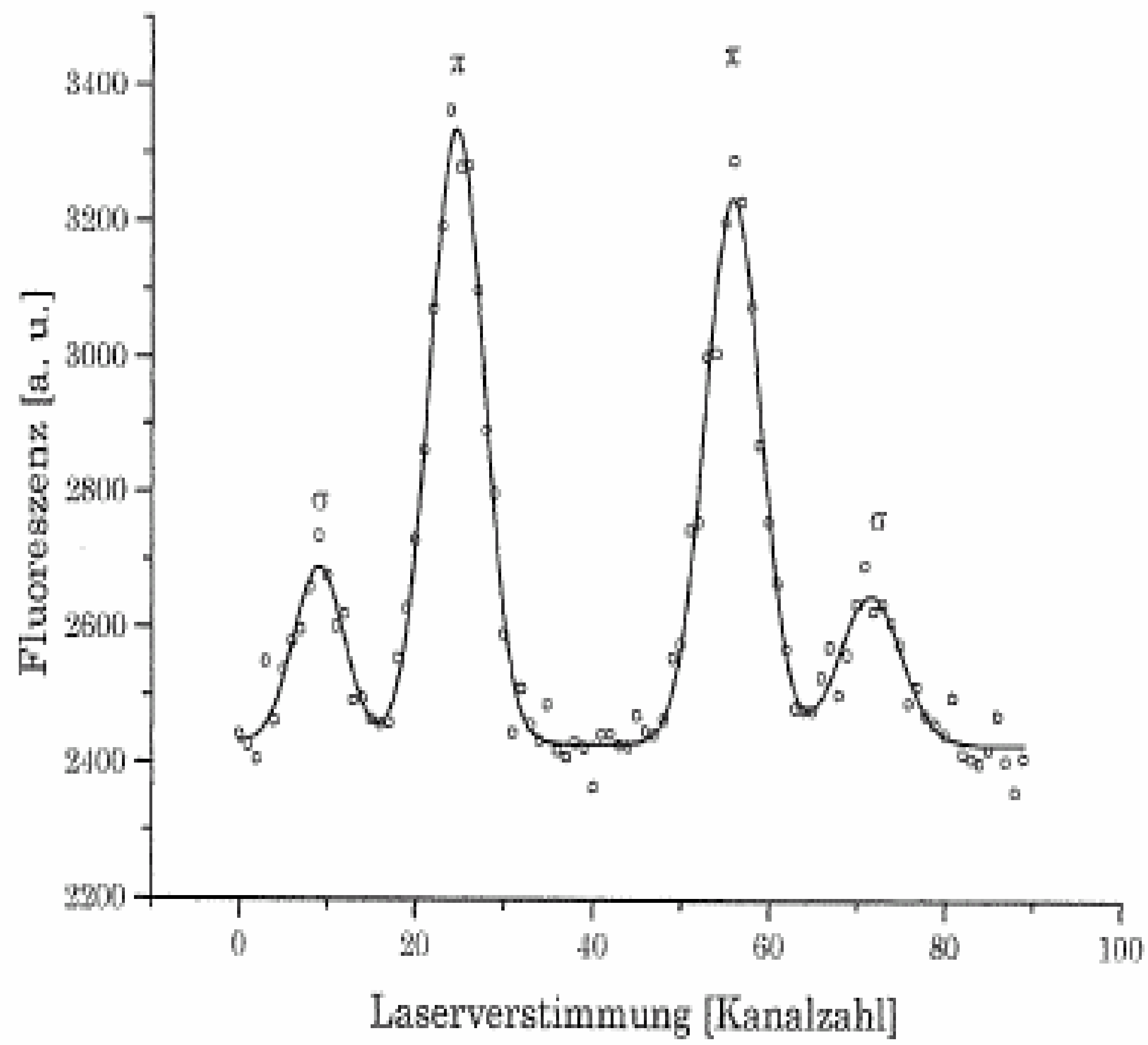
Example of g factor measurements: Ba^+

Ba^+ has alkali-like level diagram. Electronic ground state :
$6S_{1/2}$ (L=0, S=1/2, J=1/2)

In L-S coupling scheme g ≈ 2

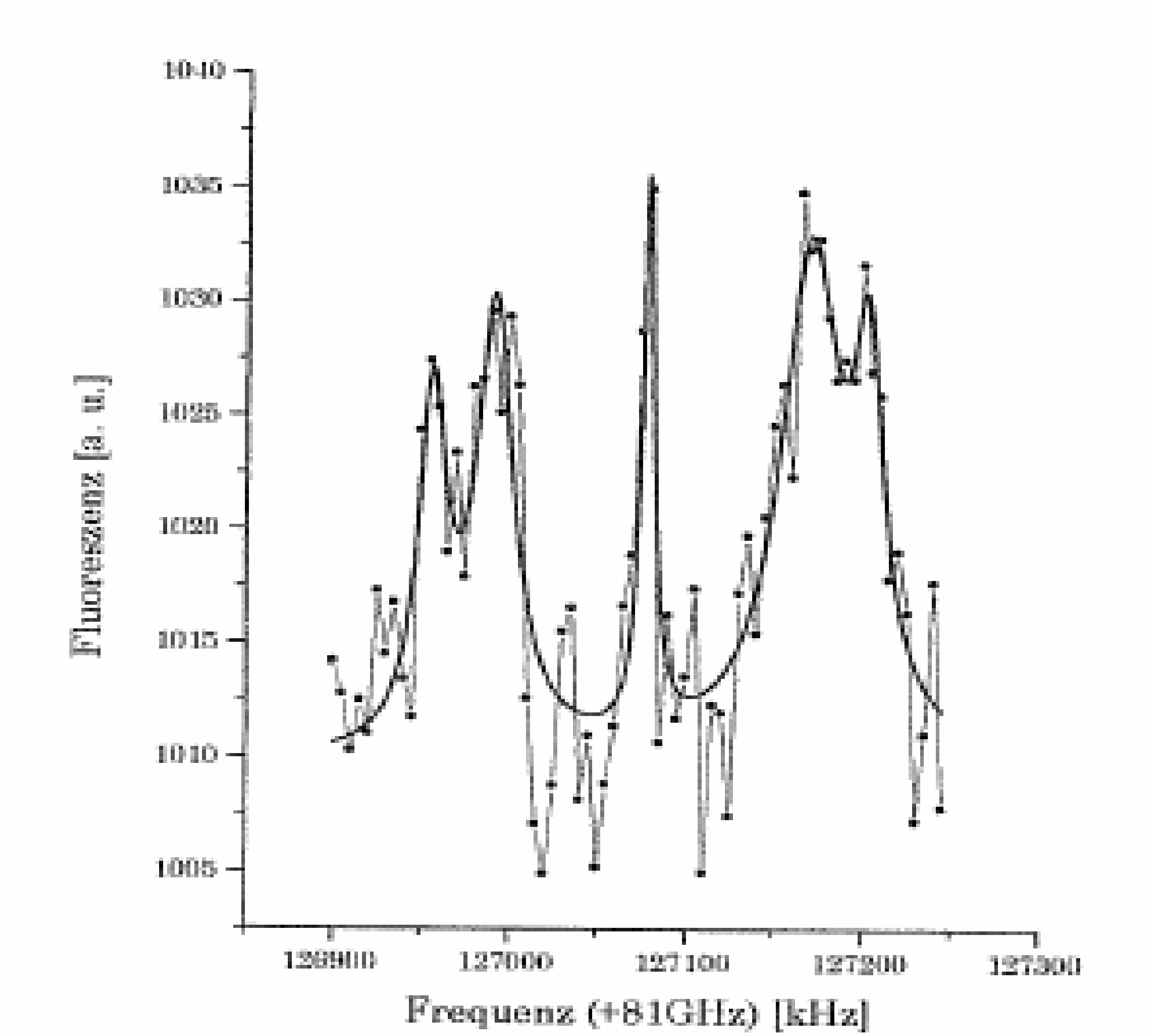


Laser excitation of Ba^{+} on $6S_{1/2}$-$6P_{1/2}$ transition



Laser can be tuned to one of the oberved line components for selective excitation of a ground state Zeeman level. This results in a decrease of the observed fluorescence intensity by optical pumping.
An induced transition to a nondepleted Zeeman level is indicated by an increase of the fluorescence at resonance r.f. transition frequency

Observed r.f. transitions between $6S_{1/2}$ (m_J=1/2 – m_J=-1/2) Zeeman levels in Ba^+

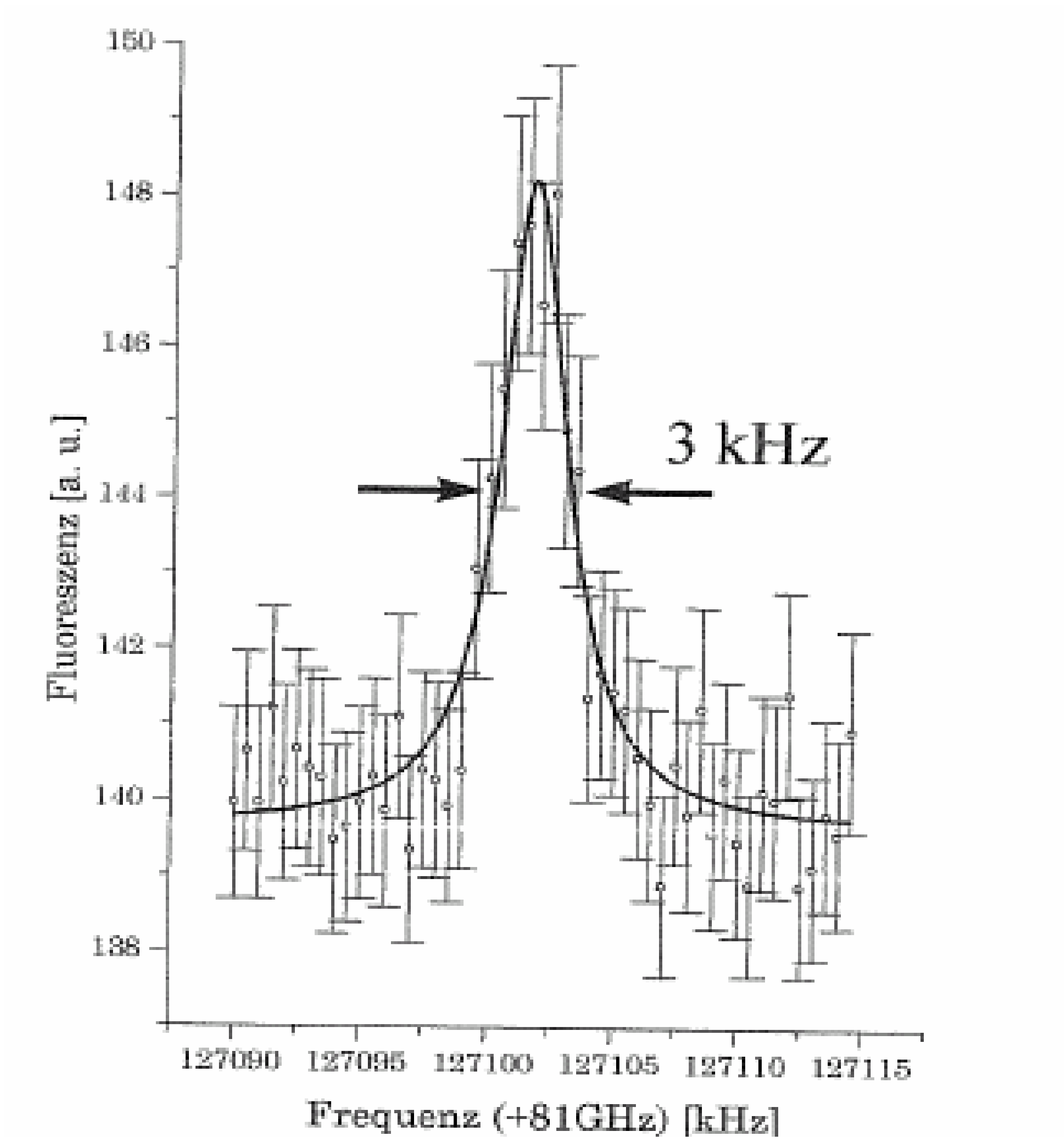


Central carrier and sidebands at combinations of the axial and radial ion oscillation frequencies

Dicke narrowing when ion oscillation amplitude is smaller than the wavelength of the r.f. radiation.

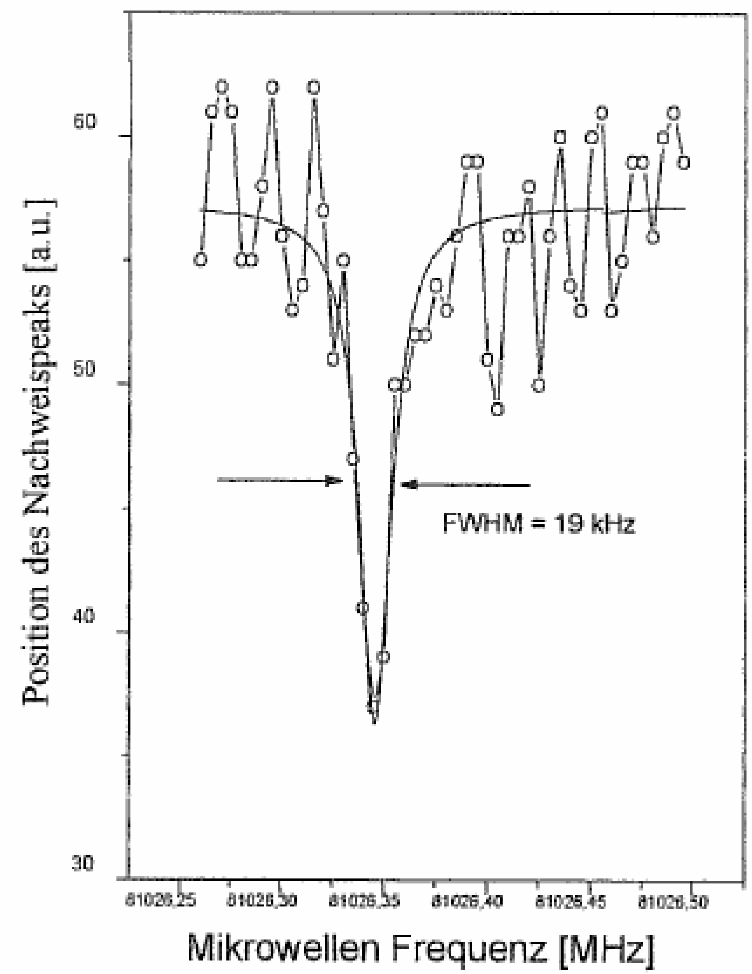
Carrier shows no first order Doppler effect

Central carrier with higher resolution



Uncertainty of transition frequency: 500 Hz of 81 GHz (6 10^{-9})
Limited by magnetic field inhomogeneities and fluctuations

Calibration of the magnetic field strength by cyclotron resonance of stored electrons



Electrons leave the trap when they are excited at their cyclotron frequency $\omega_c = (e/m)B$

FWHM of cyclotron resonance: 150 kHz at 81 GHz
Uncertainty of field calibration: 10^{-7}

Results for ground state g factors of alkali-like ions

Experiments		Theory (MCDF)	
Be^+: 2.002 262 36 (32)	(NIST)	2.002 262 8	(Lindroth 93)
Mg^+: 2.002 254 09 (30)	(NIST)		
Ca^+: 2.002 256 64 (9)	(Mainz)	2.002 262 0	(Indelicato 01)
Ba^+: 2.002 491 92 (3)	(Mainz)	2.002 491 1	(Lindroth 93)

g factor of the free electron

Dirac equation: g = 2
Quantum electrodynamics: g = 2 (1+a)
g factor anomaly a ≈ 0.001

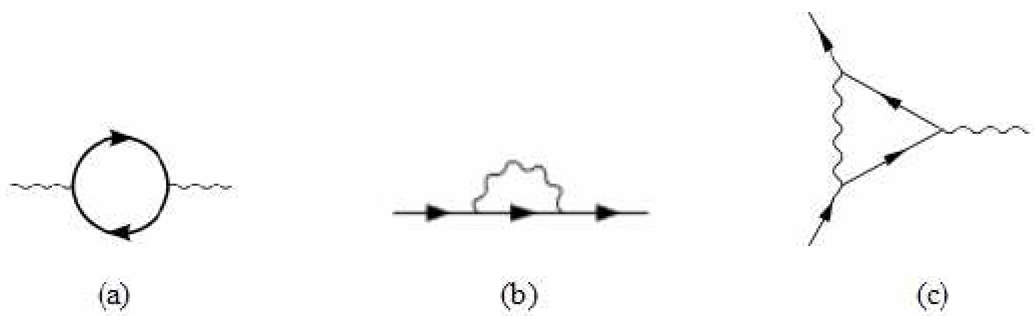


Fig. 8 Basic Feynman diagrams. (a) vacuum polarization, (b) self energy, (c) vertex correction

$$g/2 = 1 + C_2(\alpha/\pi) + C_4(\alpha/\pi)^2 + C_6(\alpha/\pi)^3 + C_8(\alpha/\pi)^4 + C_{10}(\alpha/\pi)^5 + \dots \ .$$

C2 = 0.5
C4 = -0.328 478 965 579 193…
C6 = 1.181 241 456….
C8 = -1.9106 (20)
C10 = 9.16 (58)

Kinoshita et al., (May 2012) (arXiv:1205:5368v1

C10 requires the evaluation of 12 672 diagrams

Measurement of the free electron´s g factor

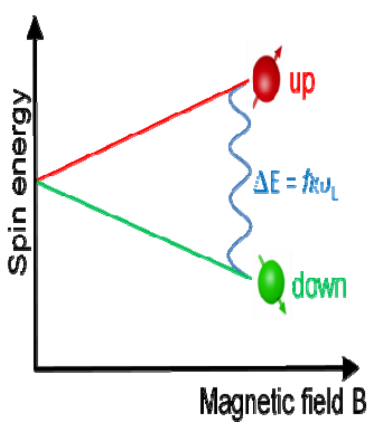
Measurement of the Larmor precesion frequency

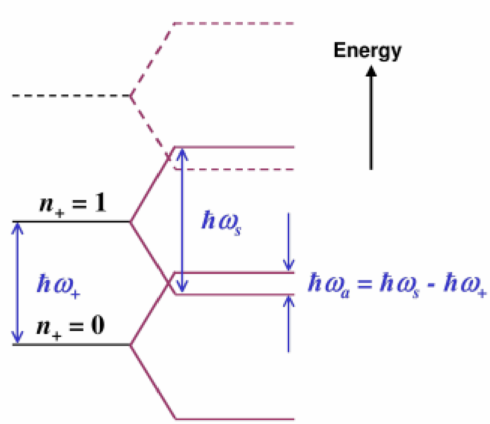
$\omega(L) = g/2(e/m)B$

Measurement of the cyclotron frequency

$\omega(C) = (e/m)B$

→ g= 2ω(L)/ω(C)





Q.M. energy levels
of a free electron
in a B-field

Particular feature of the free electron:

ω(L)≈ ω(L) : Direct measurement of difference frequency
ω(a) = ω(L)-ω(C) gains three order of magnitude in precision

Penning trap experiment

- Measurement of the cyclotron frequency: Excitation of cyclotron motion, observation by induced image currents in trap electrodes
- Measurement of spin precession frequency: „Continuous Stern-Gerlach effect „

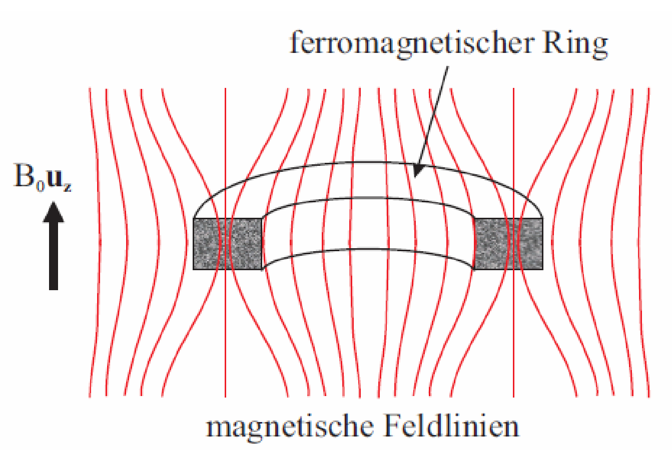
Continuous Stern-Gerlach effect

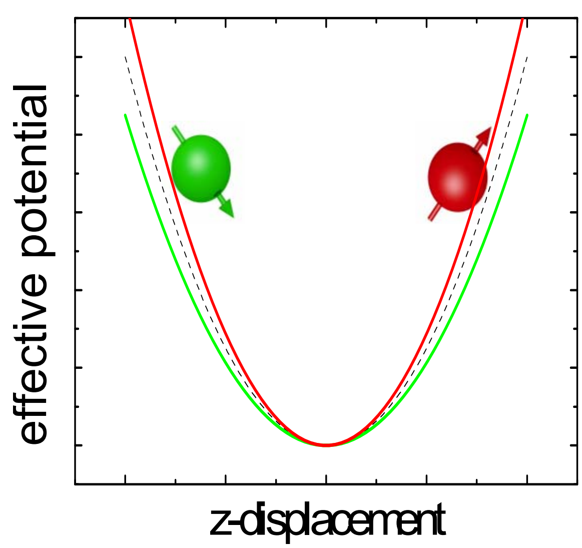
Force on the magnetic moment in inhomogeneous magnetic field → change of oscillation frequency

The magnetic force add or subtracts to the electric trapping force: Frequency difference between the two spin states.

Magnetic field inhomogenity produced by
a ferromagnetic ring electrode of the Penning trap:

$$B_z = B_0 + B_2\left(z^2 - \frac{r^2}{2}\right)$$

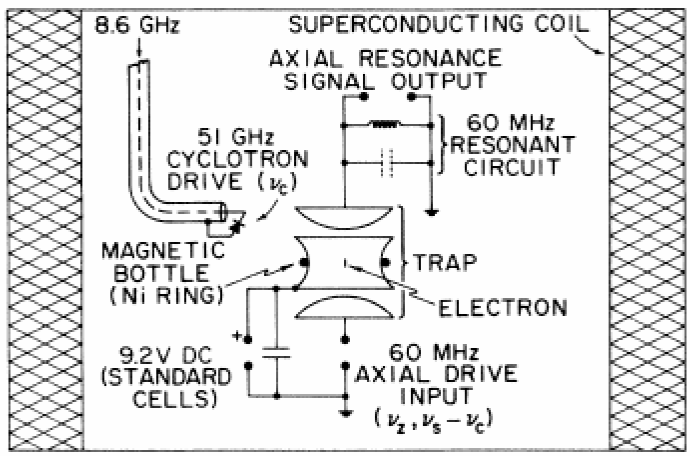


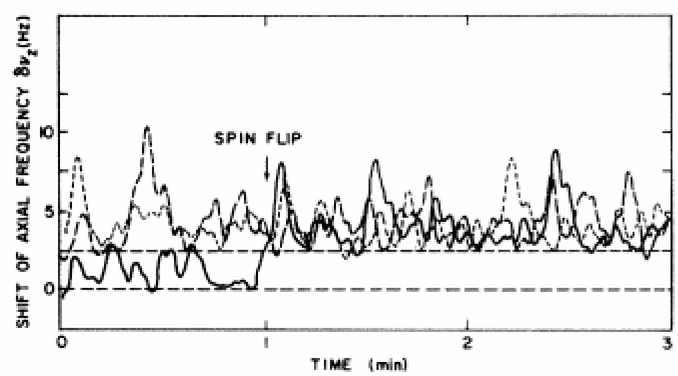


$$\omega_z = \sqrt{\frac{1}{m}\frac{\partial^2 V}{\partial z^2}} = \sqrt{\frac{1}{m}\left(\frac{\partial^2 V_{el}}{\partial z^2} + \frac{\partial^2 V_{mag}}{\partial z^2}\right)} = \sqrt{\omega_{z0}^2 + \frac{2}{m}\mu B_2}$$

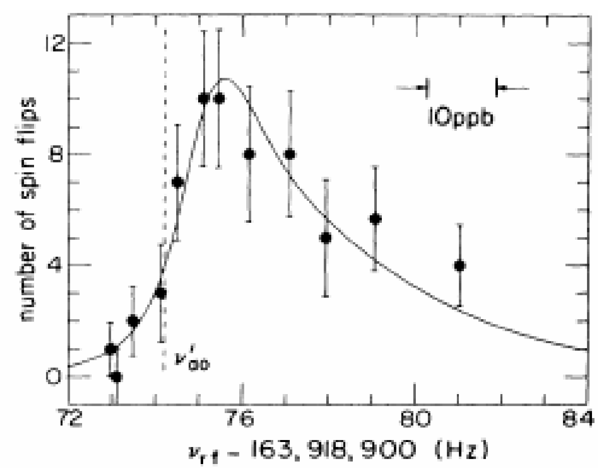
g factor experiment on free electron/positron

v. Dyck, Dehmelt, Schwinberg, PRL **59**, 26 (1987)





First recorded electron spin flip

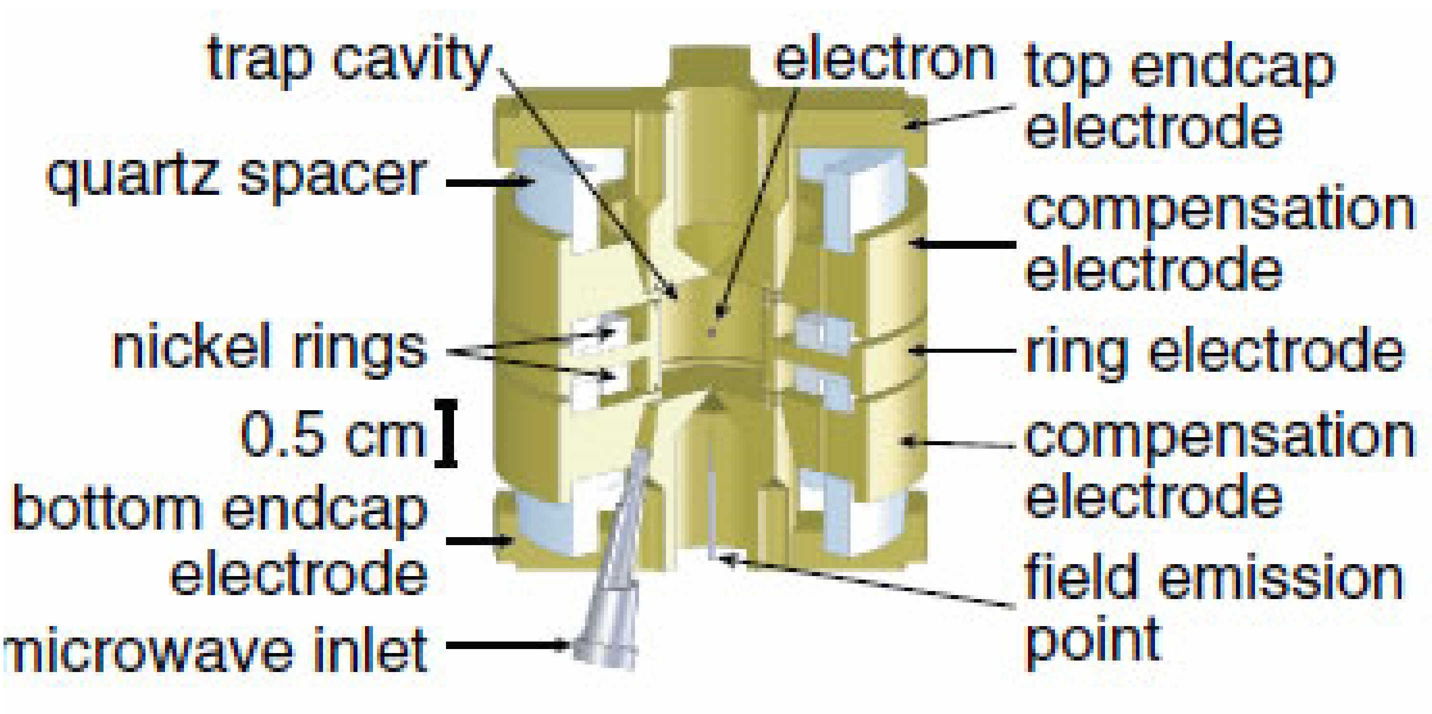


$$a(e^-) = 1\,159\,652\,188.4(4.3)\times 10^{-12},$$

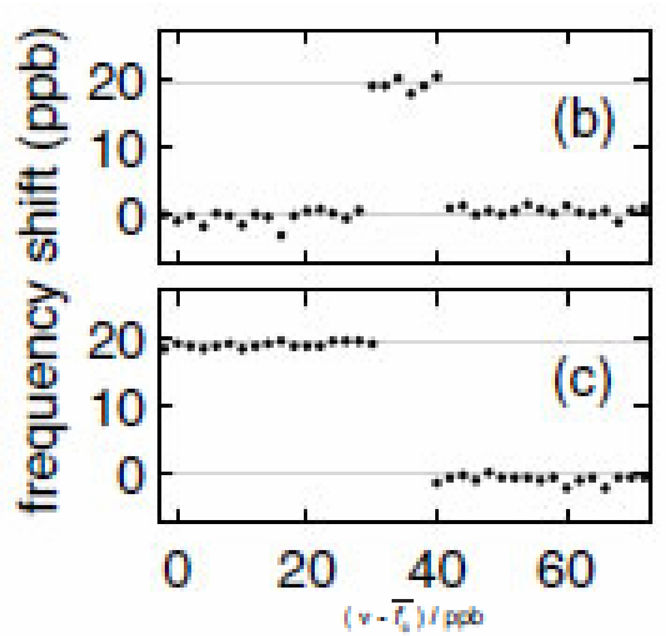
$$g(e^-)/g(e^+) = 1 + (0.5 \pm 2.1)\times 10^{-12},$$

Improvement of g factor by Gabrielse et al:

Use of cyclindrical cavity
→ better control of microwave field in trap
Reduced trap temperature
→Narrower line width, smaller influence of B-field inhomogeneity

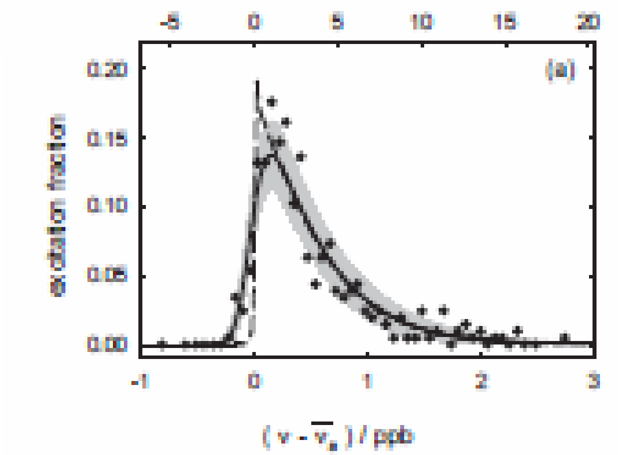


D. Hanneke et al., PRL **100**, 120801 (2008)

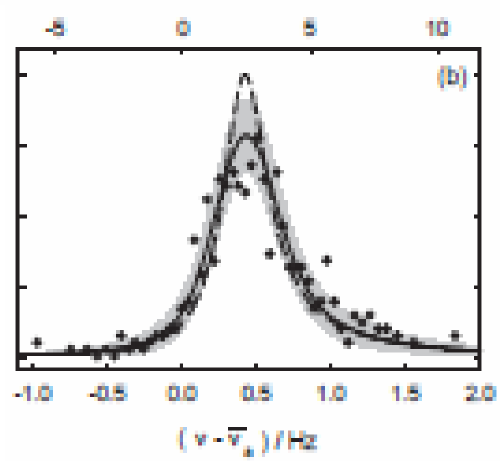


Observed cyclotron transition (above)

and spin flip (below)



Cyclotron drive



Anomaly drive

Result:

a = 1 159 652 180.73 (0.28) (0.24 ppb)

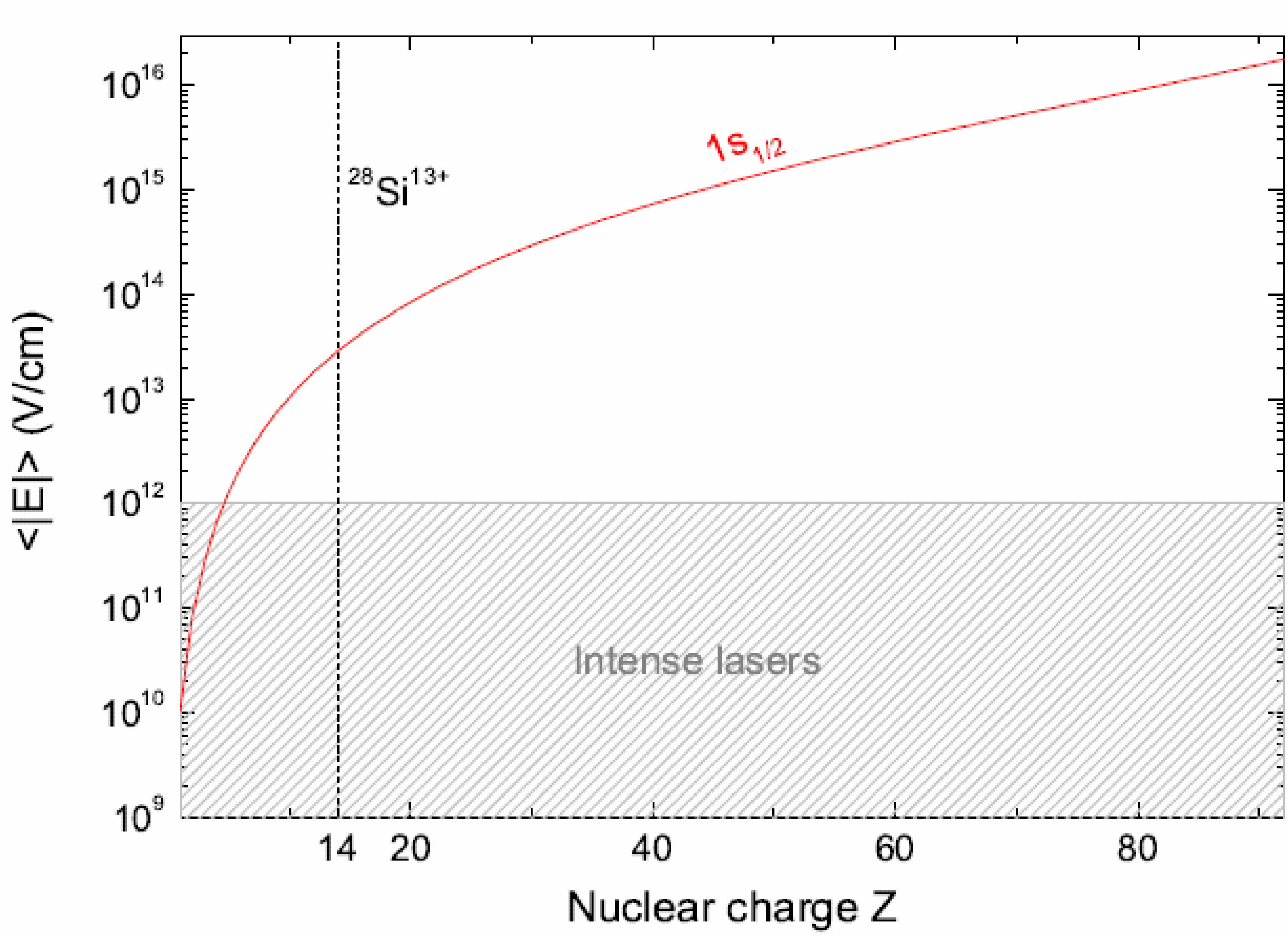
Theory:

a = 1 159 652 181.82 (0.78) (0.67 ppb)

g factor of the single electron bound in hydrogen-like ions

- **Single electron systems are calculable to high precision**
- **Electron g factor differs from free particle value by binding corrections**
- **QED corrections to electron g factor**
- **Electron near nucleus experiences strong electric field → Test of QED under extreme conditions**
- **Nuclear structure effects change the electrons g factor**

Electric field strength in H-like ions at the Bohr radius



Contributions to the electron´s g factor in hydrogen-like ions

QED of bound electron

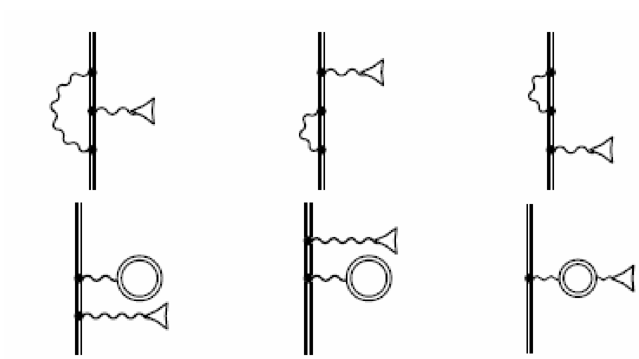
Nuclear correction

Dirac value

$$g_J = 2 + a_e + a_b + b + c$$

QED of free electron
*(*exp. Dehmelt et al.
theo. Kinoshita*)*

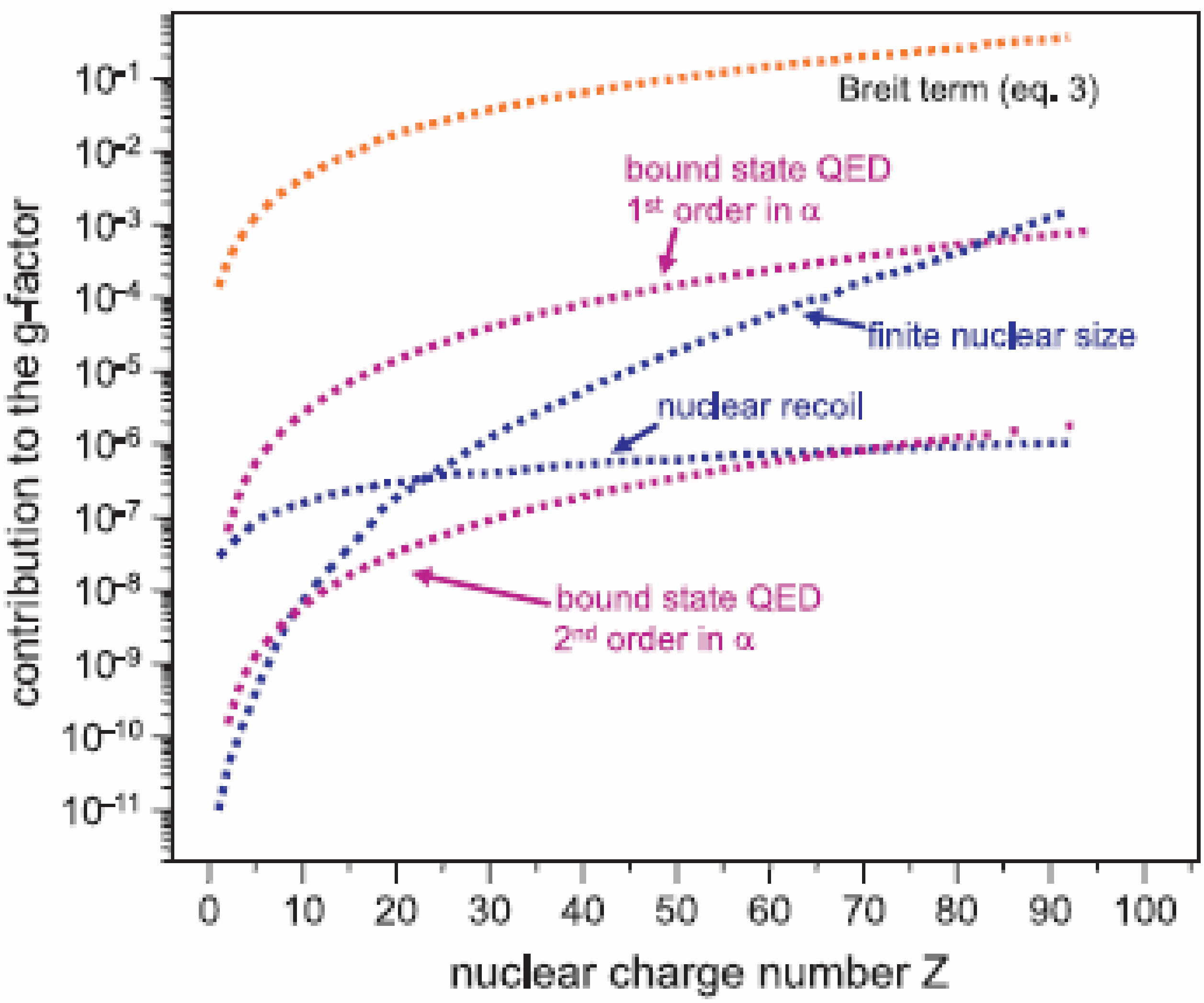
Relativistic correction



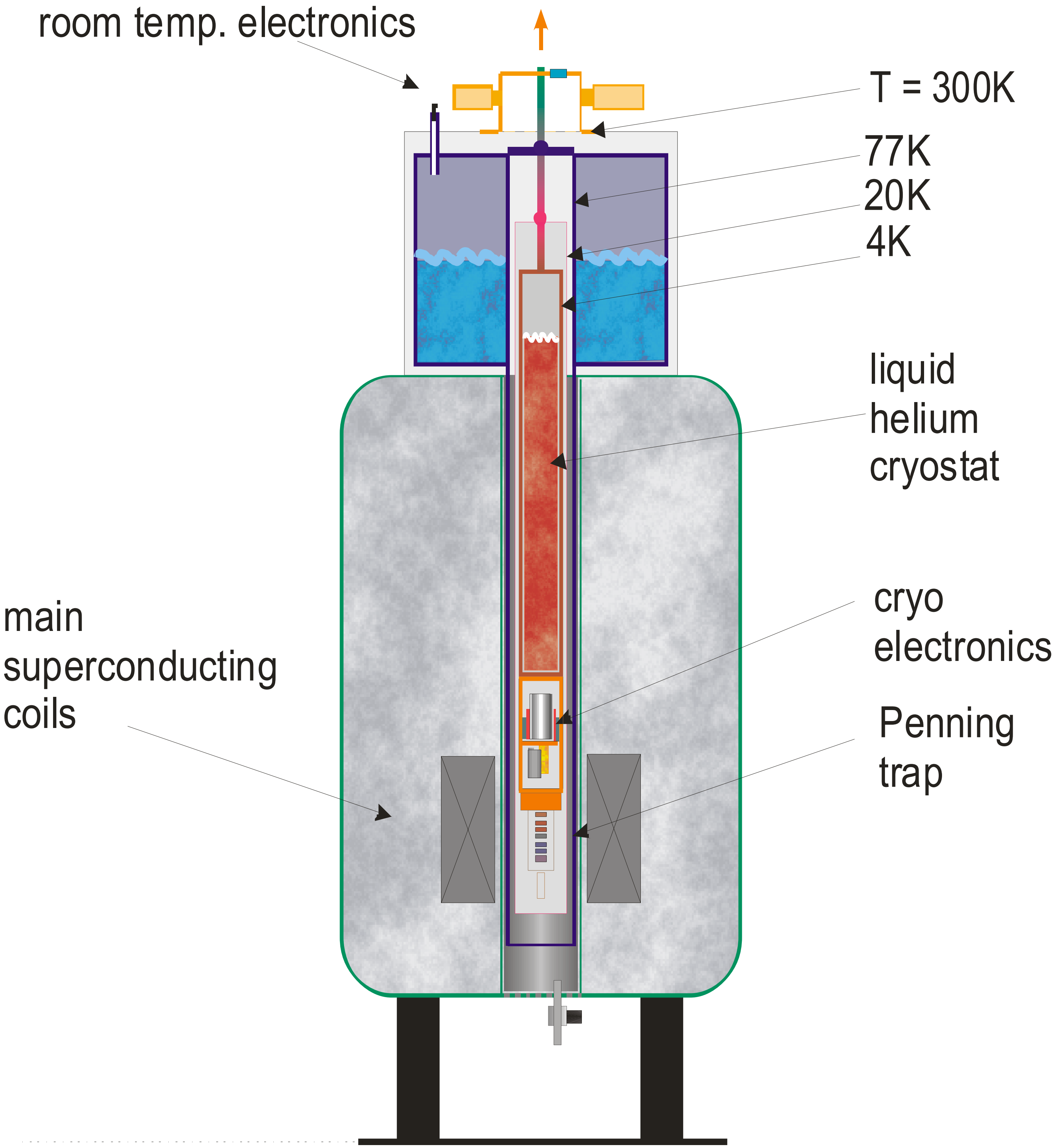
First order Feynman diagrams for the bound electron

$$g/2 = 1 + C_2(Z\alpha)(Z\alpha/\pi) + C_4(Z\alpha)(Z\alpha/\pi)^2 + C_6(Z\alpha)(Z\alpha/\pi)^3 + \ldots$$

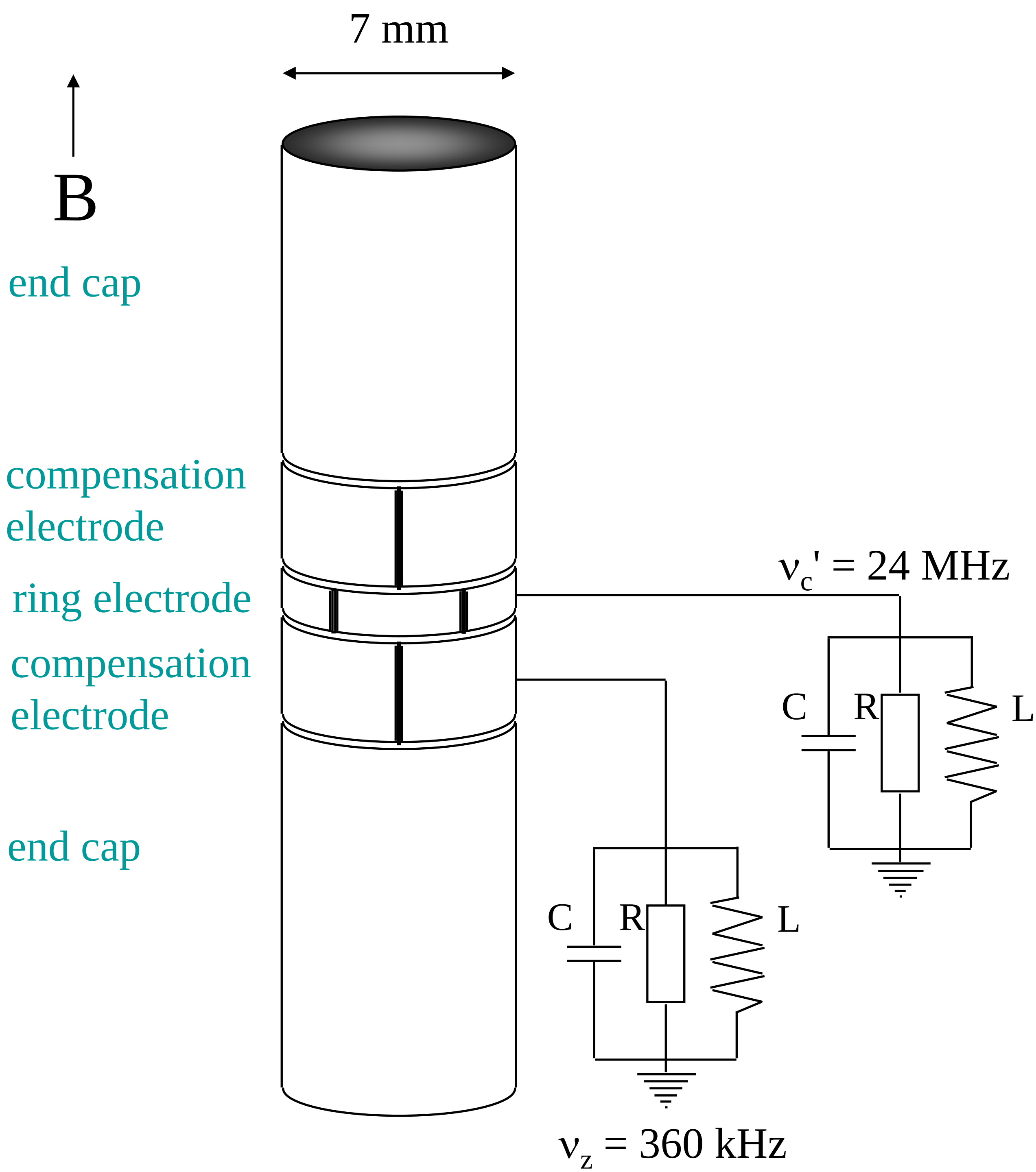
Z-dependence of g factor corrections



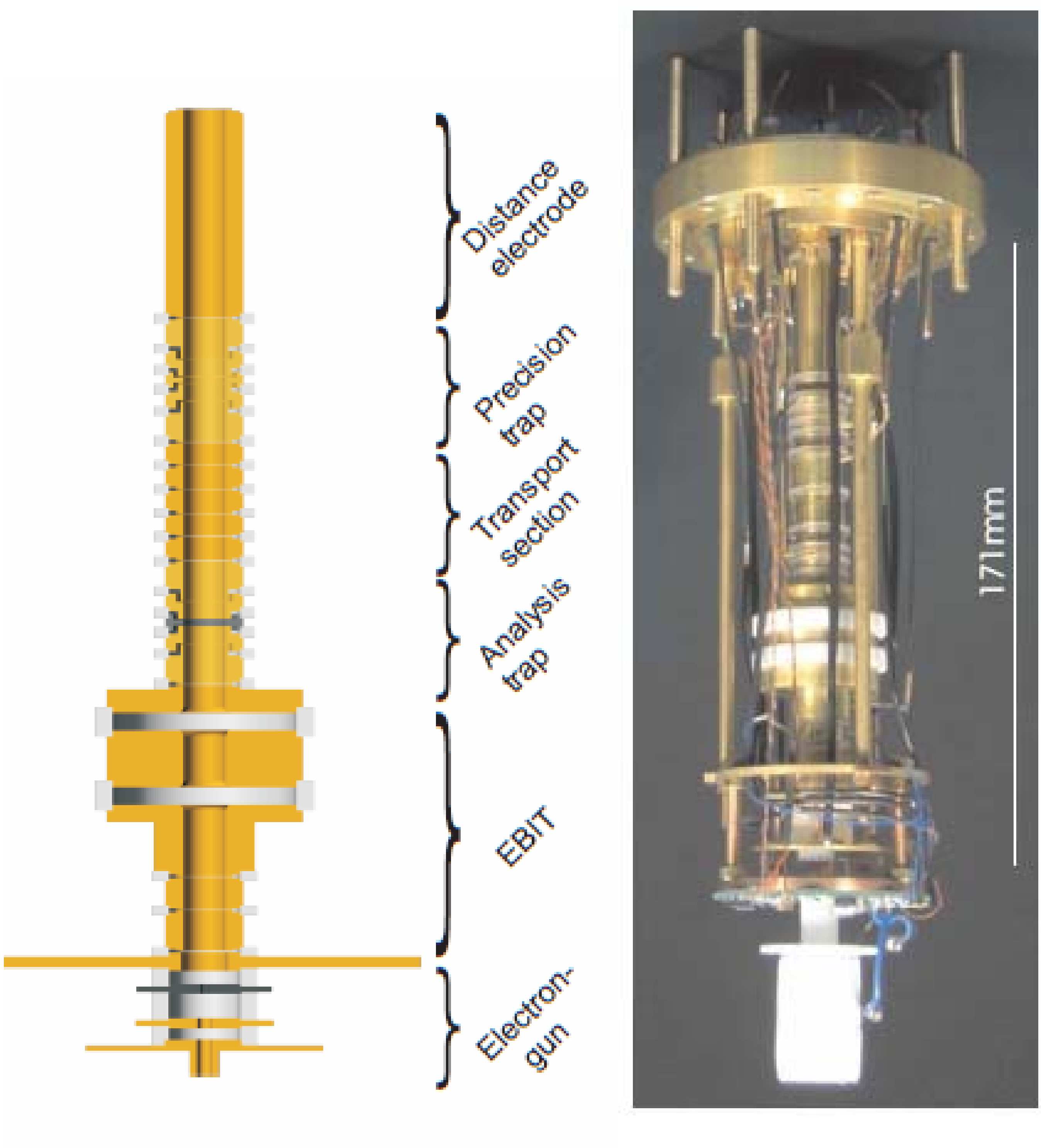
Experimental setup



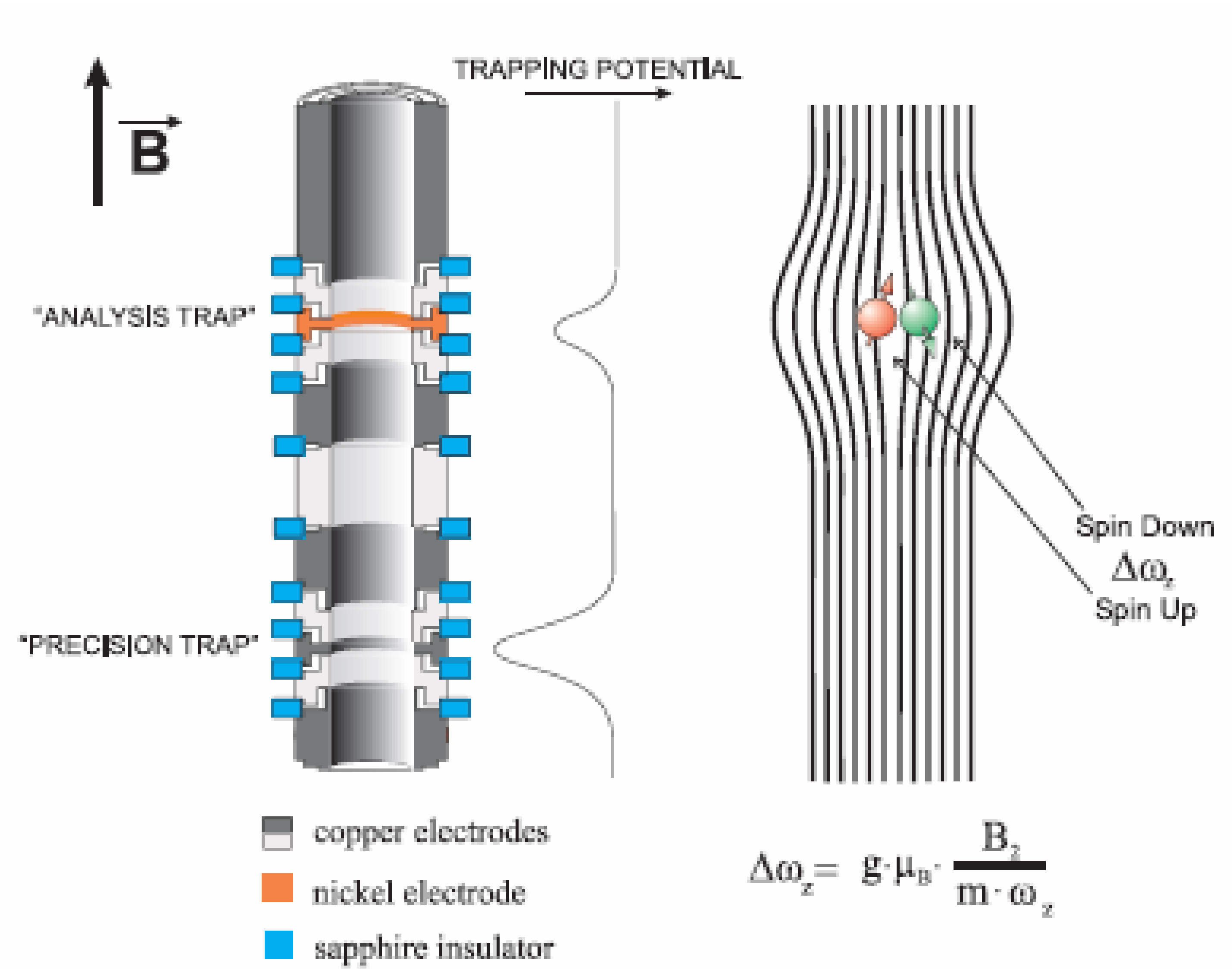
Experimental determination of electron´s g factor in a cylindrical Penning trap



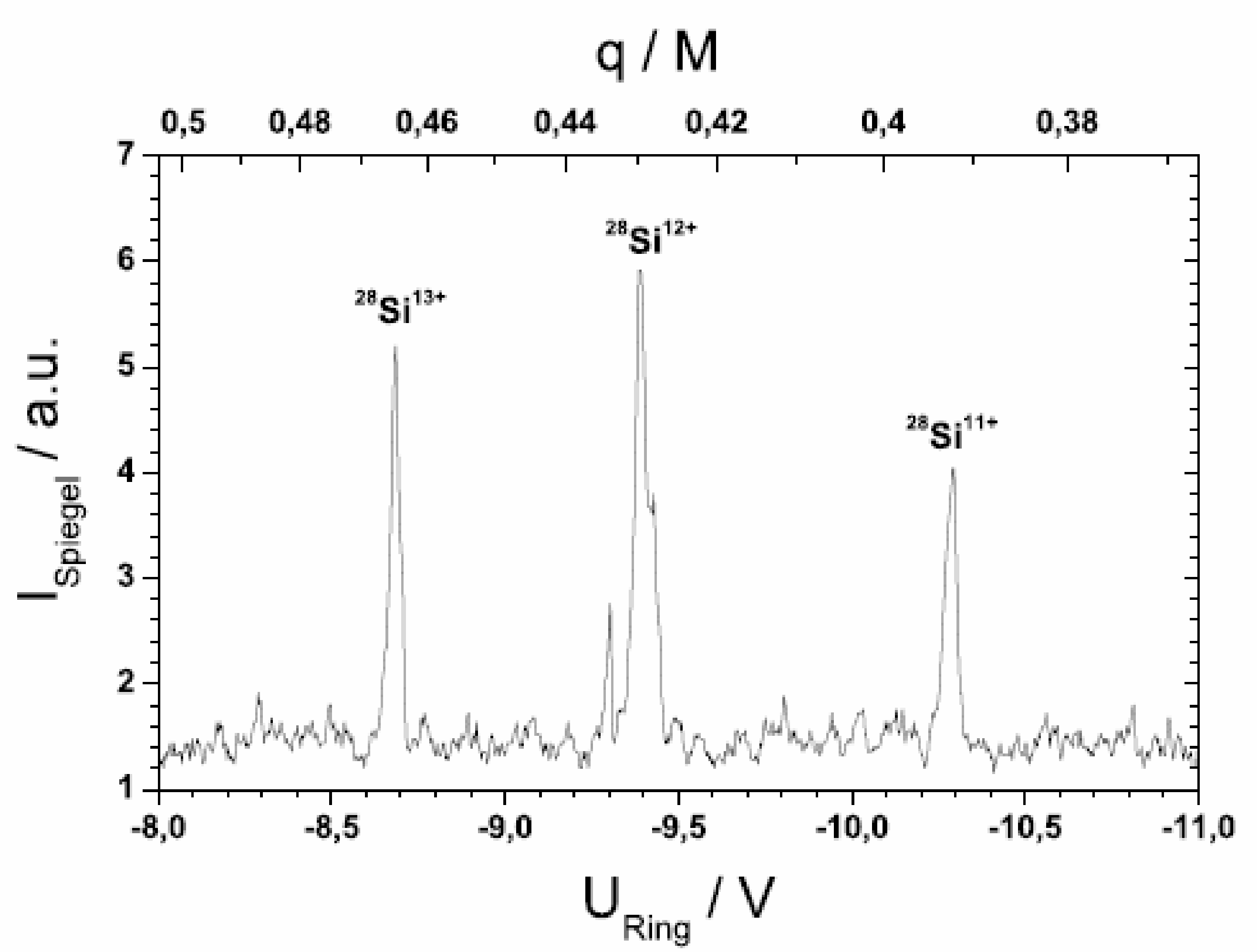
Triple trap setup



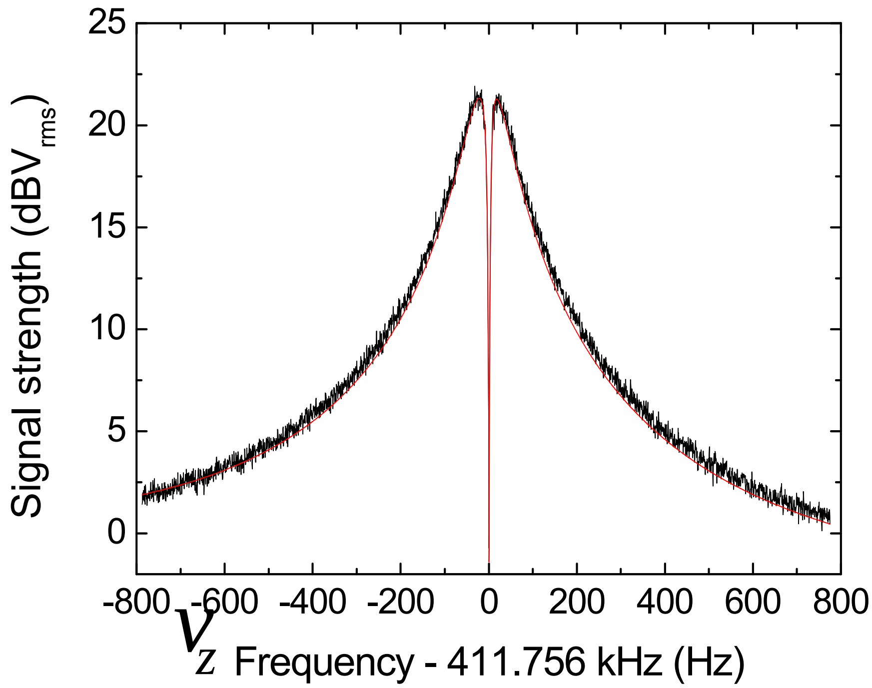
Double trap structure



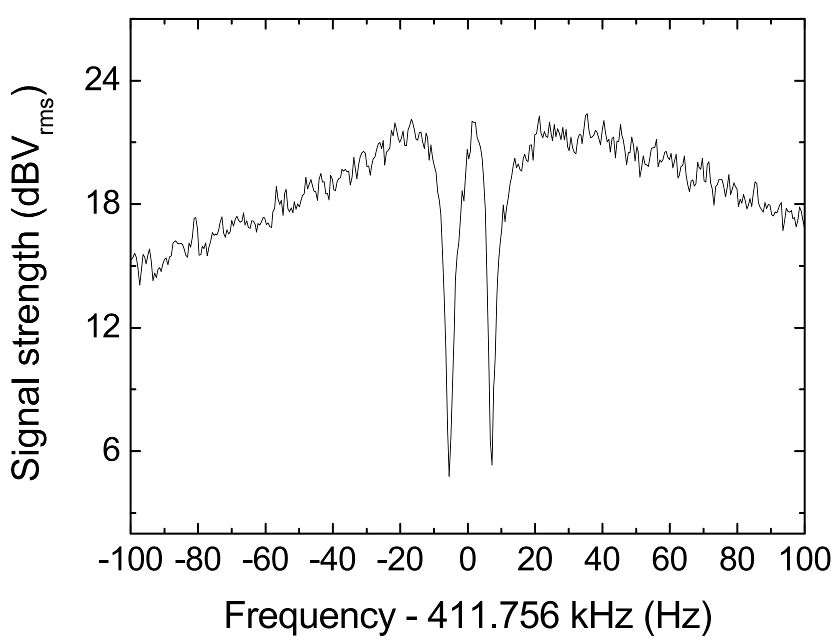
Trapped Silicon ions of different charge states



Measurement of motional frequencies

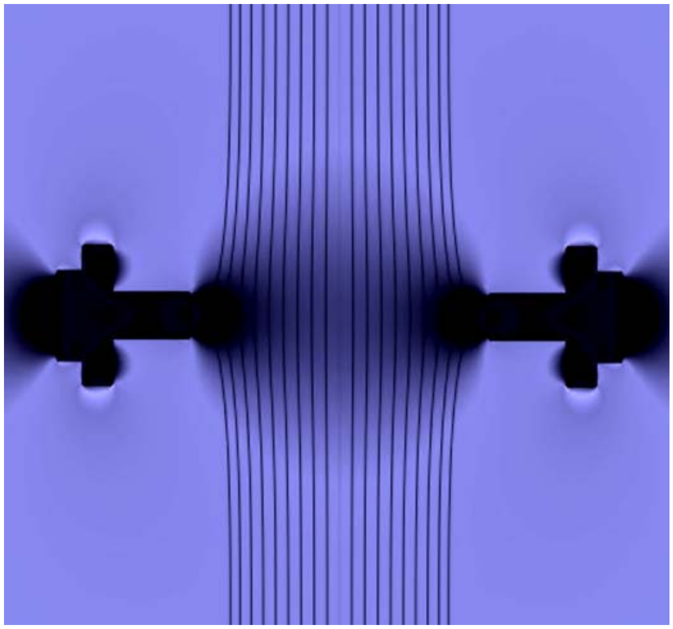


Axial frequency: Dip in axial tank circuit
Radial frequencies: Coupling to axial motion by additional rf field

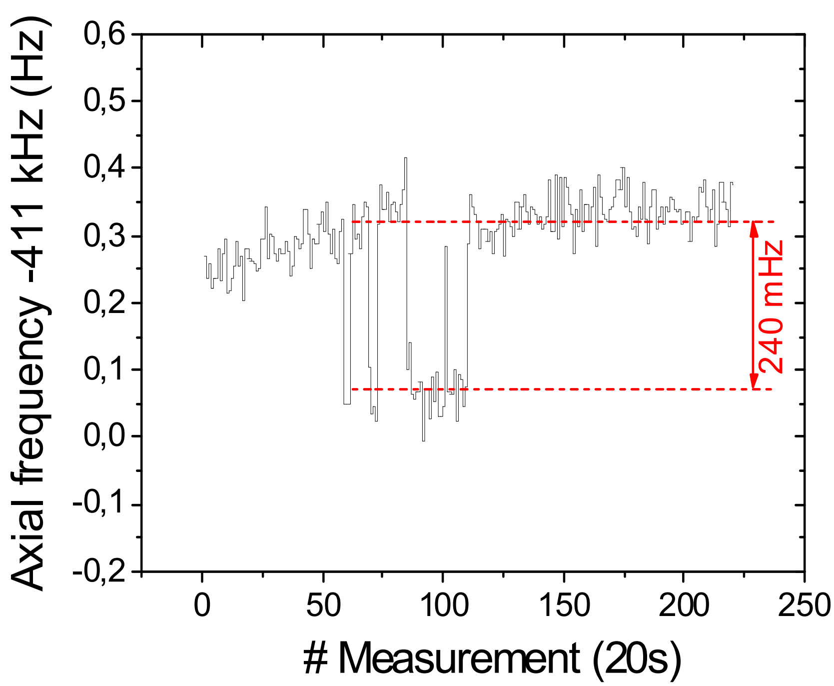


$$\nu_+ = \nu_L + \nu_R - \nu_z + \nu_{rf}$$

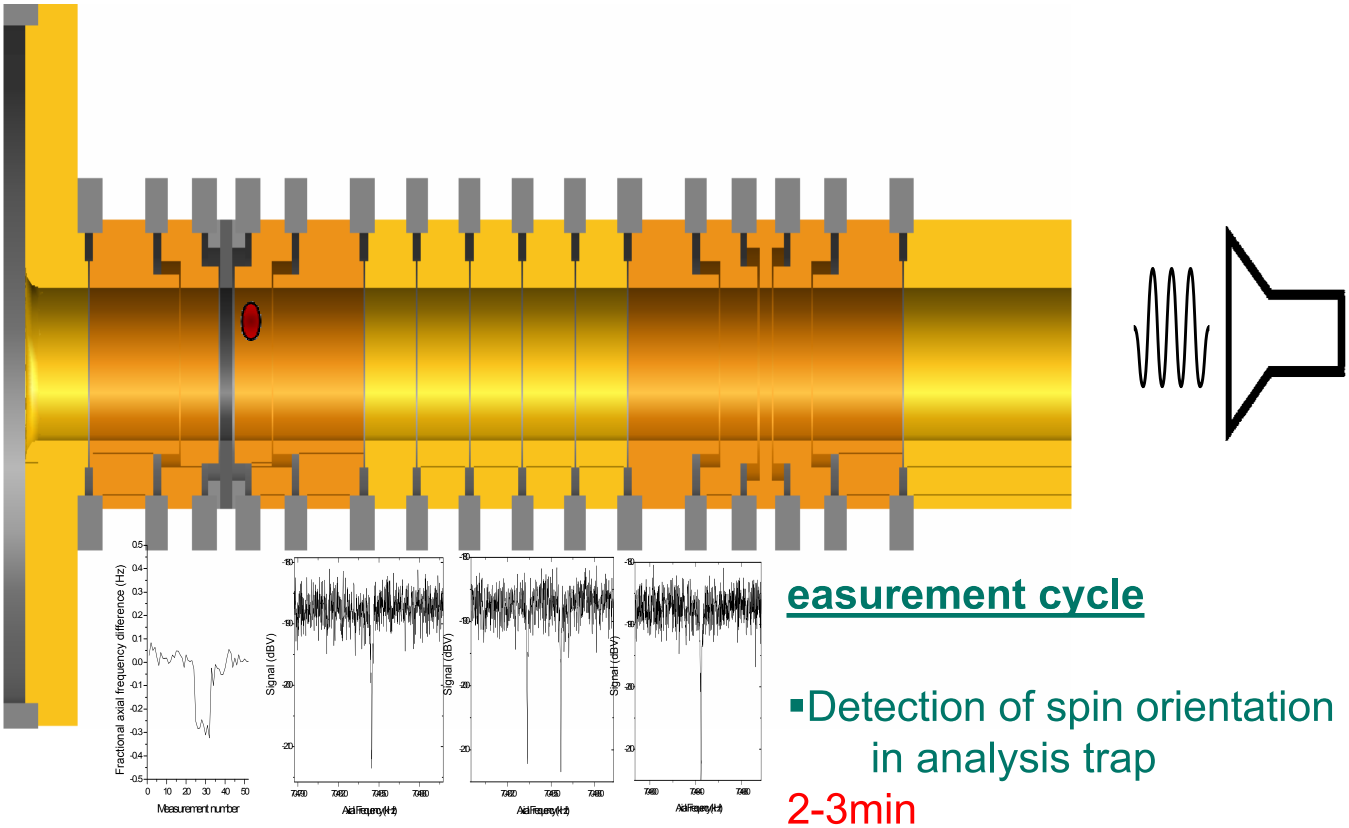
Induced spin transition detected by continuous Stern-Gerlach effect

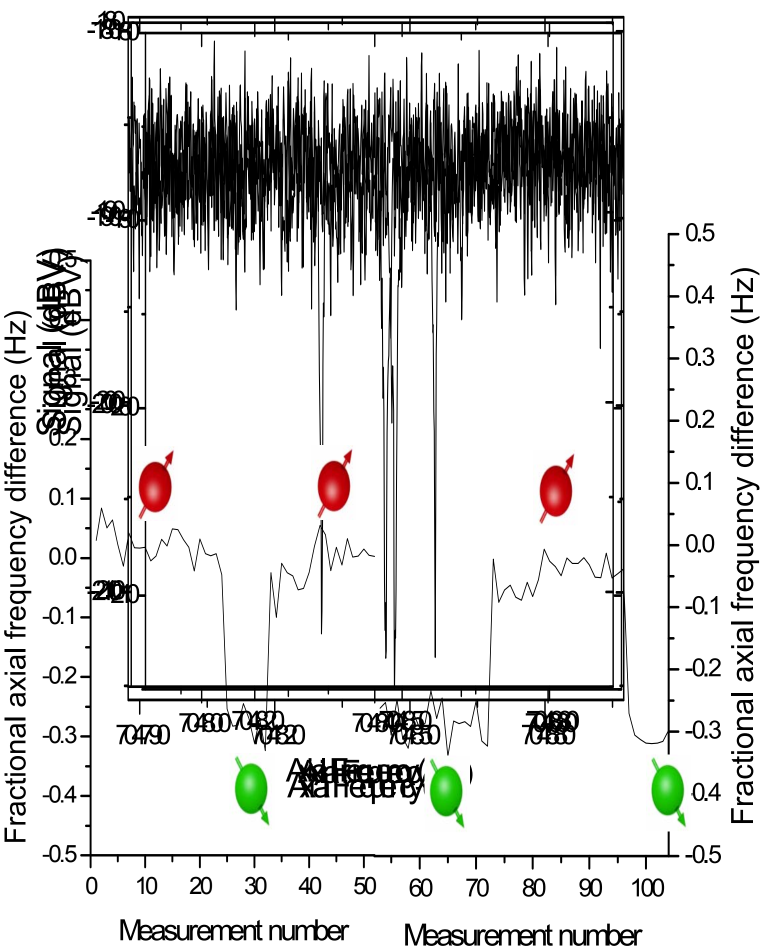


$$\Delta\nu_z = \frac{g\,\mu_B\,B_2}{4\,\pi^2\,m_{ion}\,\nu_z} = 240\,mHz$$



g-factor measurement process

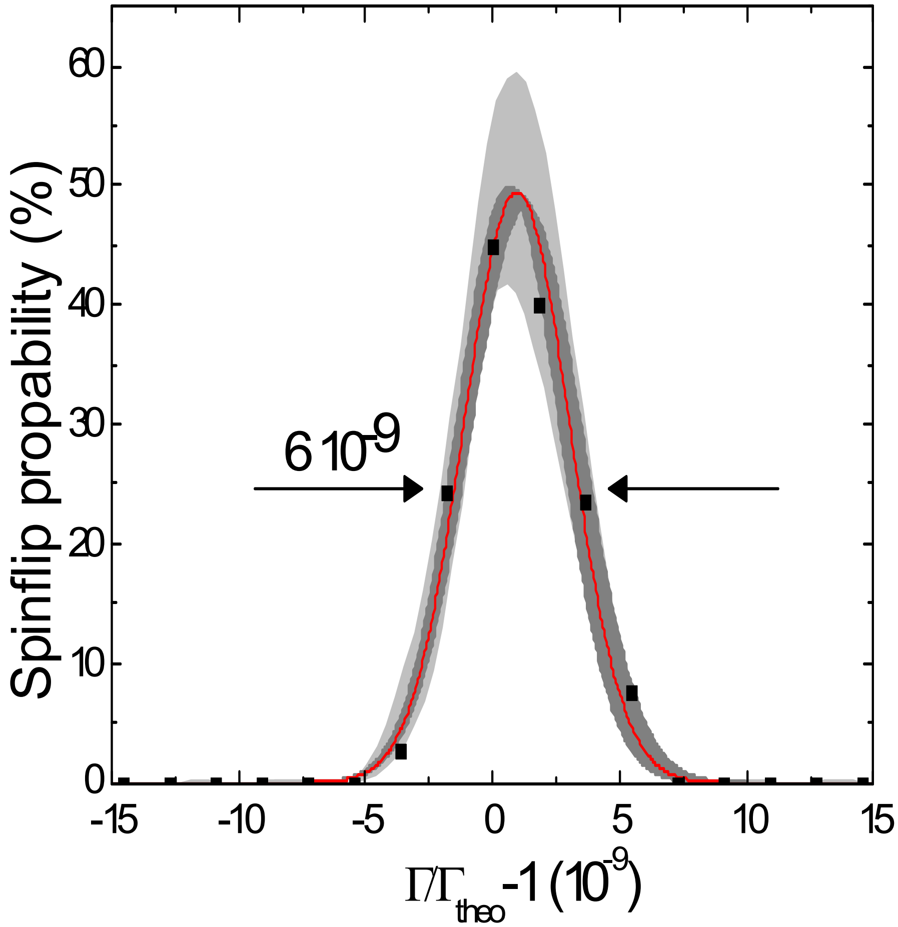




easurement cycle

- Detection of spin orientation in analysis trap
  2-3min
- Transport to precisio trap
  20s
- Measurement of eigenfrequencies and simultaneous irradiation with microwaves
  10min
- Transport to analysis trap
  20s
- Detection of spin orientation in analysis trap
  - Spin flip in the precision trap?

g-factor resonance



$$g = 2\Gamma \frac{q}{e} \frac{m_e}{m_{Ion}}$$

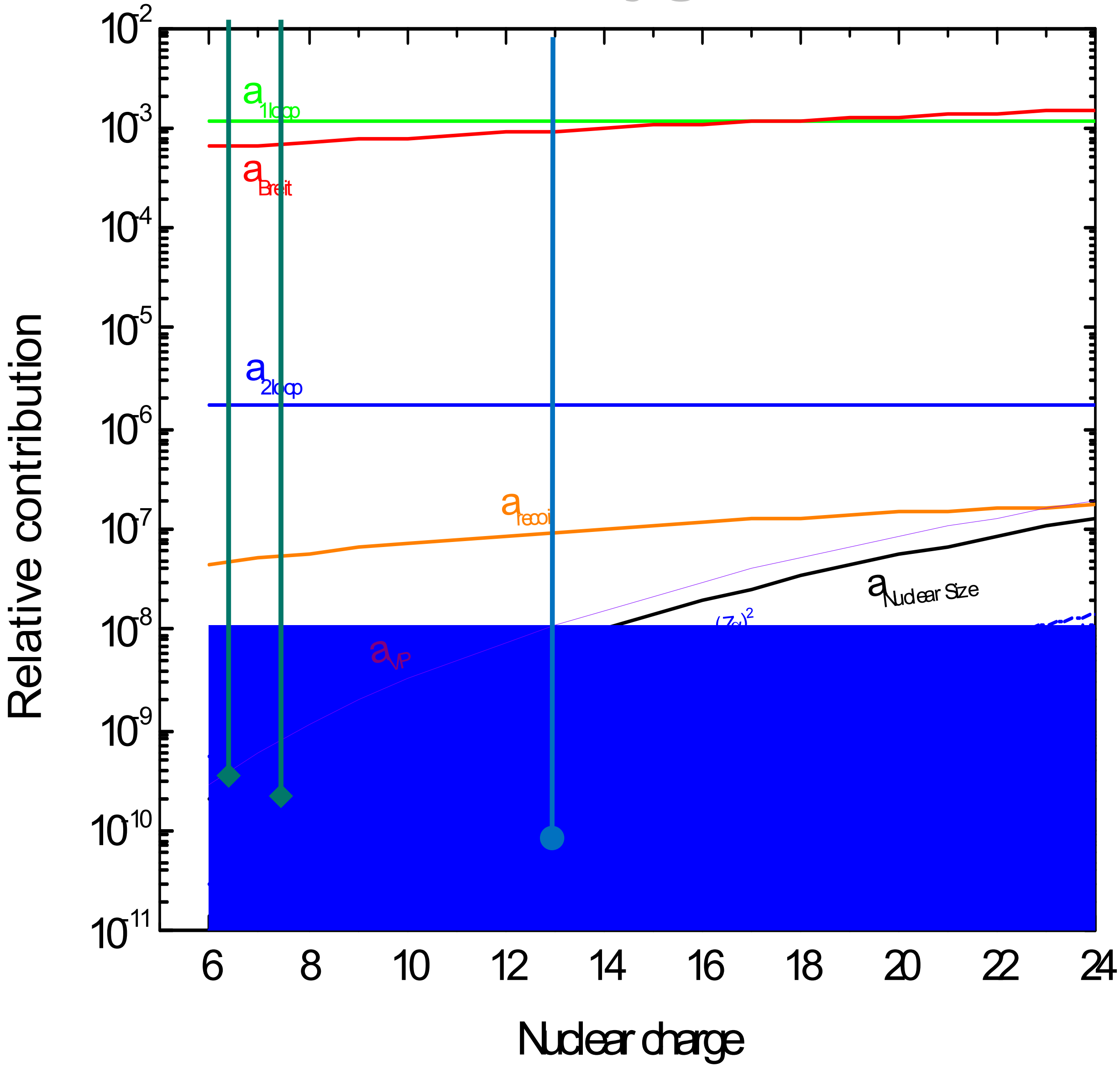
$$\Gamma = (\nu_{MW} / \nu_C)$$

g_{exp} = 1.995 348 958 7 (5)(3)(8)
g_{theo}= 1.995 348 958 0 (17)

Error dominated by uncertainty of electron mass

Theoretical value by:
[Z. Hamann, J. Zatorski, C. Pachucki et al. 2011]

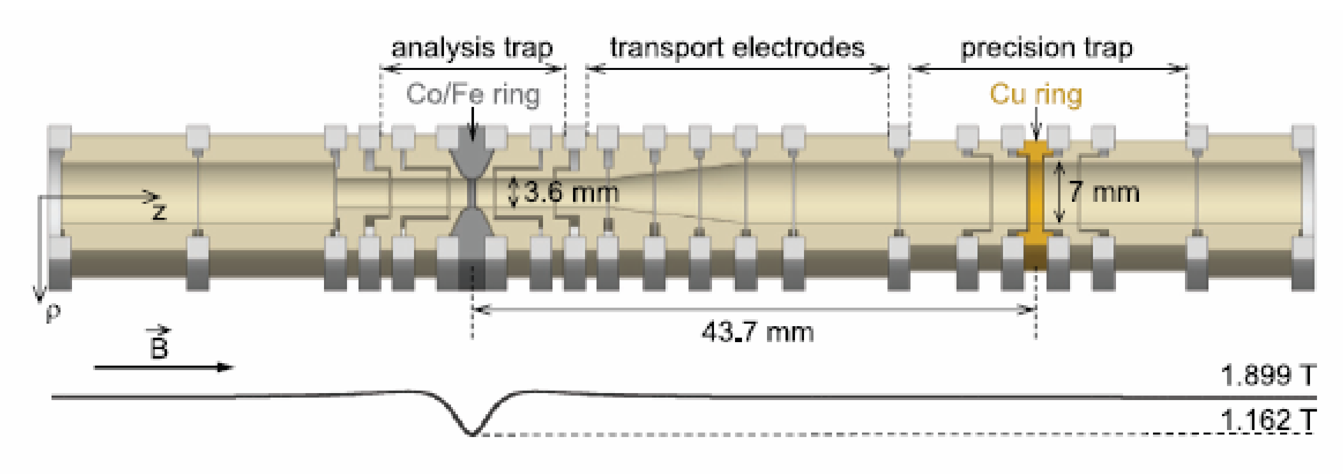
Results of g factor measurements in H-like carbon,oxygen, and silicon



- Higher order contributions to two-loop theory are relevant for the first time in Si13+
- Nuclear size effect tested, nuclear charge radius extracted: $\langle r^2 \rangle^{1/2} = 3.18\ (15)$ fm

The magnetic moment of the proton/antiproton

Small size of proton´s magnetic moment requires
high magnetig field inhomogeneity for spin flip detection



Axial frequency jump at spin flip: 190 mHz at 674 kHz