

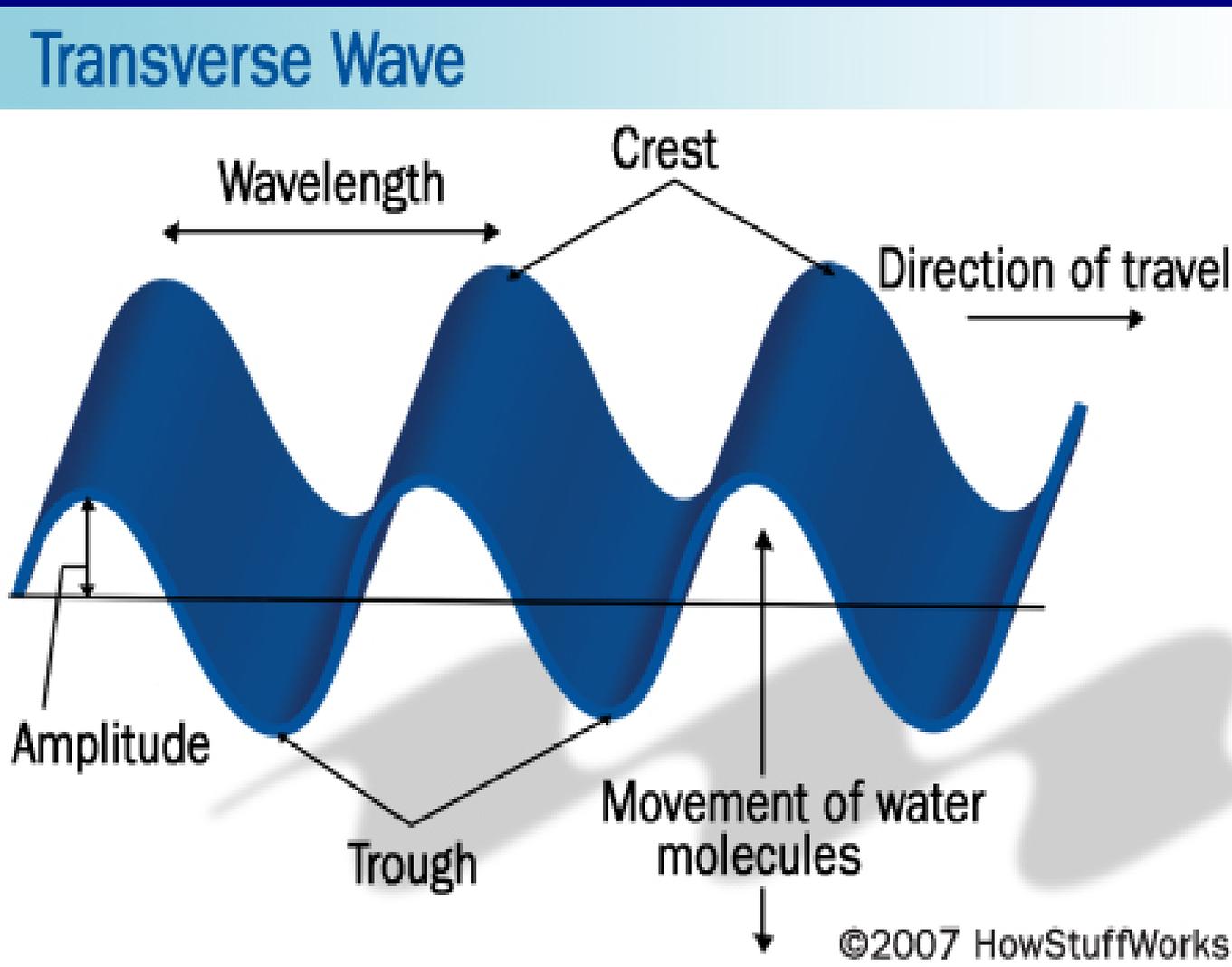
# **QUANTUM MECHANICS And Subatomic Physics**

**Stanley Yen  
TRIUMF**

**April 18, 2009**

# What have we learned in these lectures?

## 1. Basic properties of waves

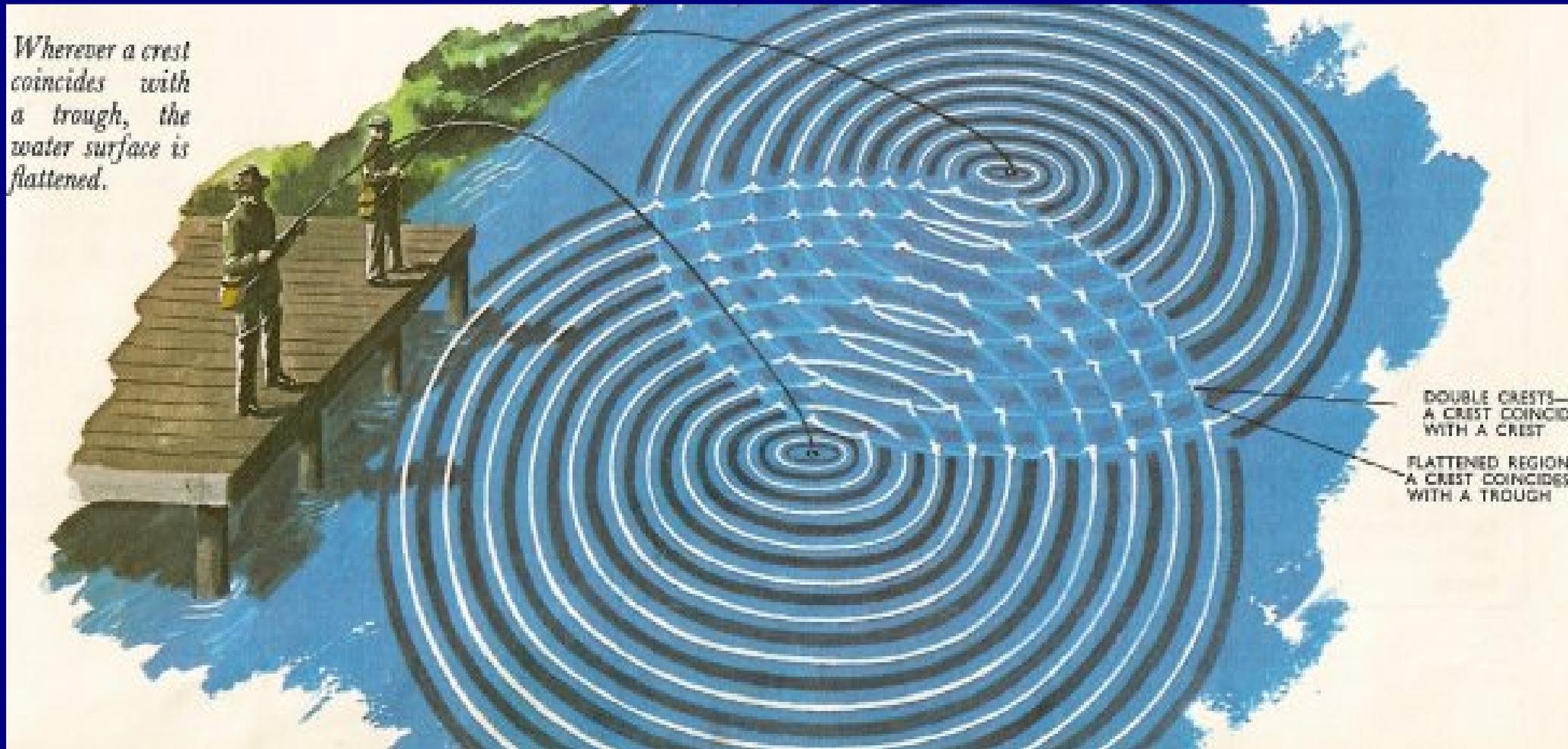


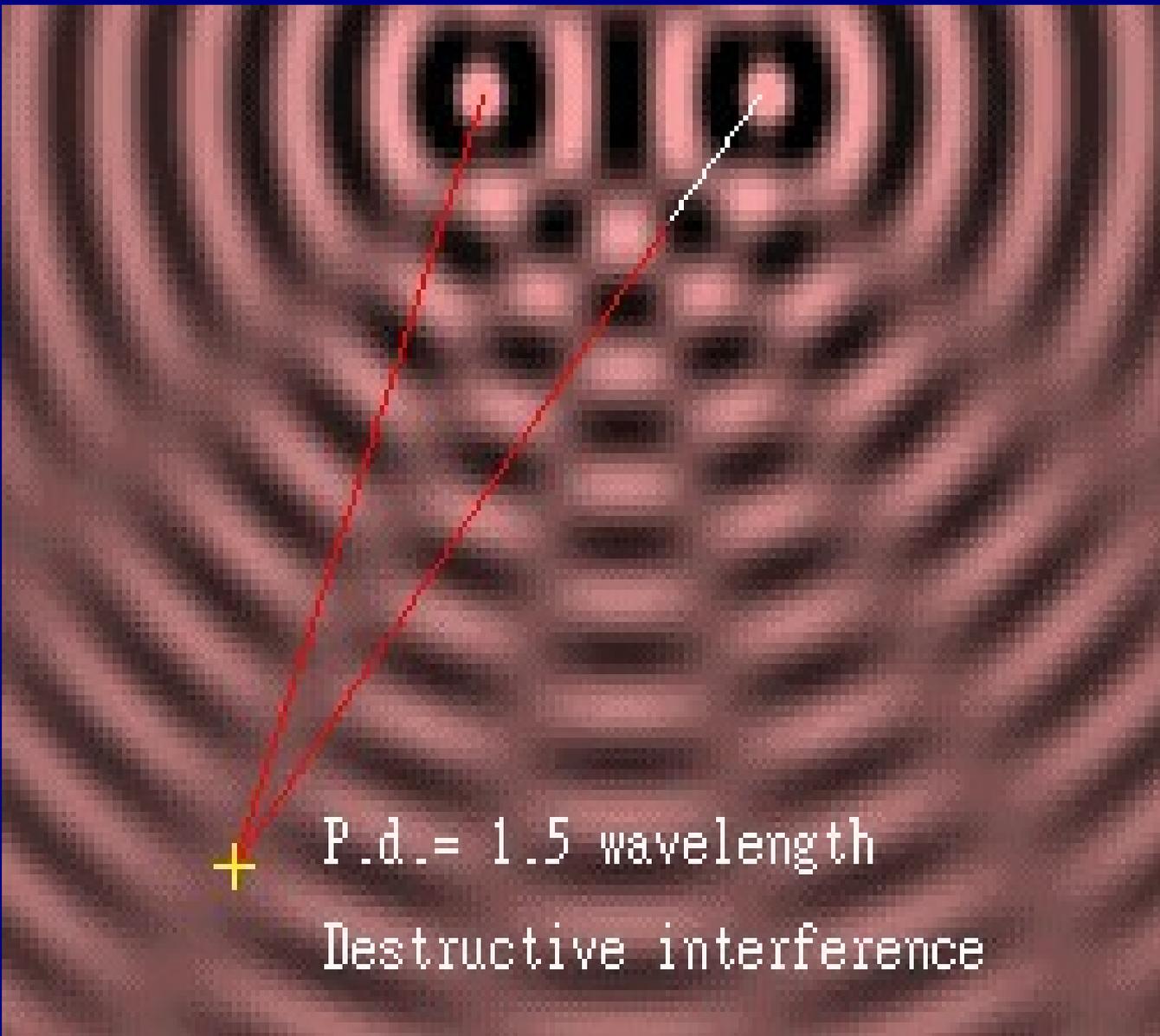
Frequency  
= number of crests  
passing by  
each second

velocity  
= frequency x  
wavelength

## 2. Waves can interfere with each other

amplitudes adding up = constructive interference  
amplitudes subtracting = destructive interference

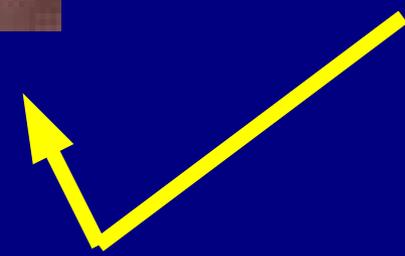




+ P.d. = 1.5 wavelength  
Destructive interference

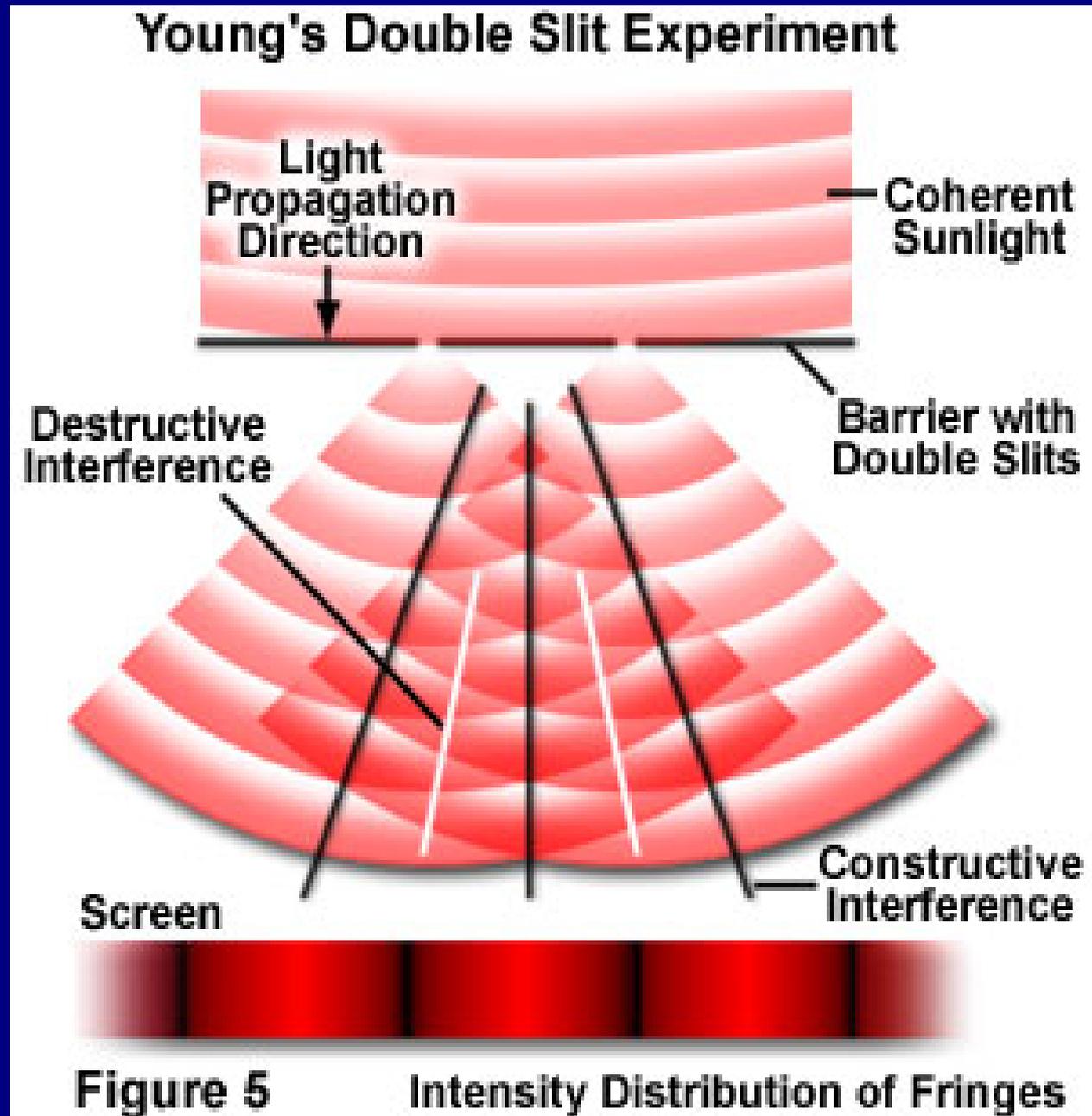


nodal lines of complete destructive interference where crest of one wave cancels out the trough of the second wave



### 3. Light is a wave, because it shows interference

destructive interference  
gives nodal lines  
(dark bands of zero light)  
just like the nodal lines  
in the case of water waves!



## 4. Light is also a particle !

The photoelectric effect shows that light is not a continuous wave, but the light energy comes in 'packets' of size

$$E = h \cdot f$$

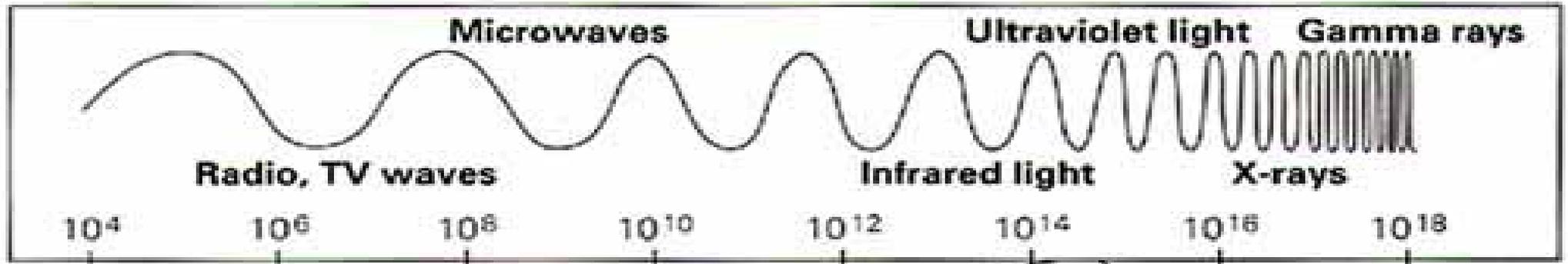
where  $h$  = Planck's constant

$f$  = frequency of the light

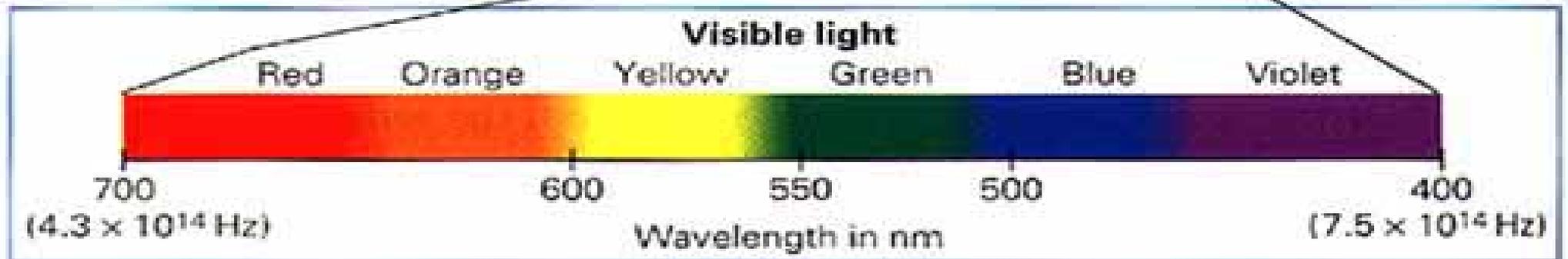
low frequency (red) light = small packets of energy  
high frequency (blue ) = large packets of energy

$$E = 10^{-10} \text{ eV}$$

$$E = 10^7 \text{ eV}$$



Frequency in Hz



$$E = 0.28 \text{ eV}$$

$$E = 0.49 \text{ eV}$$

## 6. Light is both particle and wave

This is a general property of all matter, not just of light. It is possible to see diffraction patterns made by beams of electrons, neutrons, protons, carbon atoms, and even Buckyballs !

This is the fundamental principle of wave-particle duality

de Broglie:

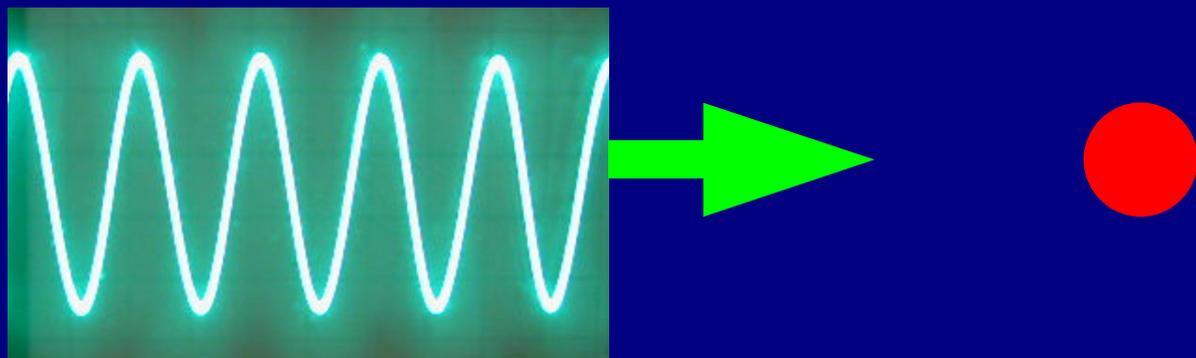
The wavelength is given by  $\lambda = h / p$

where  $h$  = Planck's constant

$p$  = momentum of the particle

## 7. The act of measurement disturbs the system being measured

e.g. measure the position of an electron by observing it with light of wavelength  $\lambda$ , momentum  $p = h/\lambda$



When the photon scatters off the electron, it imparts momentum to the electron, so....

# Heisenberg uncertainty principle

large momentum  $p \rightarrow$  short wavelength  $\lambda$

↓  
large disturbance  
of electron

↓  
precise measurement  
of position  $x$

↓  
uncertainty of electron  
momentum  $\Delta p \approx p$

↓  
uncertainty of electron  
position  $\Delta x \approx \lambda = h/p$

so  $\Delta p \cdot \Delta x \approx h$

More precisely  $\Delta p \cdot \Delta x \geq h/4\pi$

Similarly for energy and time  $\Delta E \cdot \Delta t \geq h/4\pi$

Planck's constant  $h$  sets the scale for the size of quantum effects

$$E = h \cdot f$$

$$\lambda = h / p$$

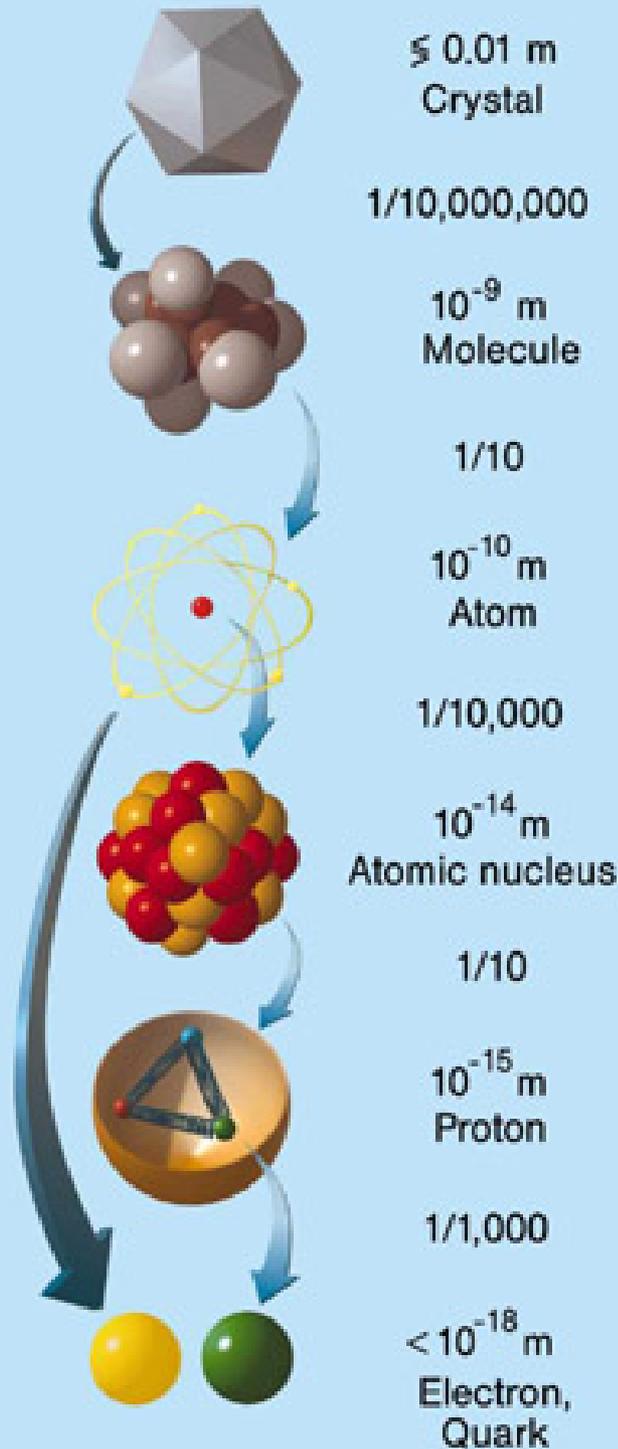
$$\Delta p \cdot \Delta x \geq h/4\pi$$

$$\Delta E \cdot \Delta t \geq h/4\pi$$

$$h = 6.627 \times 10^{-34} \text{ Joule-sec} = 4.13 \times 10^{-15} \text{ eV-sec}$$

which is tiny compared to everyday life where typical energies are many Joules, typical time are seconds

so the quantum effects are not normally apparent in everyday life because they are too small.



The deeper we go down the distance scale from

crystal  
 molecule  
 atom  
 atomic nucleus  
 protons & neutrons  
 quarks

the more apparent these quantum effects are

### Subatomic physics

= nuclear physics  
 (study of the atomic nucleus)

+

particle physics  
 (study of the elementary particles that make up nuclei and atoms)

We shall apply these principles of quantum mechanics to several topics of subatomic physics

- the sizes of atomic nuclei
- the energy scale of nuclear processes
- the existence of quarks

# **1. The sizes of atomic nuclei**

How do we see atoms and subatomic structure?  
Not with this! **WHY NOT?**

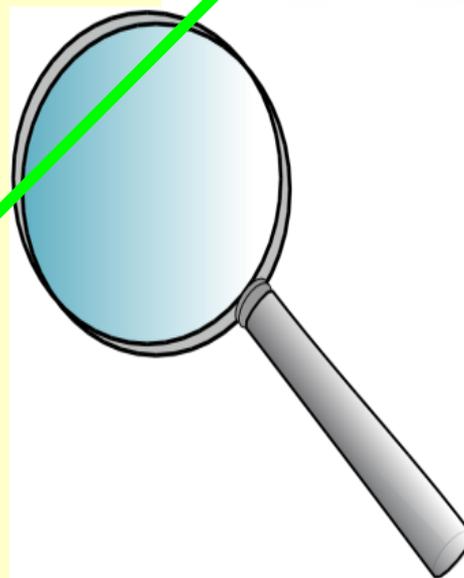
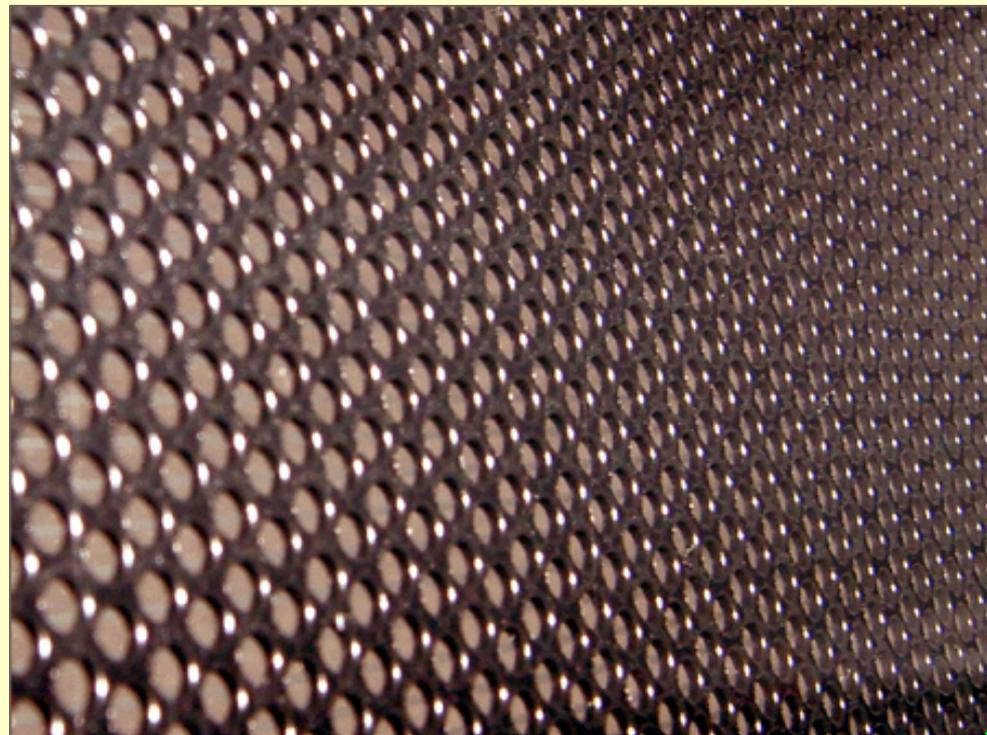


How do we see atoms and subatomic structure?  
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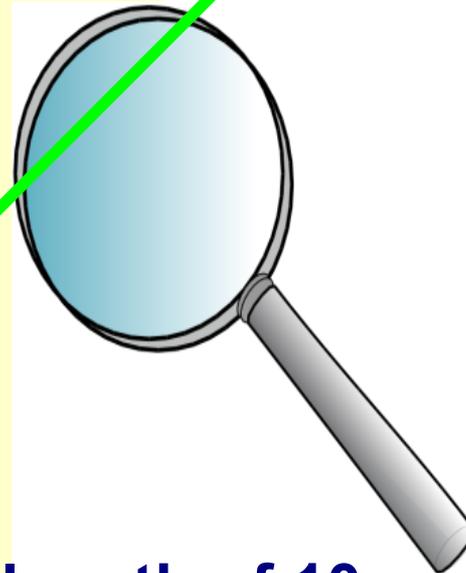
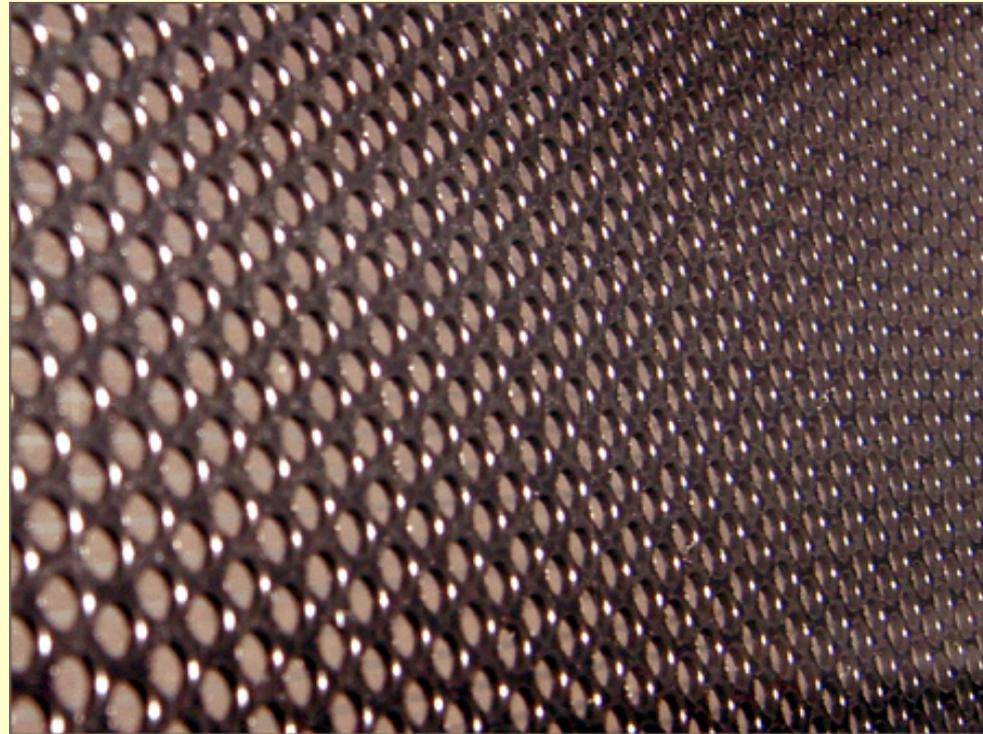


Because light is a wave, it has a “fuzziness” the size of its wavelength, and cannot resolve features smaller than that!

**Why don't microwaves leak through the metal grill on the door?**



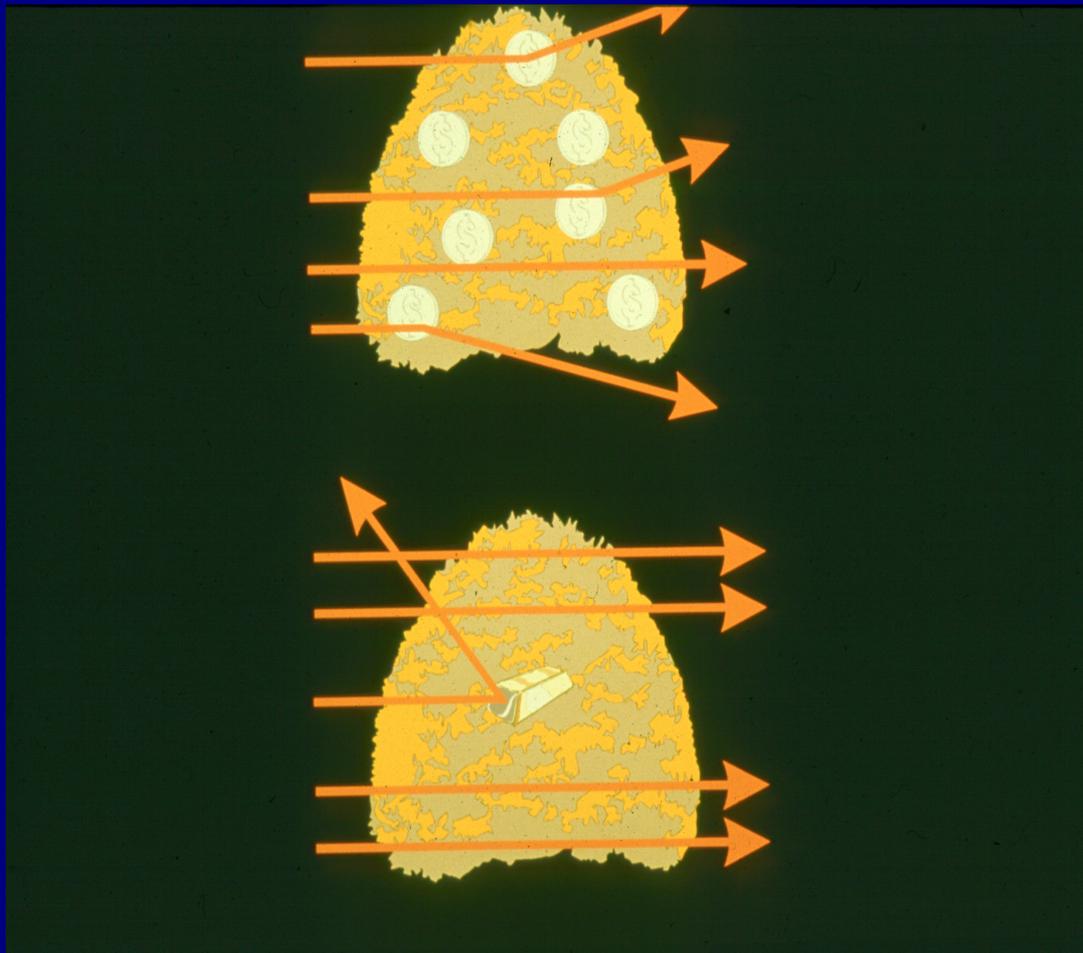
**Why don't microwaves leak through the metal grill on the door?**



**Because the waves have a wavelength of 10 cm, and cannot resolve the 2 mm holes in the grill. As far as they are concerned, the grill is solid metal!**

How do we “see” what's inside an atom?  
By means of scattering experiments.

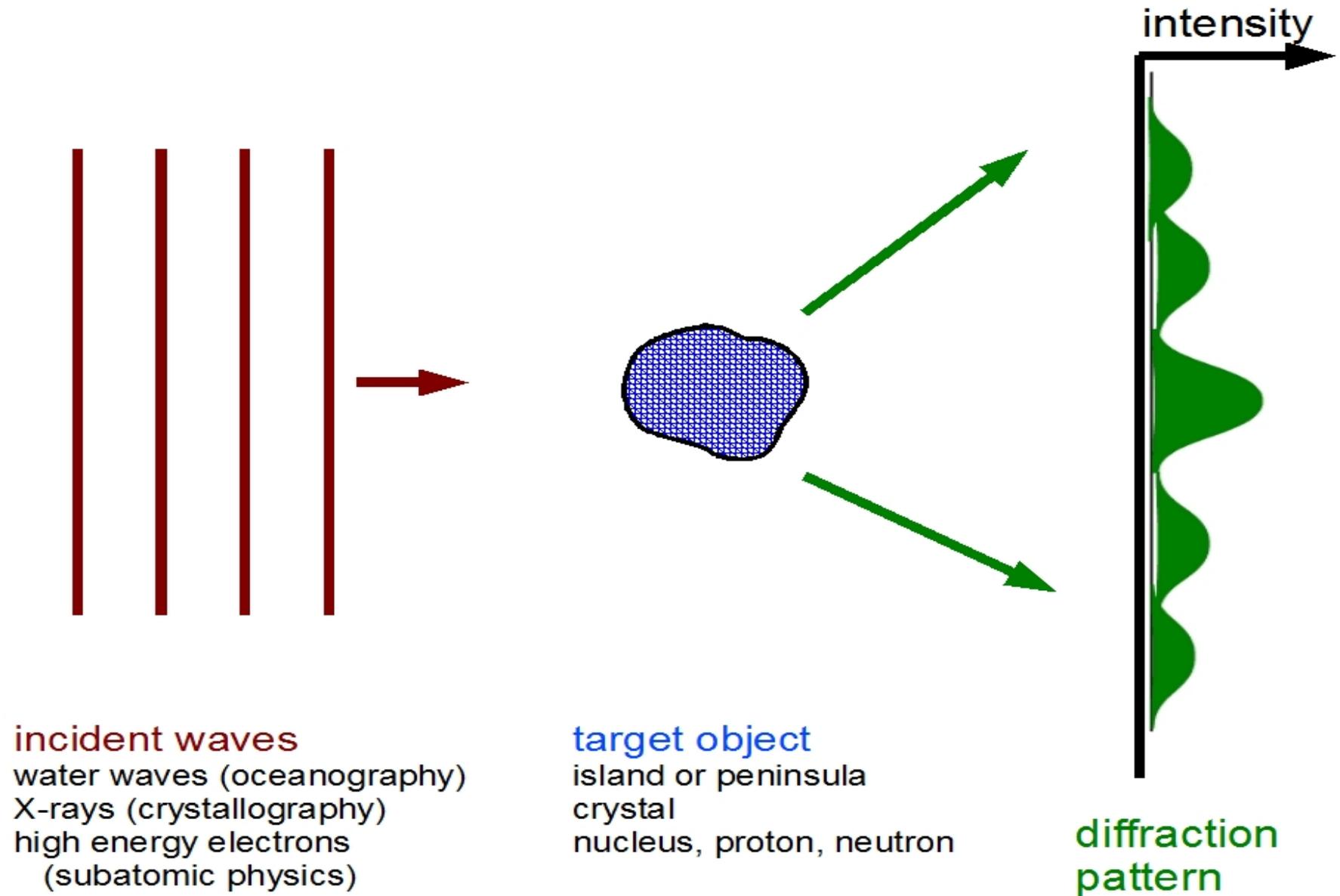
Gold coins in haystack analogy to how Rutherford discovered  
the atomic nucleus



many small  
deflections  
of the projectiles

a few large  
deflections --  
this is what the  
atom looks like!

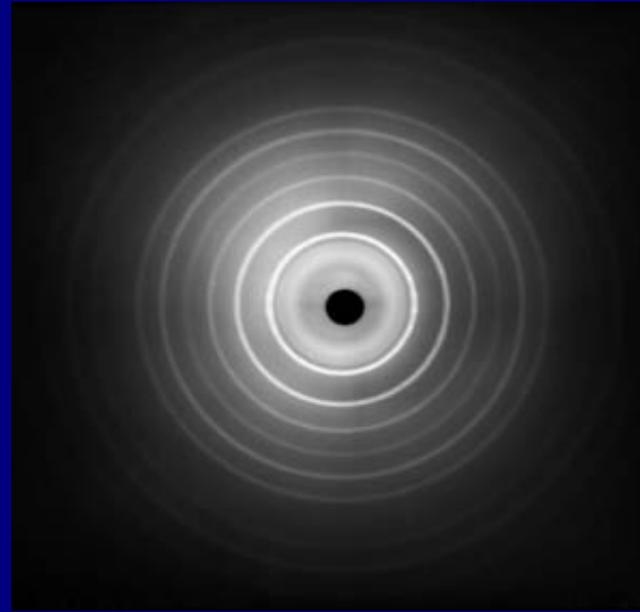
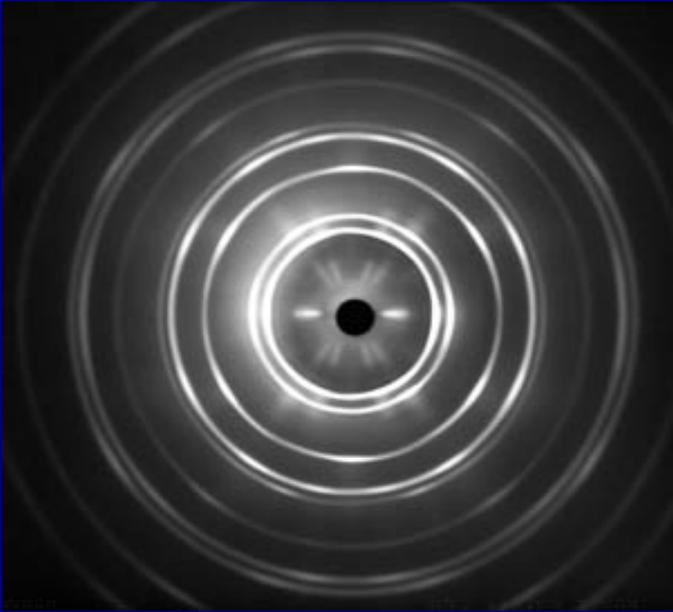
Bullets through haystack picture not accurate!  
Actually, cannot neglect the wavelike nature of the projectiles!  
In wave picture, a scattering experiment looks like this:



# Diffraction of water waves



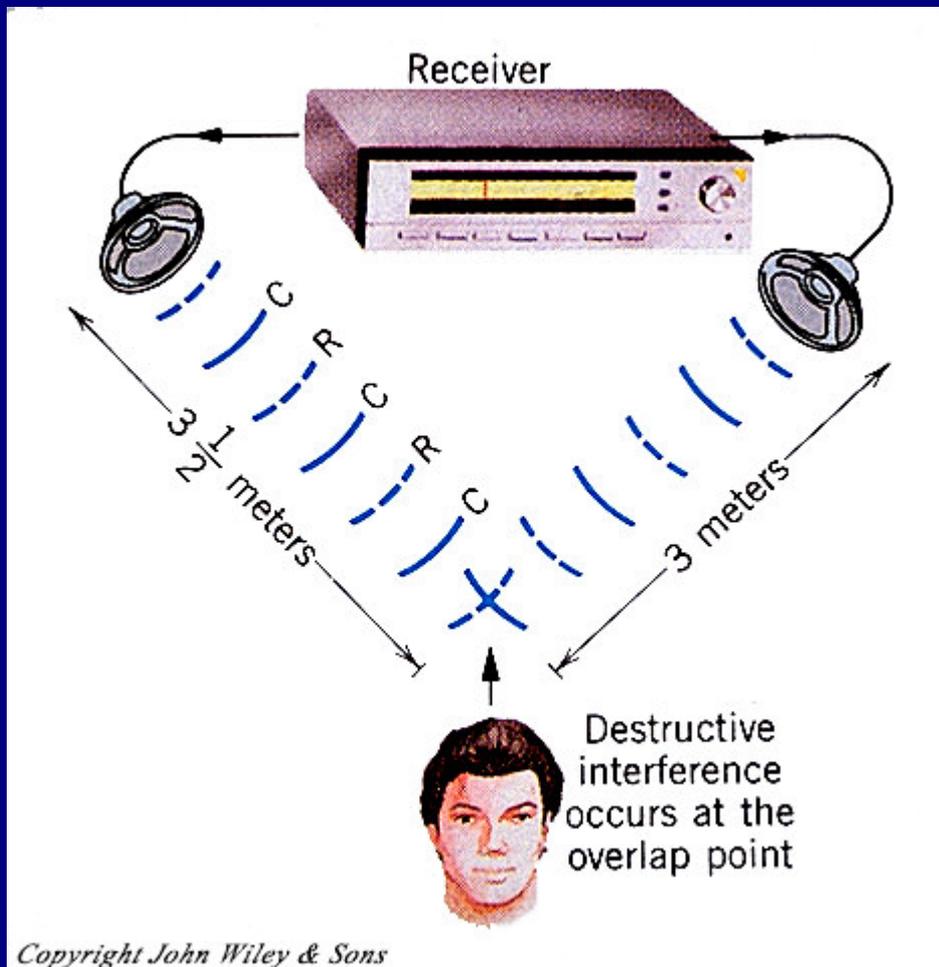
# Diffraction of X-rays from powders of Al and NaCl (powder = many tiny crystals in random orientations)



from C.C. Jones, [minerva.union.edu/jonesc/Scientific.html](http://minerva.union.edu/jonesc/Scientific.html)

Note that the intensity pattern for the two types of crystals are different. By mathematically working backwards from the diffraction pattern, crystallographers can deduce the location of the atoms in the crystal.

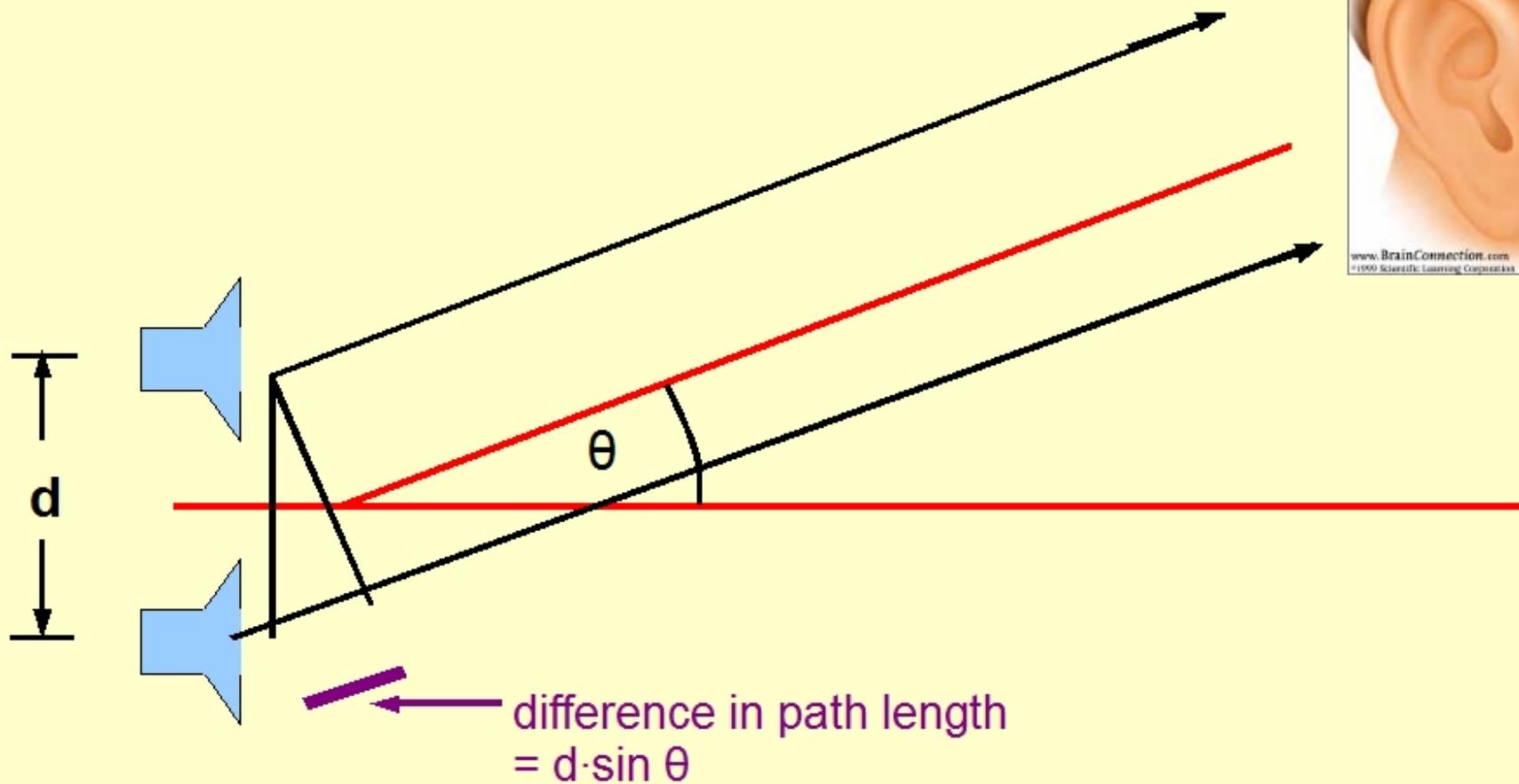
To get an idea of how the diffraction pattern can tell us the size and shape of the nucleus, consider the following demo:



If the distance from one speaker differs from the distance from the other speaker by exactly 1 wavelength or 2 wavelength, etc.

then the two crests arrive at the same time and there is constructive interference.

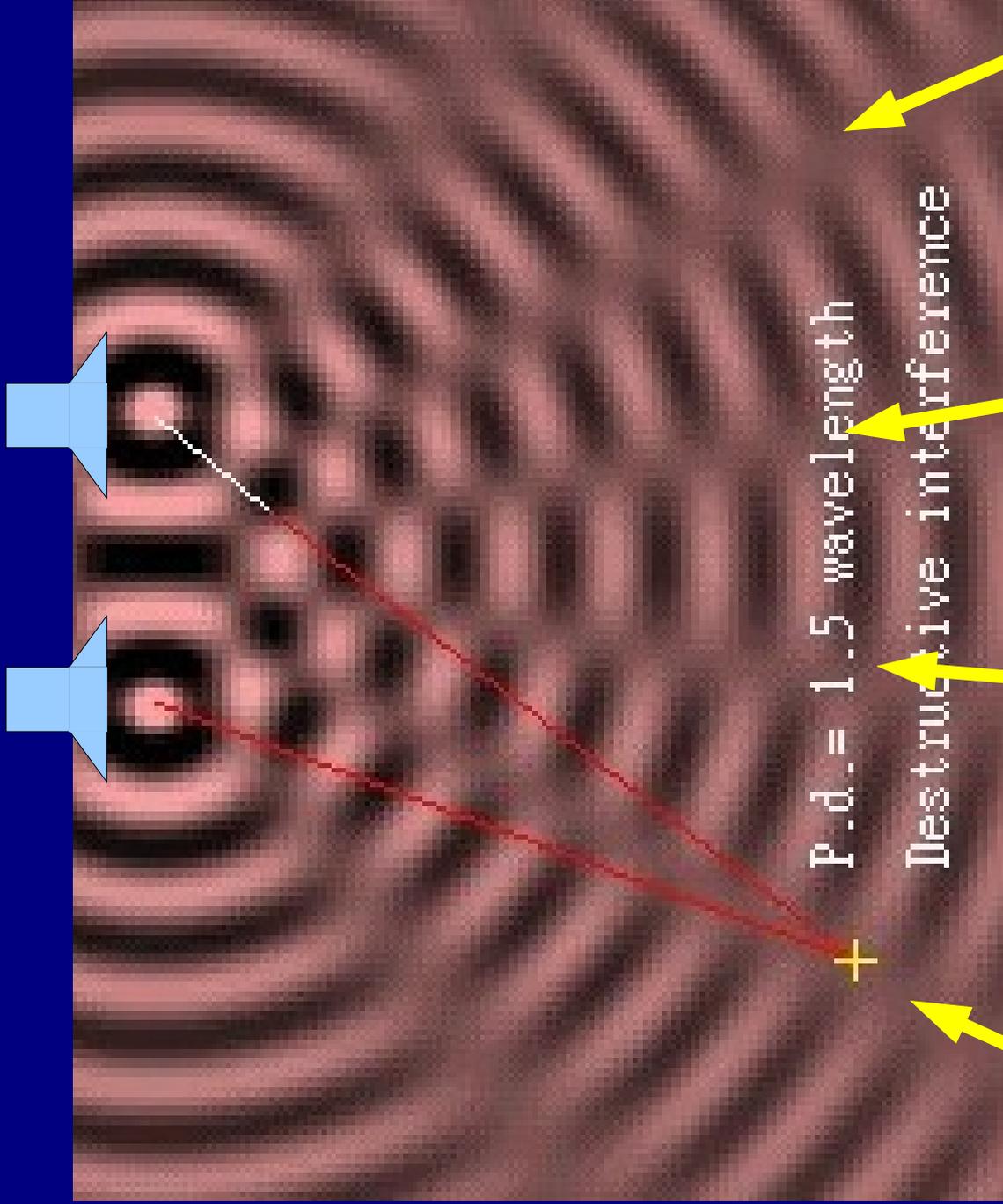
If the distances differ by a half-integer from these values, then there will be destructive interference.



For constructive interference  $d \cdot \sin \theta = n\lambda$  or  $\sin \theta = n\lambda / d$

If  $d$  is small, separation between the adjacent bands of constructive interference is large.

**Smaller separation between sources makes a wider diffraction pattern.**

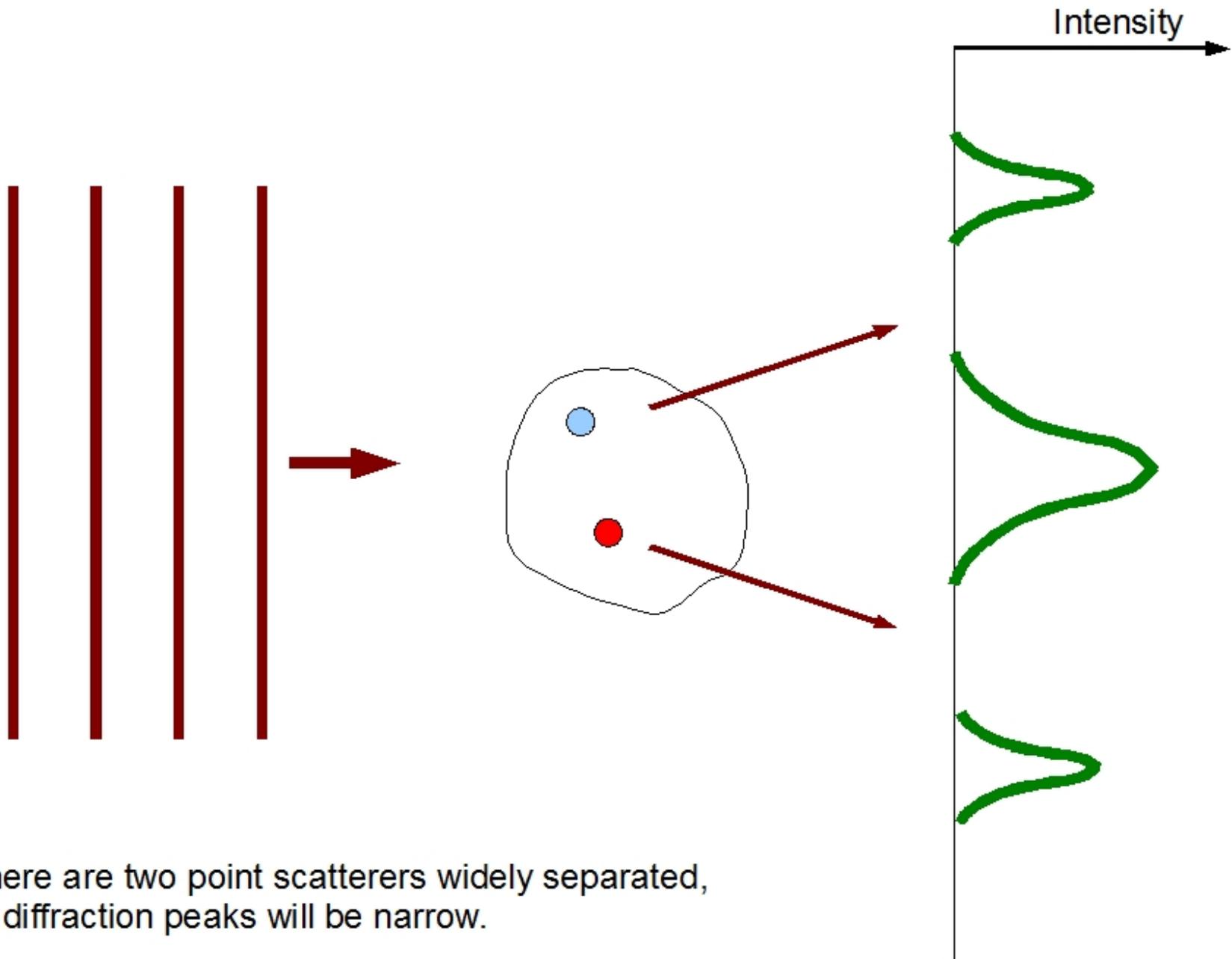


P.d. = 1.5 wavelength

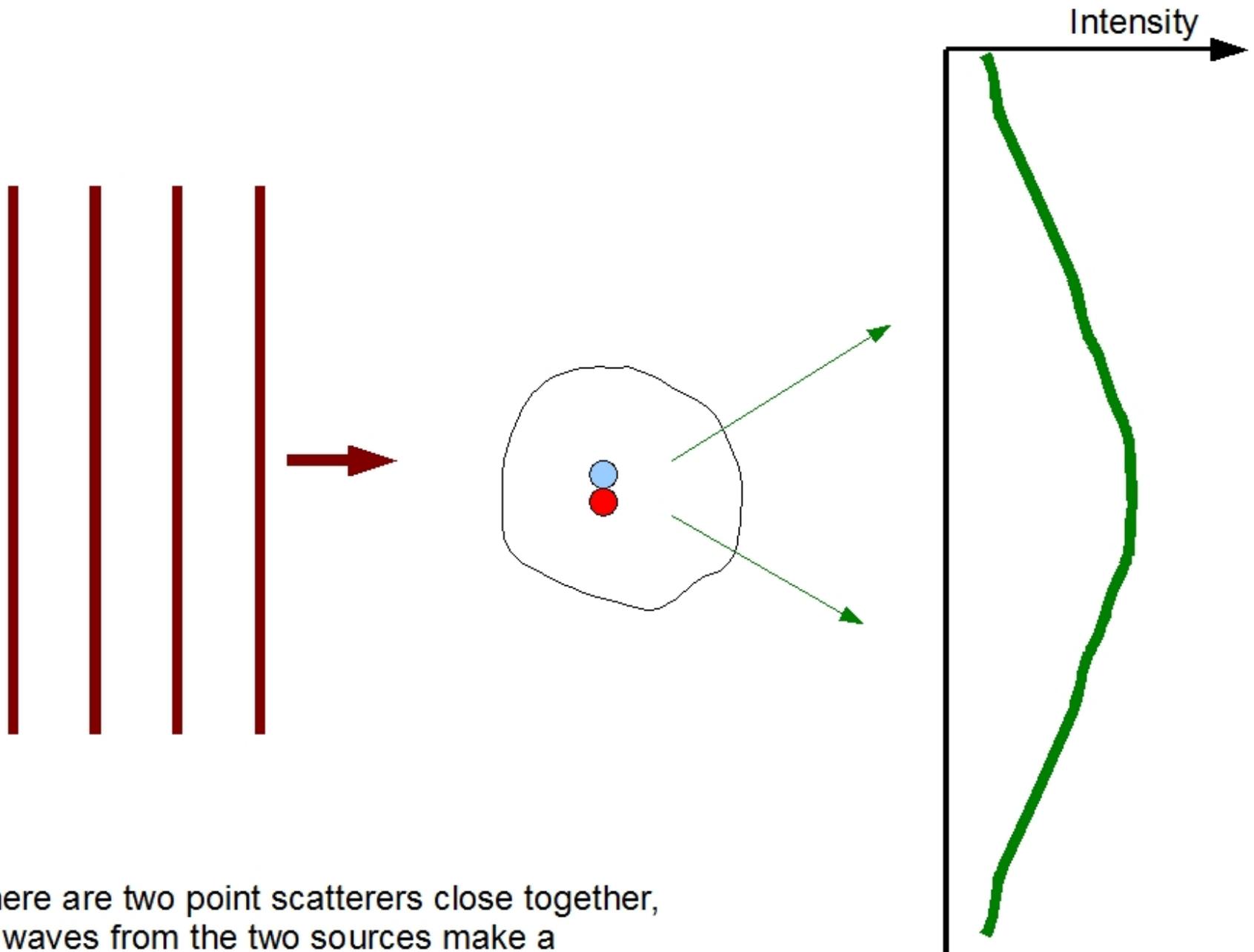
Destructive interference

nodal  
lines  
of  
complete  
silence !

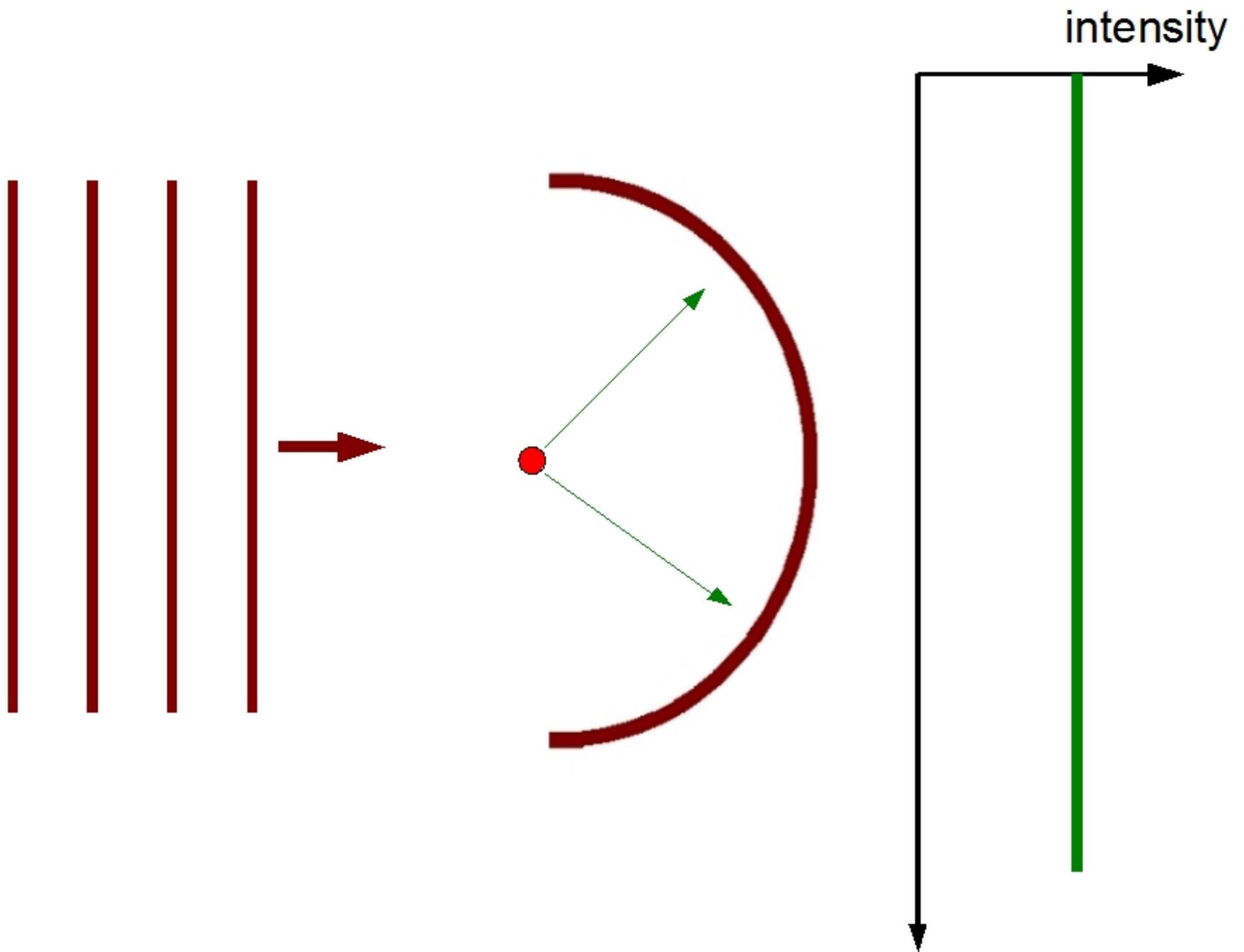
Let's try this demo with two loudspeakers!



If there are two point scatterers widely separated, the diffraction peaks will be narrow.

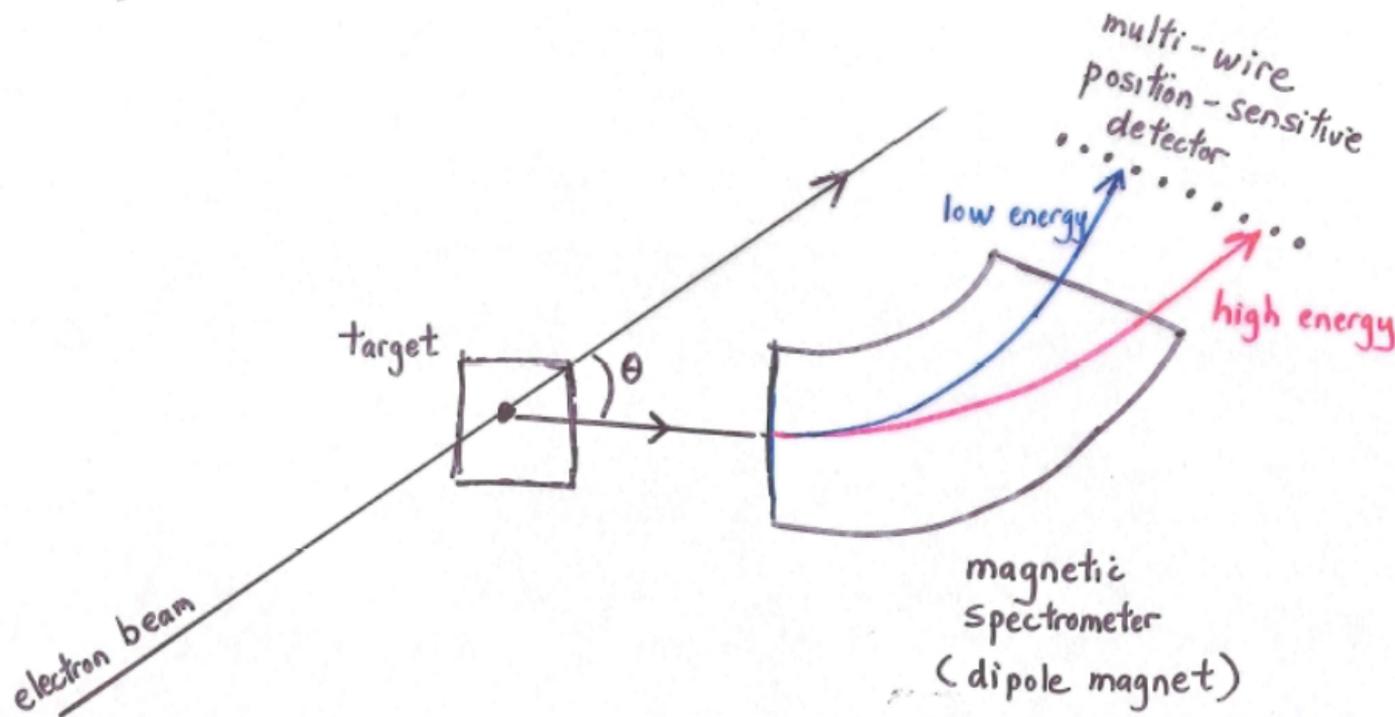


If there are two point scatterers close together, the waves from the two sources make a broad diffraction pattern



single point-like scattering centre  
results in uniform scattering intensity

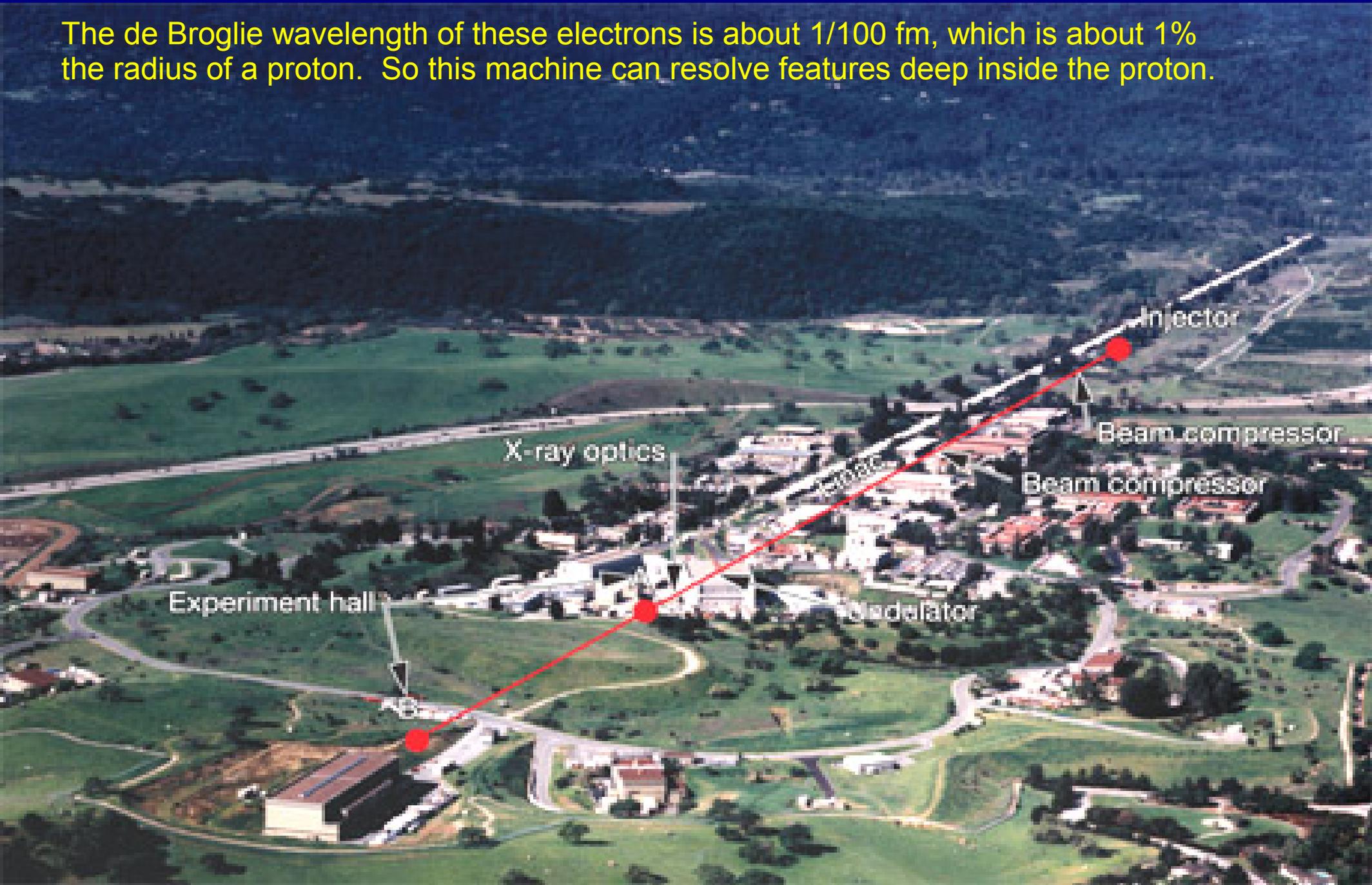
Now do the same thing with electron waves on a nuclear target.



Measure the intensity of the scattered electrons at many values of the angle  $\theta$  to map out the diffraction pattern. The width of the diffraction pattern will tell us the size and shape of the nucleus!

Stanford Linear Accelerator Centre – a 2-mile long electron accelerator, which boosts electrons to an energy of 20 billion electron volts (later upgraded to 60).

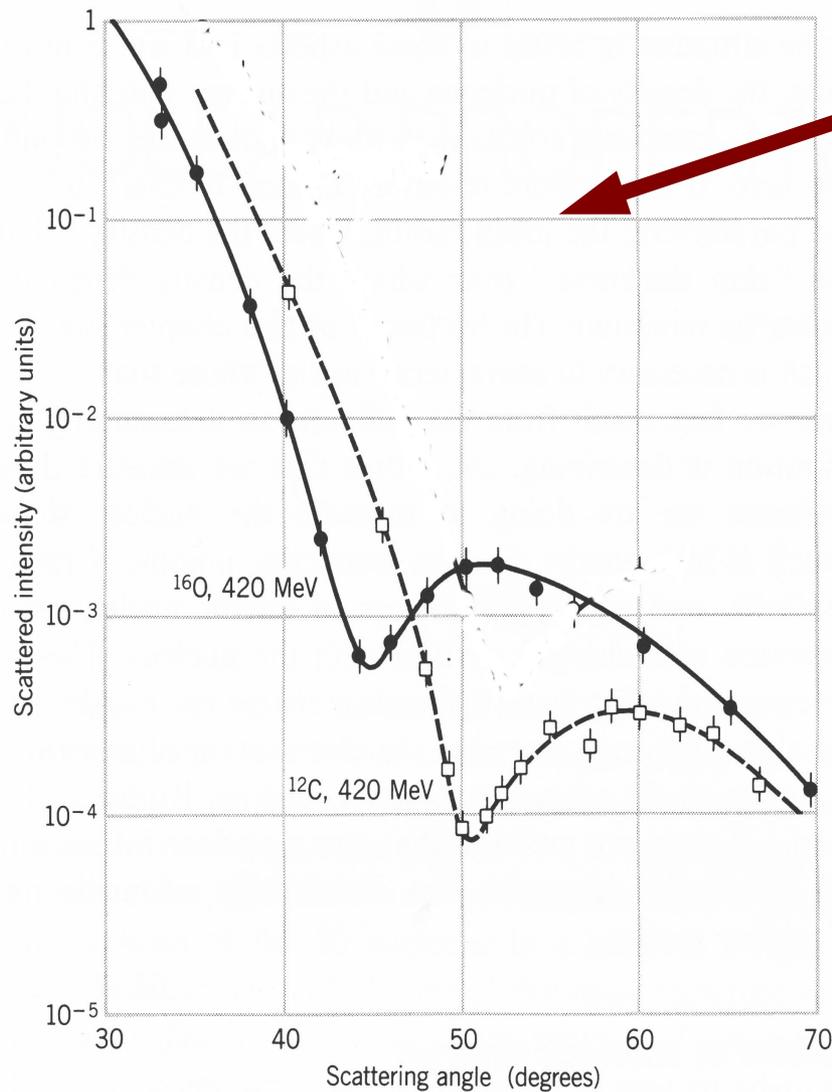
The de Broglie wavelength of these electrons is about  $1/100$  fm, which is about 1% the radius of a proton. So this machine can resolve features deep inside the proton.



A view of the gigantic magnetic spectrometer used to analyze the energy of the scattered electrons. Note the two people near the bottom for scale.



from Krane, Introductory Nuclear Physics



diffraction pattern of electron waves scattering from nuclei of carbon-12 and oxygen-16.

Note that carbon-12 is smaller, so it produces a wider diffraction pattern.

**Figure 3.1** Electron scattering from  $^{16}\text{O}$  and  $^{12}\text{C}$ . The shape of the cross section is somewhat similar to that of diffraction patterns obtained with light waves. The data come from early experiments at the Stanford Linear Accelerator Center (H. F. Ehrenberg et al., *Phys. Rev.* **113**, 666 (1959)).

From the diffraction pattern, we can mathematically work backwards using an inverse Fourier transform to get the density profile of the electric charge in the atomic nucleus:

Notice how small the nucleus is:

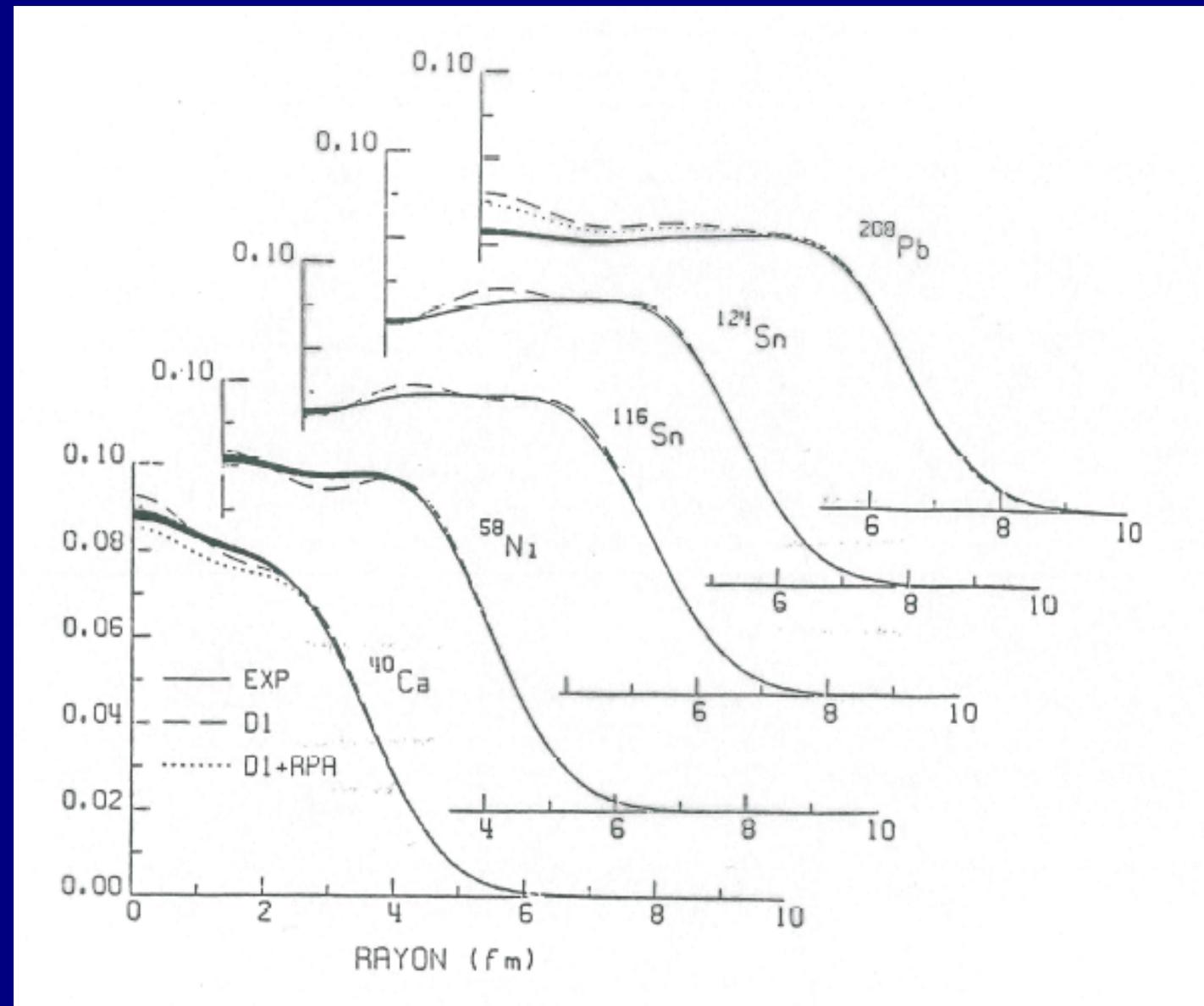
A Calcium nucleus has a radius of about 4 femto-metres) i.e.

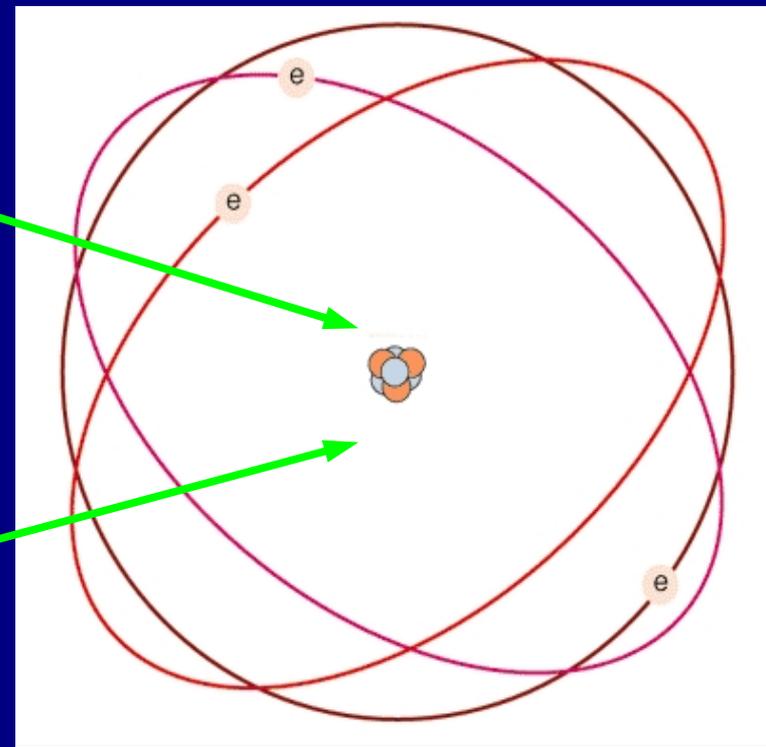
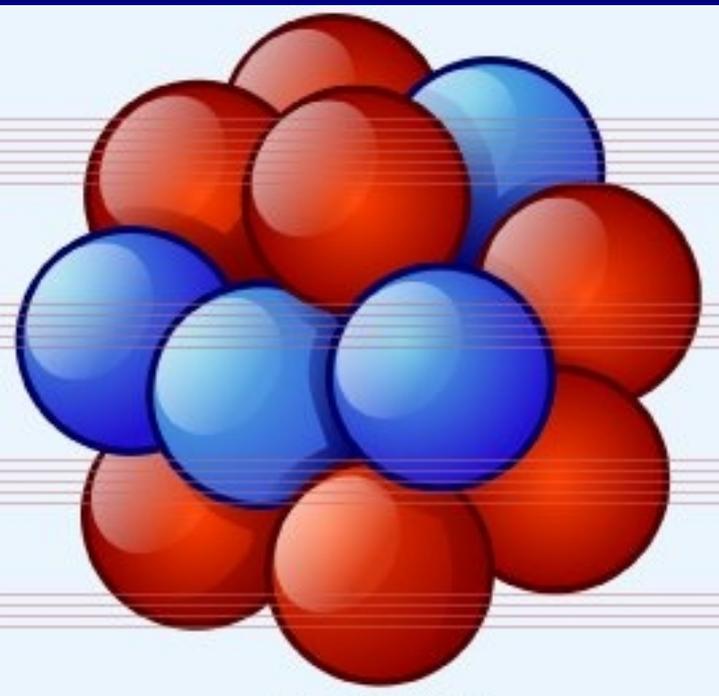
$$4 \times 10^{-15} \text{ metre}$$

while the radius of a calcium atom has a radius of

180,000 femtometres

i.e. the nucleus is 45,000 times smaller in diameter !





Nucleus in an atom is like a pea in a football stadium.



**Even though the atomic nucleus is so tiny compared to the entire atom, it contains 99.97% of the mass of the atom.**

**The nuclear matter is extremely dense – a teaspoon full would have a mass of 460 million metric tons!**

**Where in the universe can we find bulk quantities of nuclear-density matter?**

Even though the atomic nucleus is so tiny compared to the entire atom, it contains 99.97% of the mass of the atom.

The nuclear matter is extremely dense – a teaspoon full would have a mass of 460 million metric tons!

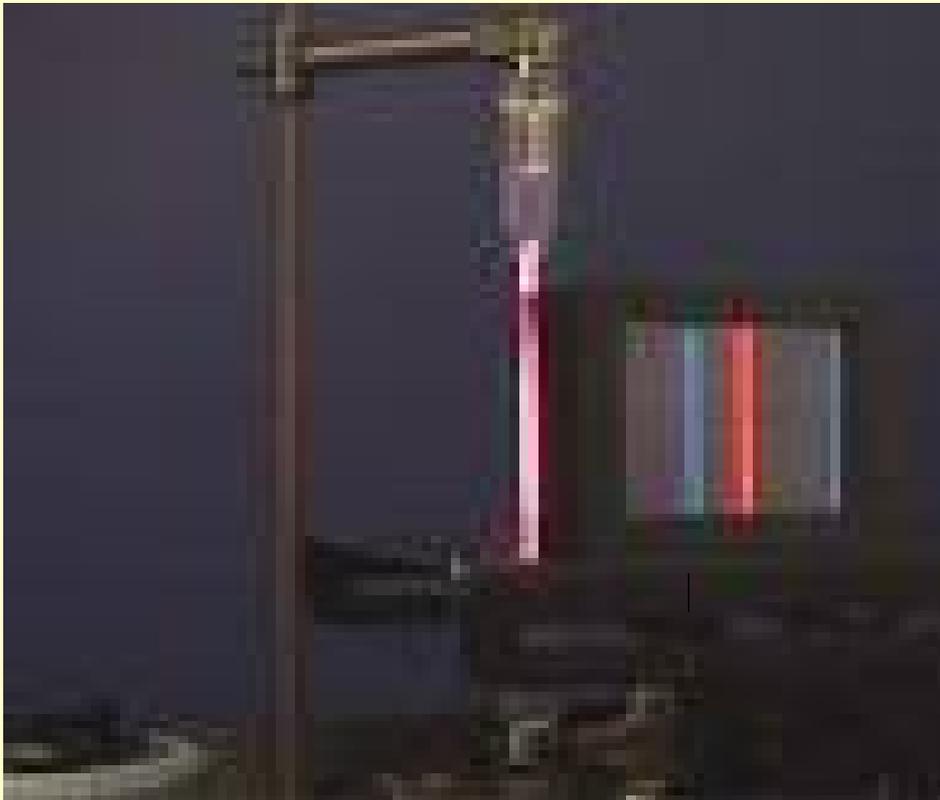
Where in the universe can we find bulk quantities of nuclear-density matter?

**Answer: In neutron stars.**

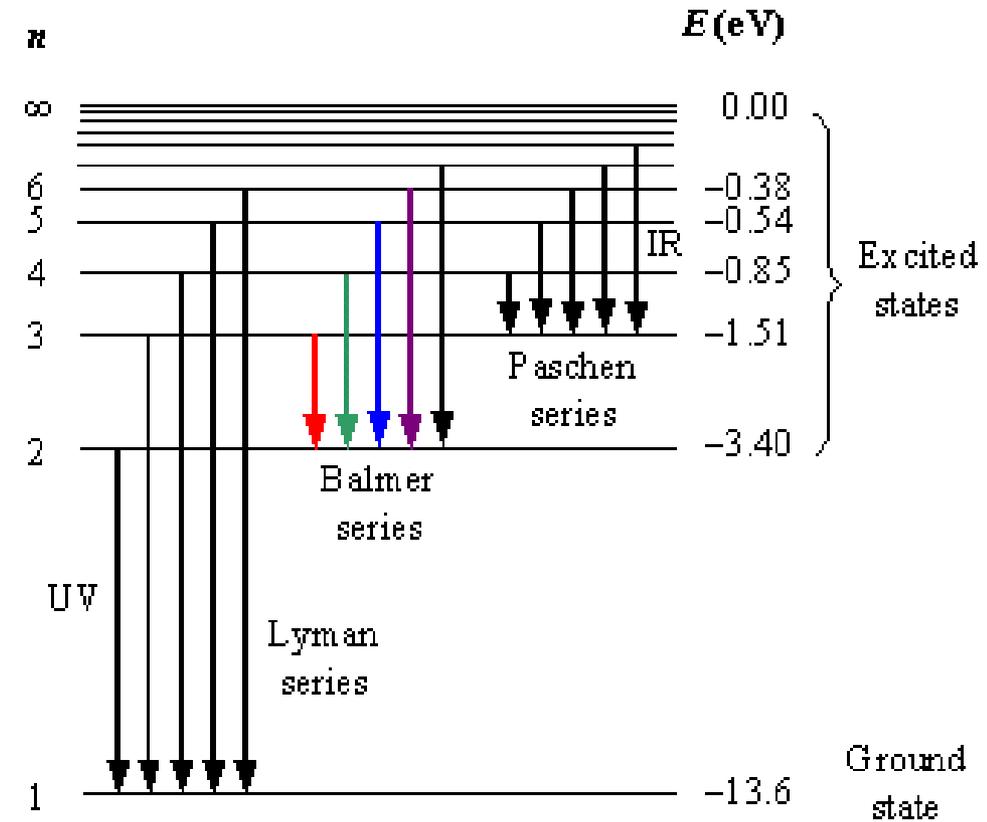
**A neutron star is formed at the end of the life of a massive star. The pull of gravity is so strong that the protons in the atomic nuclei absorb the orbital electrons, forming neutrons. This allows the whole system to become more compact, thereby lowering the gravitational potential energy. The neutron star is basically a giant nucleus, about 25 km in diameter, but having a mass of between 1.4 and 2.1 solar masses, composed almost exclusively of neutrons.**

## **2. The nuclear energy scale**

Atoms in excited states can de-excite and give off light, of energies typically around 1 electron-volt.



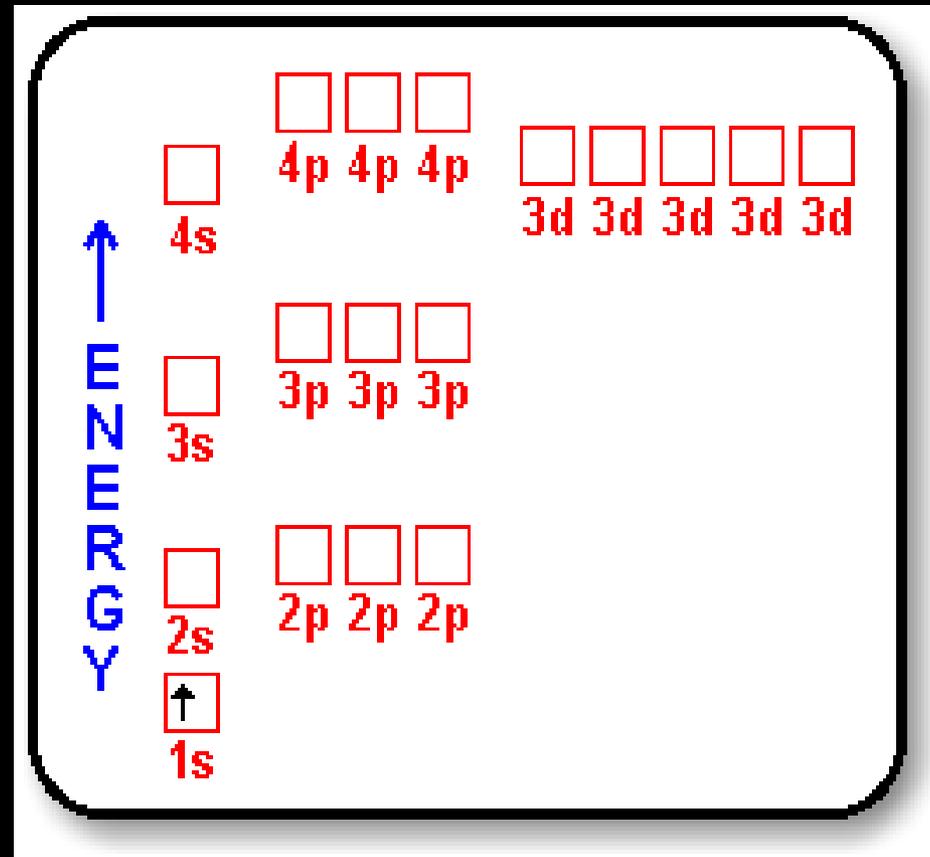
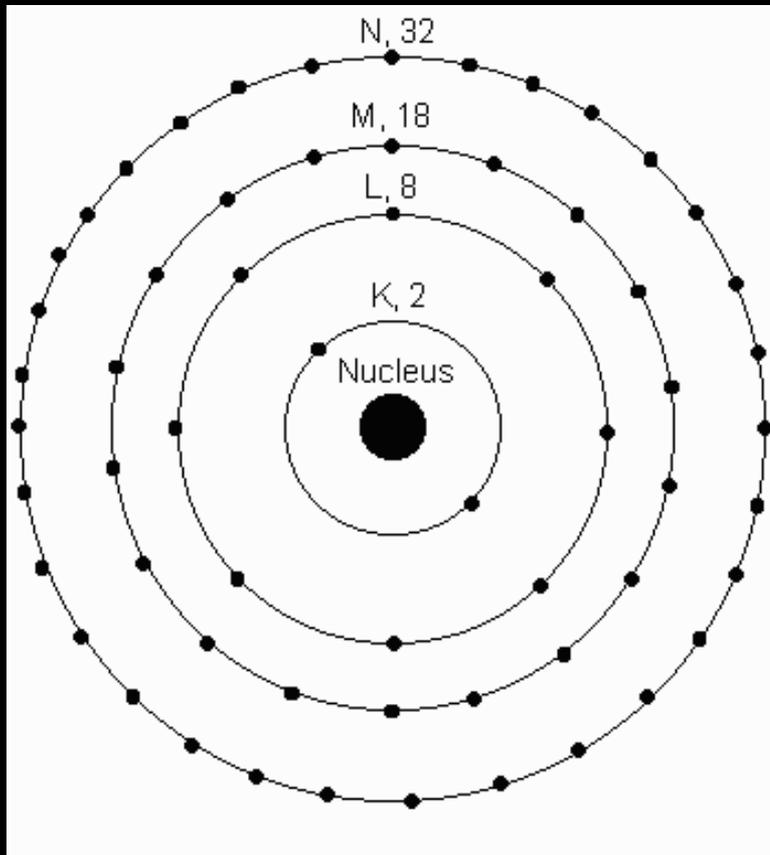
<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/grating.html>



Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

© Internet Encyclopedia of Science  
[www.daviddarling.info/images/](http://www.daviddarling.info/images/)

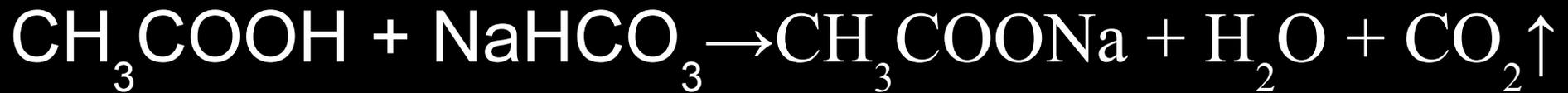
The electrons in atoms (and molecules) are arranged in shells. **Excited** atoms and molecules are made by boosting the electrons to higher shells.



Similarly, the protons and neutrons in atomic nuclei are arranged in shells, and excited nuclei are made by boosting the protons and neutrons to higher shells.

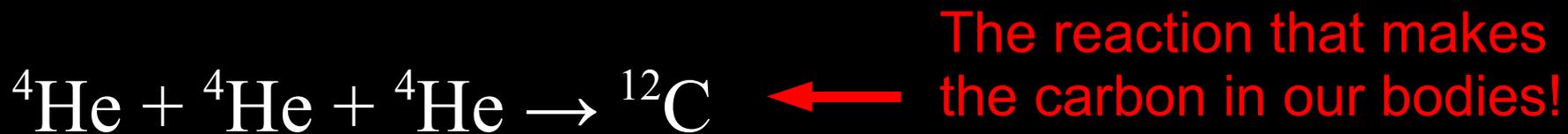
demo of atomic spectra  
and  
nuclear gamma rays

Chemical Reactions are re-arrangements of the atoms to make different molecules



**energy yield  $\Delta H = -393$  kJoule/mole**

Nuclear Reactions are re-arrangements of protons/ neutrons to make different nuclei—new elements are synthesized!



**energy yield  $\Delta H = -2.5$  billion kJoule/mole**

**Why are nuclear energies so high?**

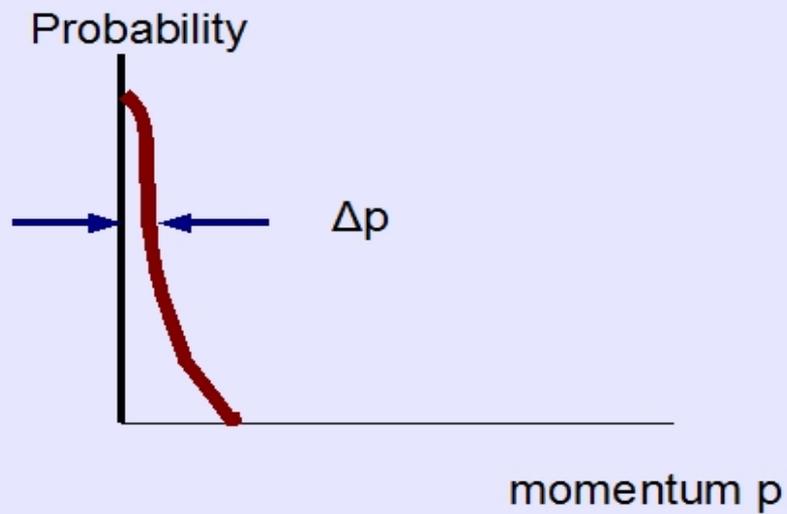
**Because the particles are packed into a very small volume**

**Using the Heisenberg uncertainty principle**

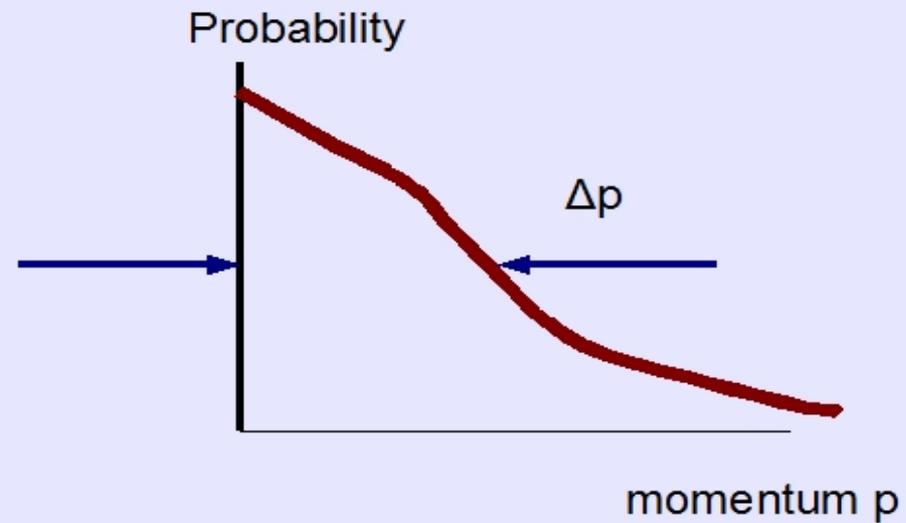
$$\Delta p \cdot \Delta x \geq h/4\pi$$

**we can calculate the extent  $\Delta p$  that the momentum is smeared out.**

**Since the nucleus is 45,000 smaller, the momentum of the protons and neutrons is smeared out 45,000 times more than the electrons in an atom. Roughly speaking, the protons and neutrons have 45,000 more momentum**



**Atom:**  
Large confinement space  $\Delta x$   
Small momentum range  $\Delta p$



**Nucleus:**  
Small confinement space  $\Delta x$   
Large momentum range  $\Delta p$

**Recall that kinetic energy  $KE = \frac{1}{2} mv^2 = p^2 / 2m$**

**and that protons are 1836 times more massive than electrons. So we get**

$$p_{\text{nuclear}} / p_{\text{atomic}} = 45,000$$

$$KE_{\text{nuclear}} / KE_{\text{atomic}} = (45,000)^2 / 1836 = 1.1 \text{ million}$$

**The kinetic energy of protons in the nucleus is about 1 million times larger than the kinetic energy of electrons in an atom, just by the Heisenberg uncertainty principle, and in good agreement with experimental data.**

**The high energy of nuclear processes is an inevitable consequence of the small size of the nucleus + quantum mechanics.**

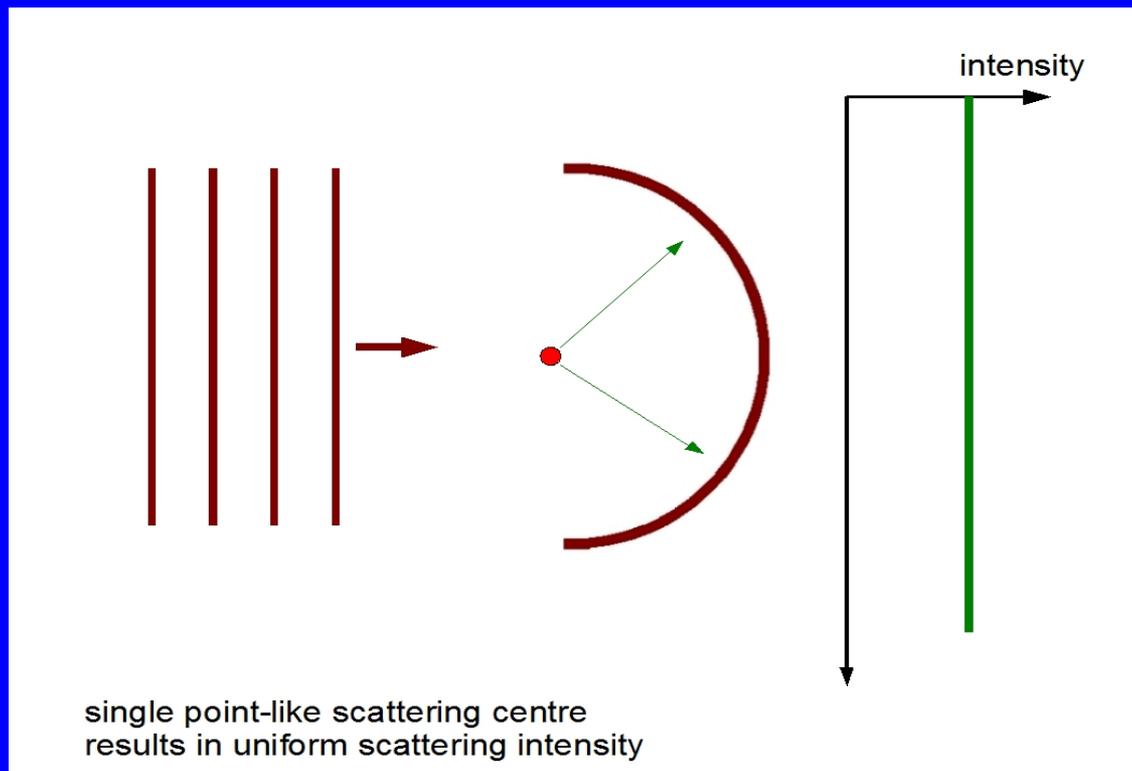
**This is not so in classical mechanics. The kinetic energy of the tiny gears in a watch is not much higher than the kinetic energy of the flywheel in a turbine of a hydroelectric generating station.**

### **3. The discovery of quarks**

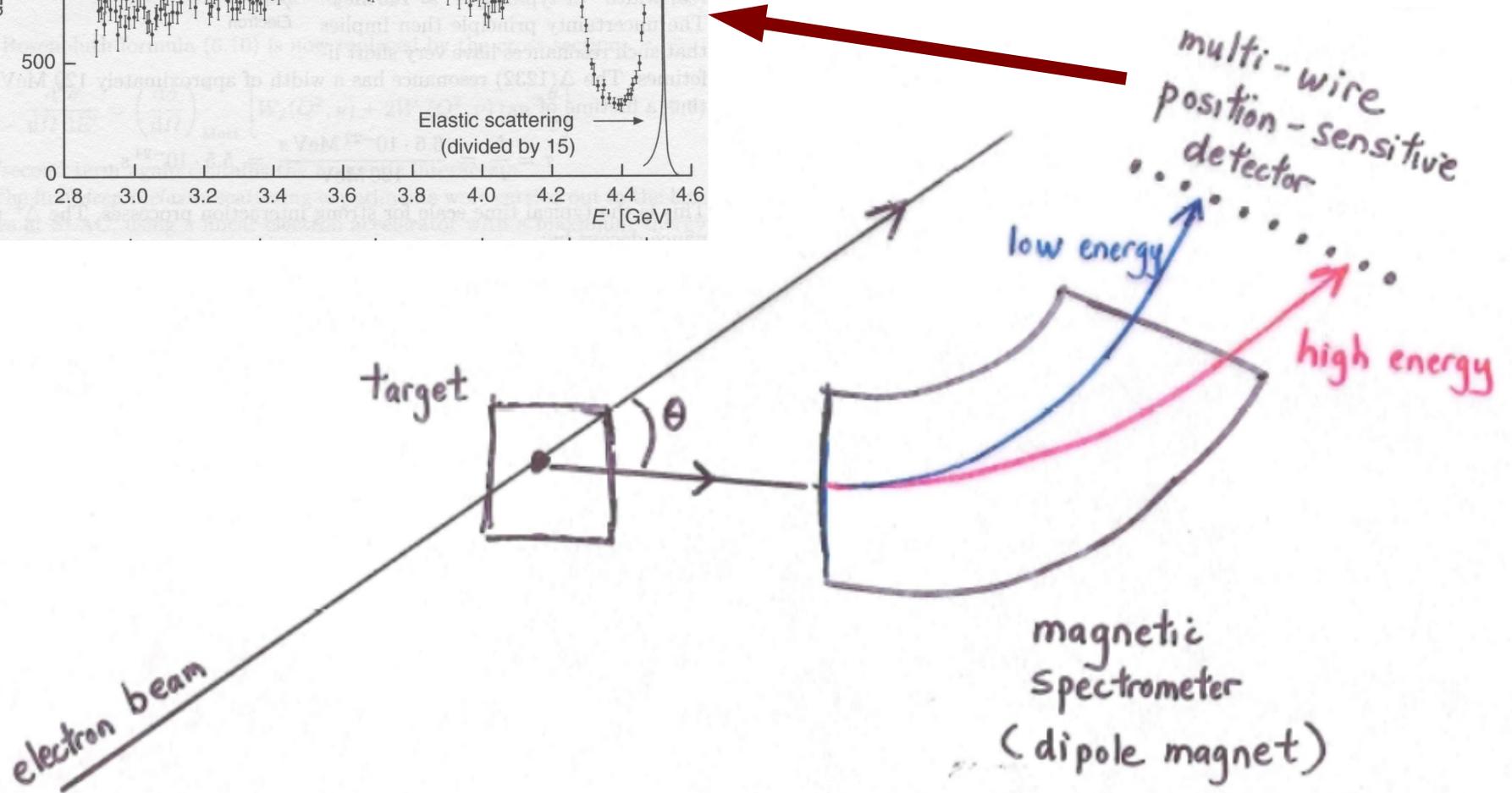
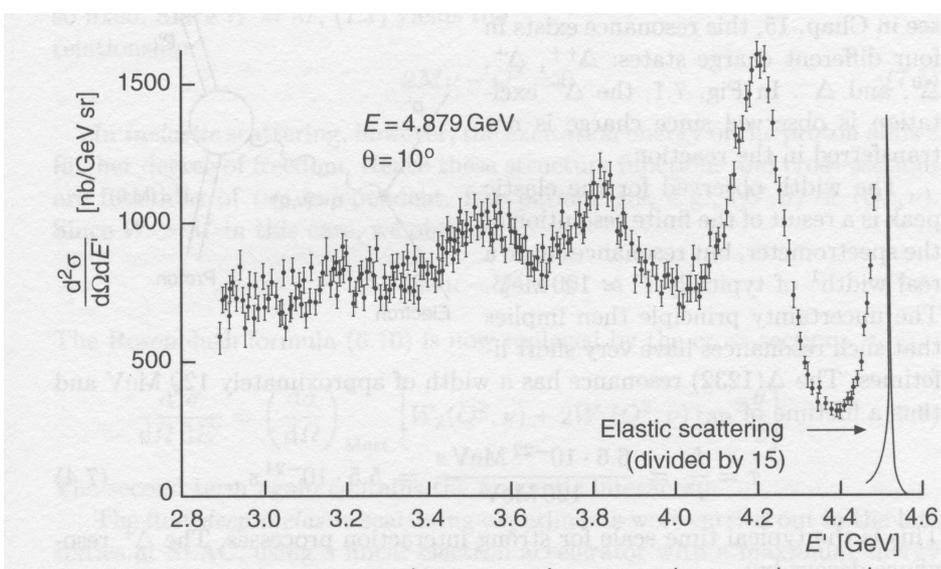
By the 1940's it was known that atomic nuclei were made of protons and neutrons. But do protons and neutrons have any size? Do they have smaller constituents inside of them?

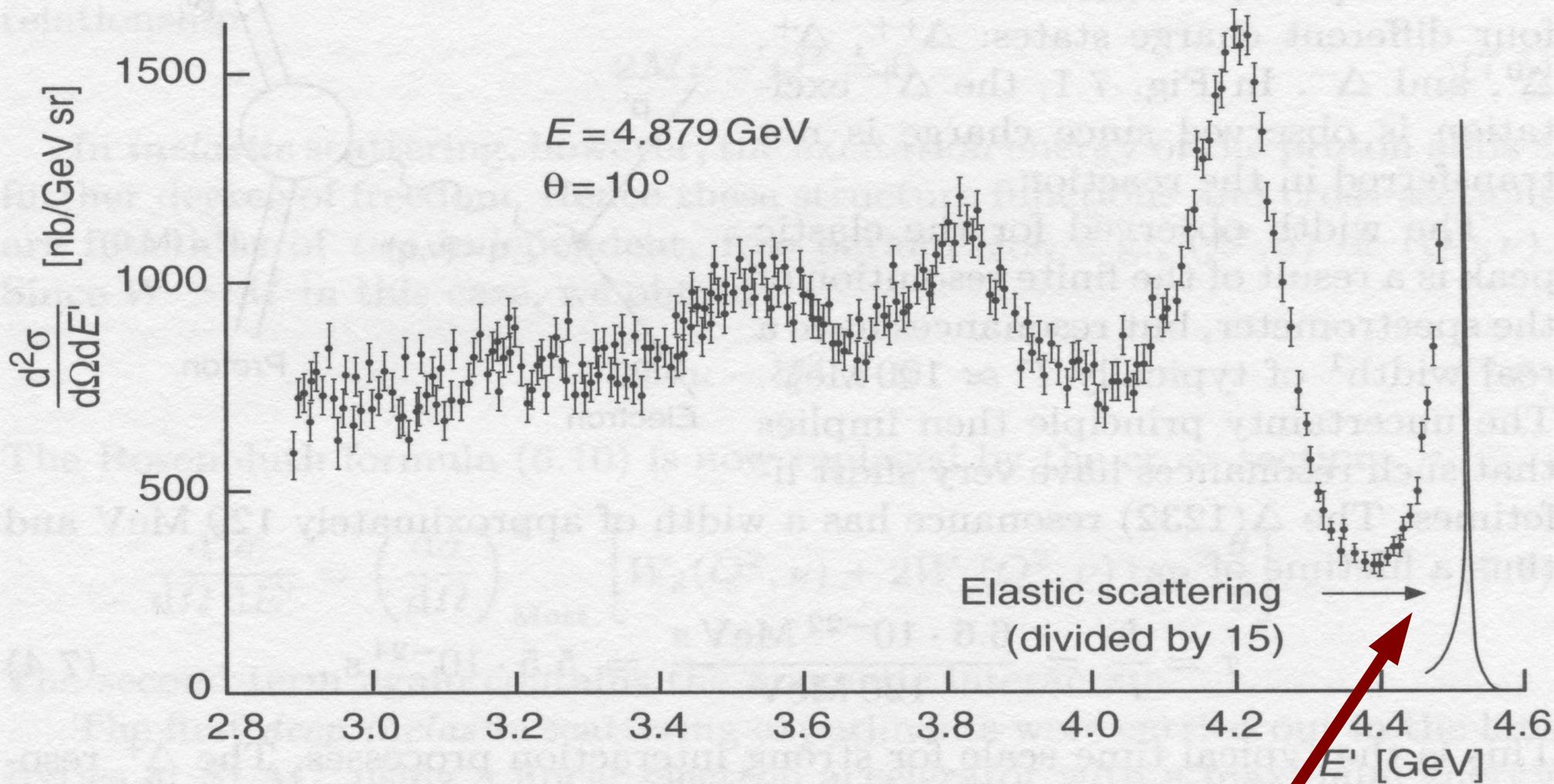
Put a liquid hydrogen target into the electron beam and see what the diffraction pattern looks like!

Recall for a point-like proton with no size, we expect this:



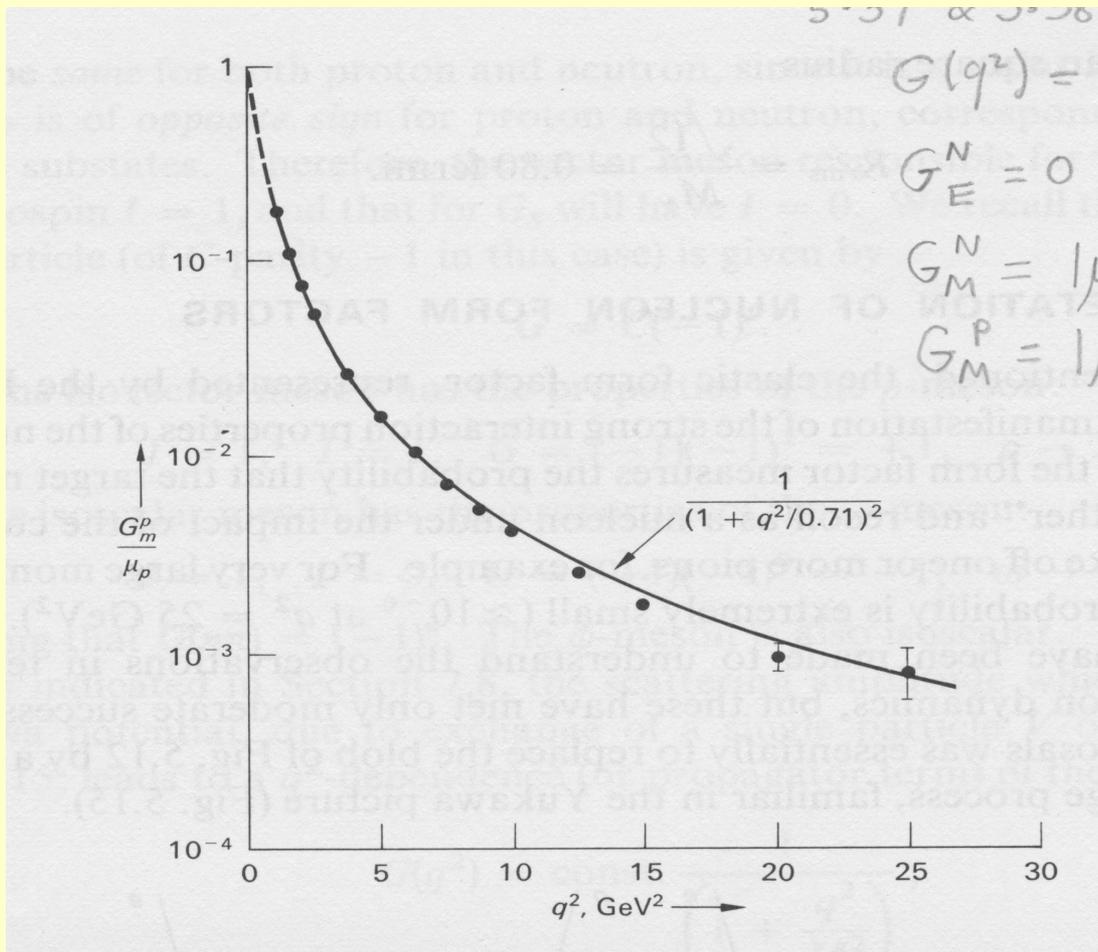
each of the peaks corresponds to different scattering processes

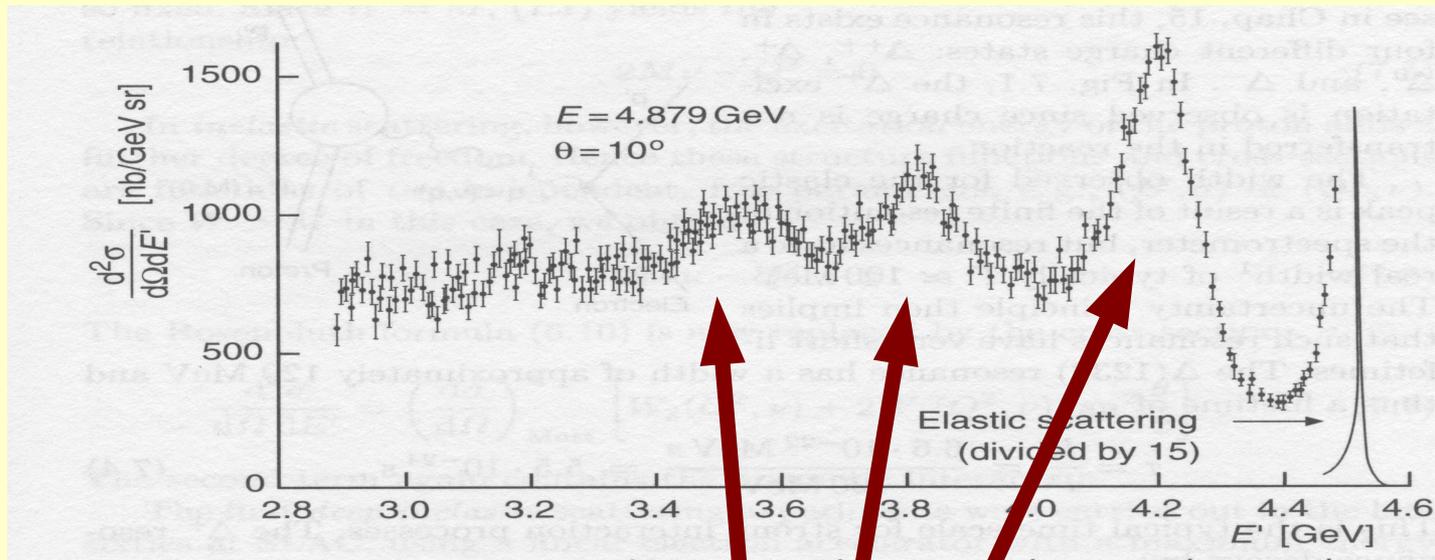




highest energy scattered electrons correspond to elastic scattering, where the electron just bounces off the proton and leaves the proton intact

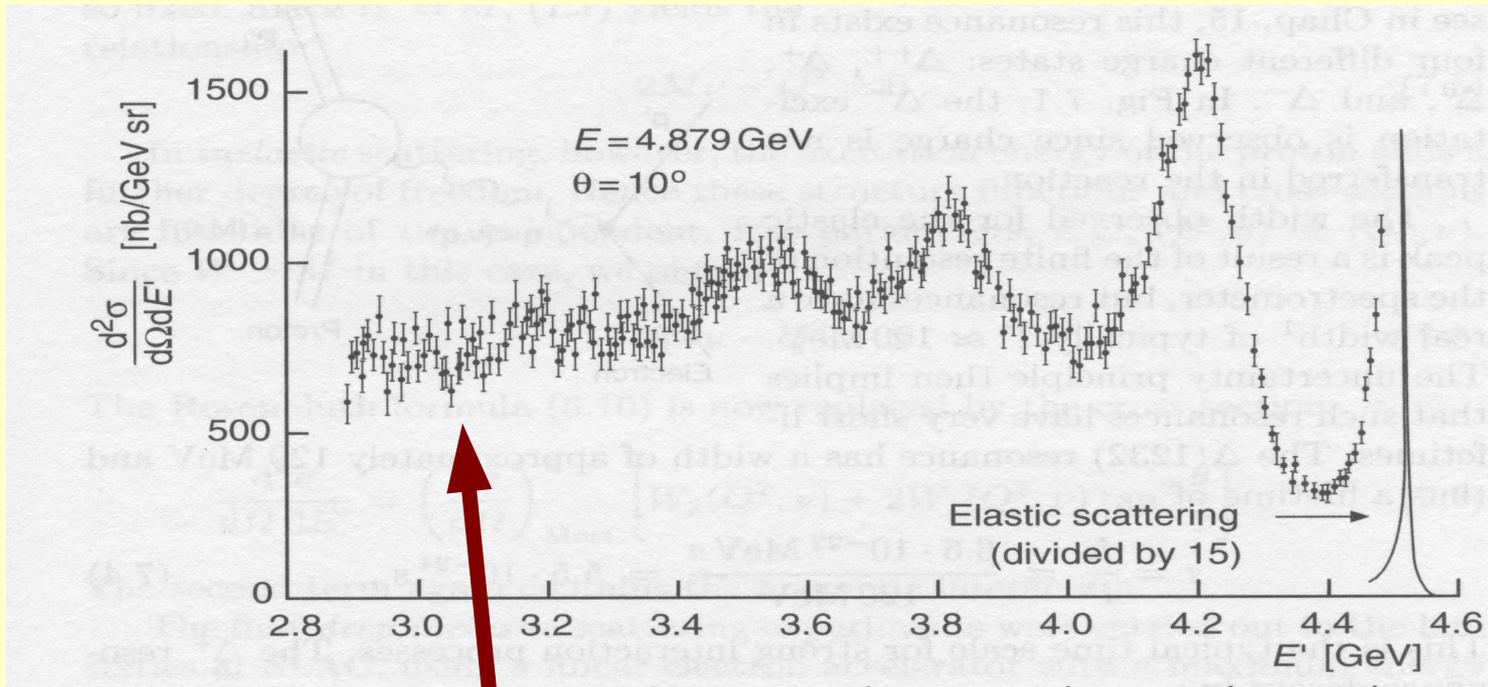
For the elastic scattering events, the diffraction pattern does not have a constant intensity. The proton is NOT a zero-size, point-like particle!



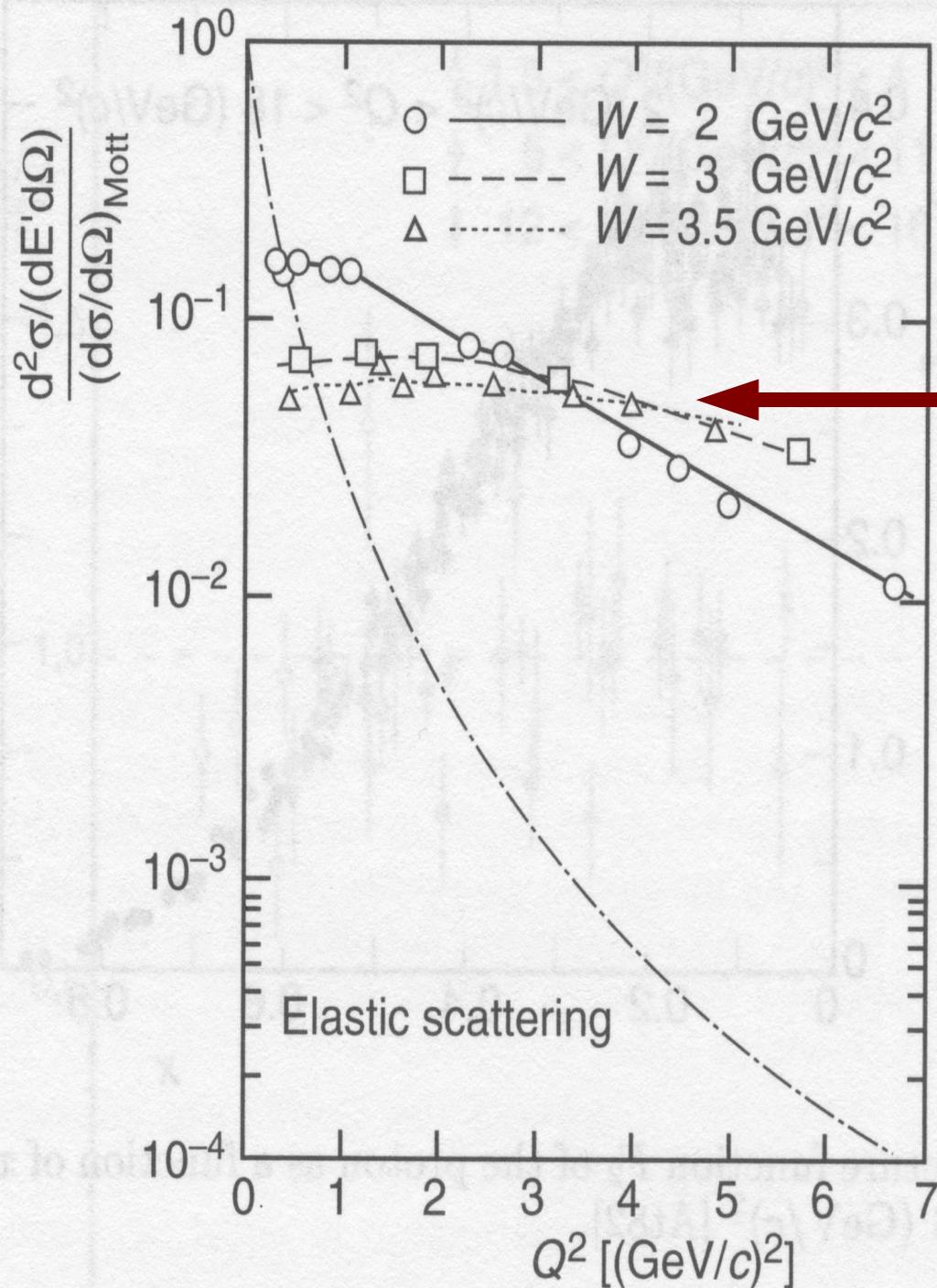


These peaks correspond to inelastic scattering where the target proton is left in an excited state (vibrating, rotating). The electron energy is less because it has given up some energy to excite the proton.

The existence of excited states indicates that the proton must have some internal parts that can be boosted up to higher energy levels.



**This is the “deep inelastic scattering” region where the electron has lost a lot of energy when it scatters from the proton. When we plot a graph of the diffraction pattern for these scattering events, we get the following:**

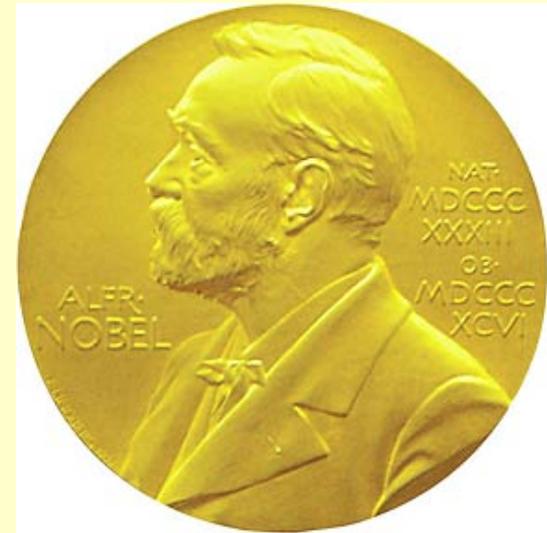
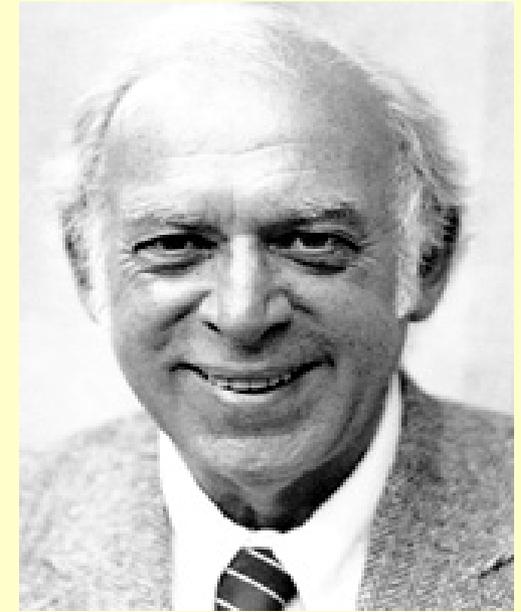
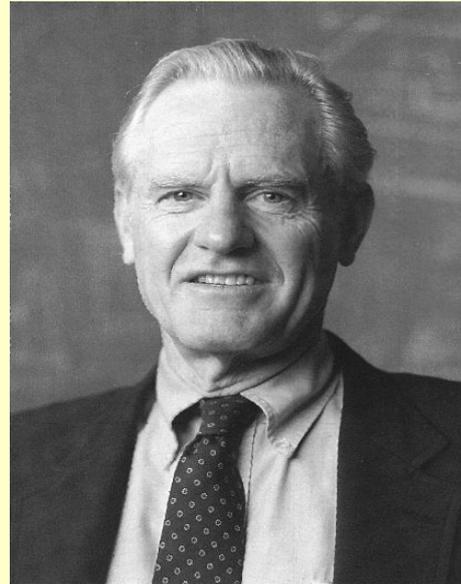
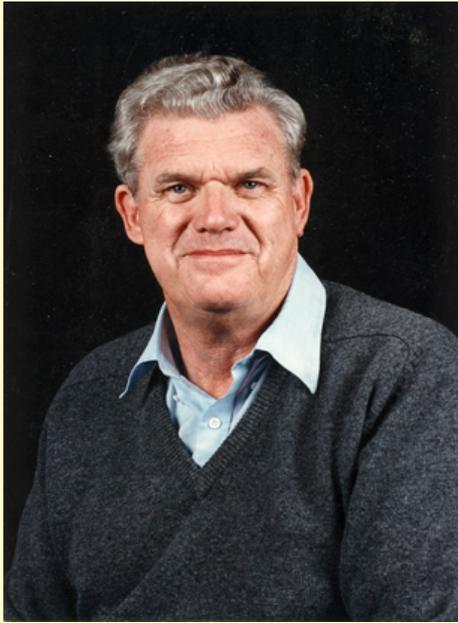


For the deep inelastic scattering events, we have a nearly constant diffraction pattern.

This indicates that we are scattering from point-like objects inside the proton. These are quarks.

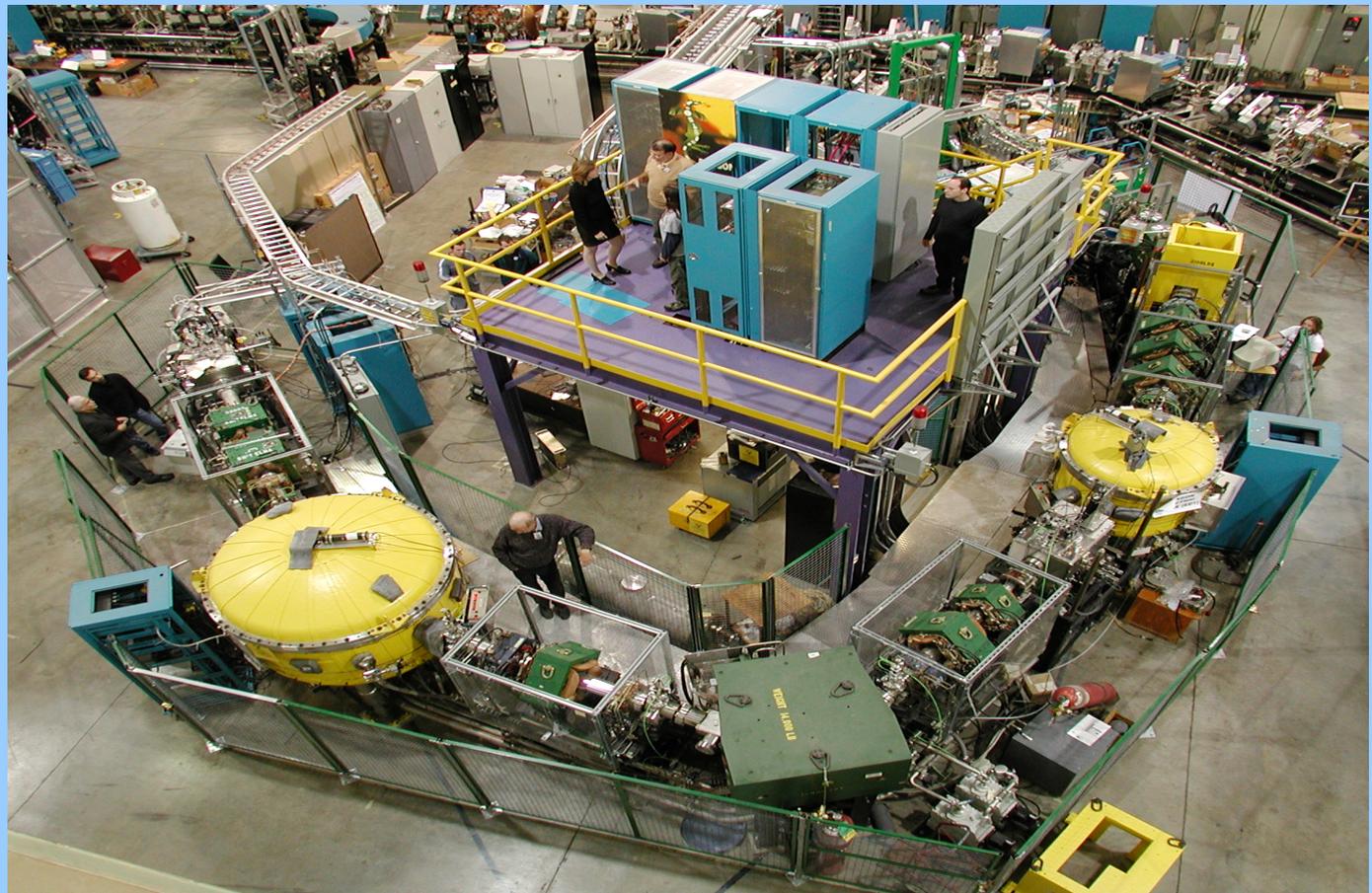
(called “scaling” in particle physics)

# This work won the 1990 Nobel Prize in physics for Richard Taylor, Henry Kendall and Jerome Friedman

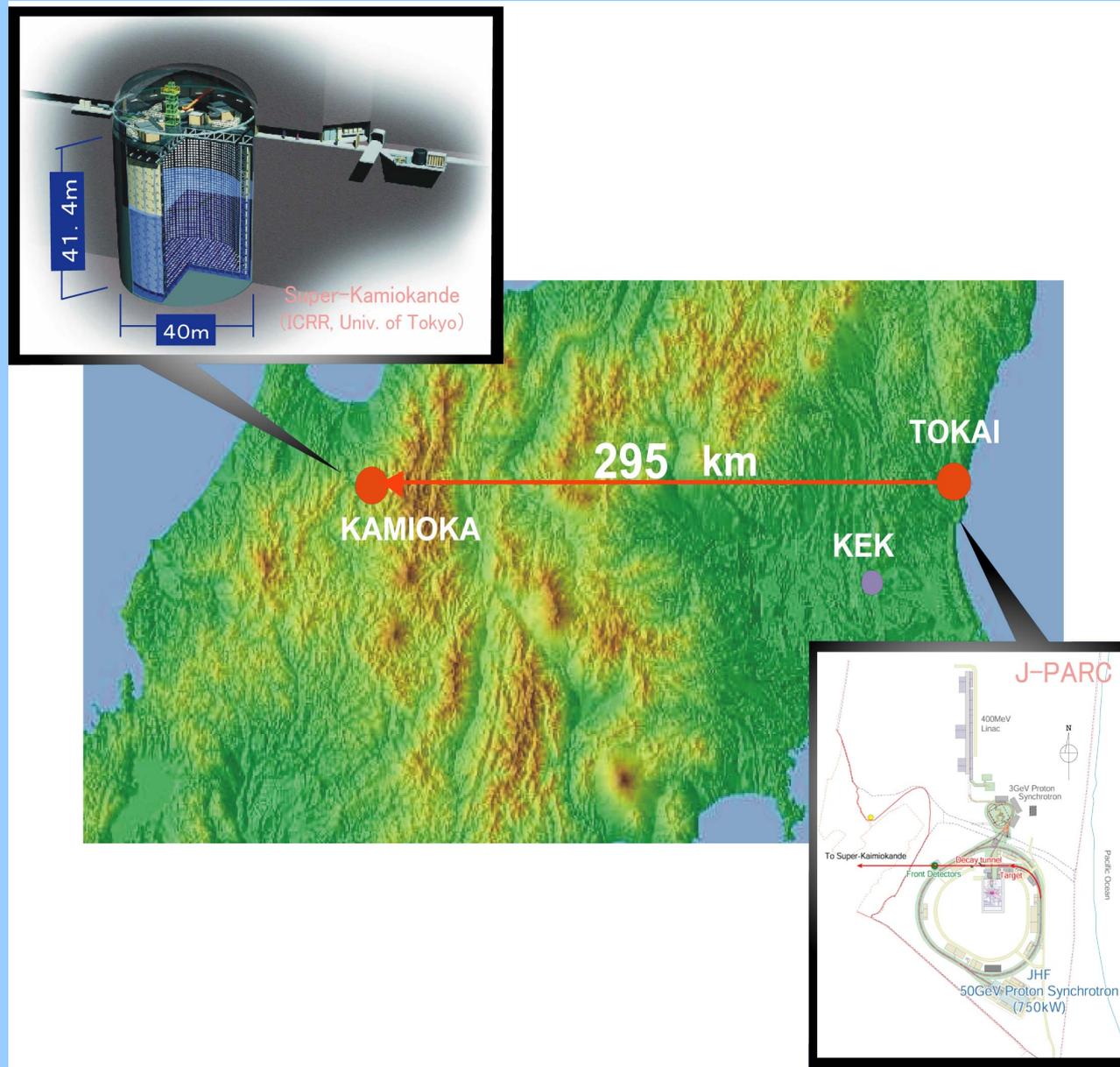


**These are exciting times in nuclear and particle physics. My colleagues here at TRIUMF are working on projects such as:**

- studying the nuclear reactions that occur in the Sun and the stars, and which synthesize the elements that we are made of**

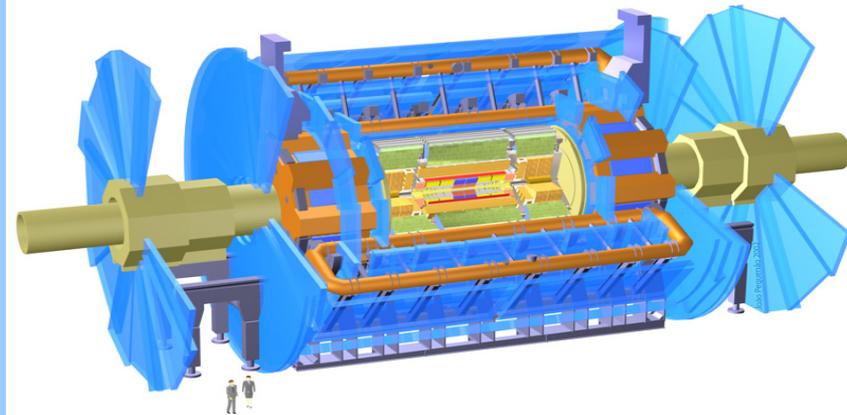


- shooting a beam of artificial neutrinos from the east coast of Japan to a 50 kiloton water tank on the west coast of Japan, to study how the neutrinos morph from one species to another in flight





At the Large Hadron Collider (LHC) near Geneva, we are colliding protons at 7 trillion electron volts to reproduce the high energy conditions in the first milliseconds after the Big Bang

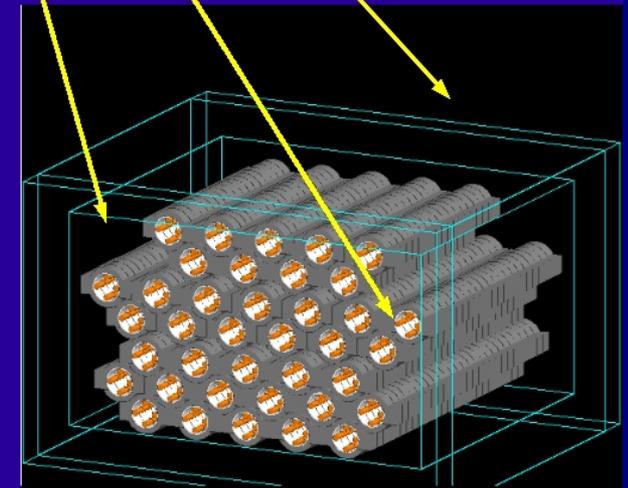


A mile underground  
in a nickel mine in  
Sudbury, we're building  
detectors sensitive  
to neutrinos coming  
from supernova  
explosions in the galaxy.



Helium  
And  
Lead  
Observatory

for supernova  
neutrinos



**To high school students:**

**consider a career in subatomic physics  
research**

**We're looking for bright young minds  
to tackle the next generation of problems.**